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THE LAW OF ATTRACTION IN RELATION TO SOME VISUAL AND TACTUAL ILLUSIONS.

BY PROFESSOR HAYWOOD J. PEARCE, PH.D.,

From the Psychological Laboratory of Brenau College.

In working over the results of some experiments which were published under the title, 'Ueber den Einfluss von Nebenreizen auf die Raumwahrnehmung,'¹ in which was demonstrated a tactual illusion similar in nature to the Müller-Lyer visual illusion, I observed that the influence of a secondary stimulus (Nebenreiz) in producing an elongation of a primary linear stimulus was directly proportional to the intensity of the secondary stimulus and inversely proportional to the square of the distance between the two stimuli. The number of specific instances upon which this observation was based was somewhat small and the number of variations in distance and in intensity of secondary stimuli were very limited. Moreover, data for determining the influence upon the result which might be occasioned by the variation in intensity of the primary stimulus were almost wholly lacking. On account of these and other similar deficiencies, it was not thought wise at the time of the former publication to propose a hypothesis of such apparently far reaching significance.

Starting, however, with this hypothesis in mind I have directed numerous other experiments, designed to reveal the exact relation between primary and secondary stimuli and the effect in perception of the one upon the other. Inasmuch as it was so clearly shown² that the tactual illusion was similar in

¹ *Archiv für die gesamte Psychologie*, Vol. I., pp. 31-109.

² *Ibid.*

almost every detail to the visual illusion, I selected the latter for the investigation. The visual figure lends itself more readily to small and numerous variations, the mean variation in any series of judgments is much smaller and it seems to me that the operation of a law such as that indicated would be more easily detected because more uniform in its manifestations.

The present paper, therefore, is a report of some experiments made for the purpose of determining quantitatively the influence exerted by secondary visual linear stimuli upon a primary visual linear stimulus, or perhaps more accurately stated, the attempt is to determine in mathematical terms the attractive force operative between two visual stimuli. It will be understood, of course, that the limitations of language confine us to the use of the term 'stimulus.' If there be, in truth, any attractive force operating between the actual objective stimuli, it is certainly not our present purpose to determine that. Stimulus, as I have used the term, represents an 'impression' made upon a sense organ. It is not yet, necessarily, sensation; it is certainly no longer stimulus. It is rather a middle state, viz., a state of the nervous system occasioned by objective stimulus and the conditioning element in sensation.

I began, first, a series of experiments with a figure of the Müller-Lyer type. The projecting arms were turned outward, and separated from the central line, or primary stimulus, by small open spaces as represented in Fig. 1 (Plate I.).

In general, the results were of the sort which I had expected, but it soon became manifest that another factor in addition to distance and sensation intensity was playing a part in the results. It is a well-known fact that the Müller-Lyer illusion varies with the cosine of the angle formed by the projecting arm and the central line.¹

In order to eliminate this third variable factor, I abandoned the Müller-Lyer figure entirely and constructed a figure with one central linear stimulus and two other simple linear stimuli, which I have termed the secondary stimuli. The secondary stimuli were constructed exactly in the line of direction of the

¹ Heymans, *Quantitative Untersuchungen ueber das optische Paradox in Zeitschrift f. Psychol.*, Vol. IX., p. 221.

central, or primary, stimulus but separated from it by small open spaces.

The accompanying Fig. 2 makes clear this construction. AB is the primary stimulus, EC and DF are the secondary stimuli. The distance between primary and secondary stimuli is measured of course from center to center and is xy in the figure.

It will be observed that several radical variations of this figure may be made. We may vary (1) the distance xy , or (2) the stimulus AB , or (3) the stimuli EC and DF , retaining in each case all other factors constant. The results of our experiments, therefore, fall naturally into three groups: (1) The effect of secondary stimuli at different distances, (2) the effect of secondary stimuli when the primary stimulus is varied, (3) the effect of secondary stimuli of different intensities upon primary stimuli of a constant intensity and at the same relative distances.

METHOD OF THE EXPERIMENT.

Preliminary experiments were conducted in order to determine the best method. (1) A card upon which was drawn the figure to be judged was given the subject and, in addition, a series of cards containing each a single line but of different lengths. The subject was required to select from the series of cards the one containing the line apparently equal in length to the primary stimulus in the figure. (2) Instead of a series of cards, a single card upon which was drawn a series of lines of different lengths was used. The subject was required to designate in the series of lines the one which appeared equal to the primary stimulus. (3) Instead of a series of lines, a single long line was drawn upon a piece of cardboard, and the cardboard was adjusted to slide back and forth through a slit cut in another piece of cardboard. By this means it was possible for the subject to make the line longer or shorter until it seemed to him equal in length to the primary stimulus.

The method finally adopted and which I think will be recognized as the most convenient and accurate of the four methods which were tried, was as follows: A frame, three feet square, was hung upon two upright posts which projected three feet

above a low table. In this frame was fixed a square of cardboard containing near the center two rectangular openings. On the rear side of the frame and parallel to the openings were tacked wooden runners or grooves, so adjusted that one could slide certain cards, containing the figures to be judged, into their proper positions filling the rectangular openings just referred to. When in position and ready for the experiment, the frame appeared to the subject as shown in Fig. 3. The upper single line could be lengthened or shortened by sliding the card back and forth. On the back of this card was a millimeter scale, so arranged that the experimenter could read immediately the length of the line as it appeared to the observer. The experimenter, seated behind the screen at the table, could move the card easily back and forth and record immediately the reading of the millimeter scale, which registered the judgment of the observer.

The observer was seated in front of the screen at a distance of 80 cm. His task was to observe the moving upper line and the lower stationary figure at the same time and to say 'stop' as soon as the *difference* between the upper line and the lower primary stimulus *ceased to exist*. This form of instruction to the observer was adopted because it was noted that if told to say stop when the two lines appeared to be equal, the subject adopted, somewhat irregularly, either of two courses: (1) She said 'stop' when the difference *ceased*, or (2) having allowed the variable to pass the point where the difference ceased, she said 'stop' not until she began to perceive a difference in the other direction. In order to secure relative constancy, the former type of reaction was insisted on. In half the experiments constituting a series, the moving, variable line was gradually lengthened and in the other half of the series this line was gradually shortened.

METHOD OF ESTIMATING THE INFLUENCE OF THE SECONDARY STIMULUS.

It seemed natural at first thought to estimate the influence of the secondary stimuli as equal to the difference between the length of the primary stimulus as given by objective physical

measurement and the length of a second line which is judged by the subject to be of the same length.

Preliminary experiment, however, showed clearly that when a subject attempts to estimate the length of a single line (without secondary stimuli), using the method above described, the judgment is always too small, *i. e.*, the line is always judged to be shorter than it actually is. Consequently, in order that the secondary stimuli may produce subjectively an elongation of the line objectively given, the tendency to shorten just observed, must first be overcome. Inasmuch as the addition of a secondary stimulus accomplishes this, we must include this in our estimate of the influence of the secondary stimulus. Accordingly in every series of judgments of a line accompanied by secondary stimuli, I have required evenly distributed judgments of the same line without secondary stimuli. The results, therefore, which appear in the tables as 'influence of the secondary stimulus' always represent the difference measured in centimeters between the judgment of the line without secondary stimuli and the judgment of the length of the same line with secondary stimuli.¹

The foregoing is in general the method of experimentation and computation of results employed in each of the three groups of experiments which follow. In connection with each group some further details of method must be pointed out.

¹ Professor Judd (*Genetic Psychology for Teachers*, p. 11) has attempted to explain the fact that if one tries to draw upon paper a line equal in length to a copy upon the blackboard, he invariably makes it too short, as due to the larger environment represented by the blackboard as compared with the smaller environment represented by the sheet of paper. The facts brought out in my experiments seem to throw serious doubt upon the adequacy of Professor Judd's explanation. In my experiments, the environment of the two lines judged to be equal was the same, and moreover, the error remained the same when the relative position of the two lines was reversed.

The explanation of the error is probably as follows: When I am comparing two lines, one standard and the other variable, the latter is the one which is kept most prominently in the foreground of attention. The eye wanders to the standard only to renew the memory of its length. What actually happens is a comparison of a present vivid, intense sensation with a fading memory image. I may fixate the standard but by the time my eye reaches again the line which I must make equal to the standard, the latter has become a memory image, or at best appears upon the periphery of vision and consequently has less sensation value than the same image upon the fovea centralis or in the focus of attention.

THE EFFECT OF TWO SECONDARY STIMULI UPON A PRIMARY
STIMULUS WHEN THE DISTANCE IS VARIED.

I present first three tables, I., II. and III., showing results for a primary stimulus of 16.0 cm., 17.0 cm. and 18.0 cm., respectively, in length. The experiments with the three different primary stimuli, though recorded in different tables, were conducted simultaneously. For example, a card containing a primary stimulus 16.0 cm. and secondary stimuli 9.5 cm. distant was presented to the subject. Five successive judgments of this same line were required, the variable line being first lengthened and then shortened and so on alternately for the five judgments. Then a second card containing primary stimulus 17.0 cm. was presented and five successive judgments of this line in a similar order were required. Then primary stimulus 18.0 cm. was presented and the same judgments required. Now we return to primary stimulus 16.0 cm. but one in connection with which the distance of the secondary stimuli has been slightly increased, viz., 10.0 cm. Then the series 17.0 cm. and 18.0 cm. with similar increase in distance of secondary stimulus are taken, and then back again to primary stimulus 16.0 cm. with distance of secondary stimuli still further increased and so through the entire series of five variations in distance for each of the three primary stimuli. Including the three cards which contained only a single line each, to which reference has previously been made, there were eighteen different cards and five judgments of the stimulus on each were required. Such a series could be made in about a half hour, which was the length of a setting for each subject.

At the second sitting, the experiment was conducted in a similar manner, except that the detail in every particular was reversed. Two sittings afforded a series of ten judgments each for each of the eighteen primary stimuli. The tables show results for ten different subjects and each result given is the average of ten individual judgments made at two different sittings. There are two such series for each subject and the general average for the ten subjects represents in each case two hundred individual judgments.

The subjects used were of varied age and character. One is my colleague, Professor Essary, of the department of biology, to whom I am under especial obligation; another was a student in the department of psychology; a third was a special student of painting, and a fourth was a special student of music. The remaining six were taken indiscriminately from the preparatory school of Brenau College and vary in age from ten to fourteen years. All except the first mentioned are female.

TABLE I.

INFLUENCE OF TWO SECONDARY STIMULI, EACH 2.0 CM. LONG UPON A
PRIMARY STIMULUS 16.0 CM. LONG AT DISTANCES 9.5,
10.0, 10.5, 11.0 AND 13.0 CM.

Distances.		9.5 cm	10.0 cm.	10.5 cm	11.0 cm.	13.0 cm
Subject.	Series.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm	Influence of Secondary Stimulus in cm.	Influence of Second- ary Stim- ulus in cm.
E.	1	1.49	1.18	1.11	.46	.04
	2	1.61	1.16	.81	.32	.08
Pa.	1	1.17	.81	.72	.18	— .01
	2	1.18	.71	.90	.06	— .18
B.	1	1.06	1.10	.78	.46	.35
	2	.86	.54	.31	.31	— .04
C.	1	1.49	1.46	1.35	.84	.47
	2	1.75	1.47	1.27	.92	.56
H.	1	2.16	1.59	1.51	1.00	.73
	2	1.92	1.90	1.81	1.56	1.08
Pi.	1	1.57	1.42	.92	.52	.29
	2	2.00	.82	.69	.98	.18
Pr.	1	3.30	3.07	2.57	1.78	1.10
	2	1.29	.87	.55	.09	— .22
G.	1	2.09	1.56	1.29	1.03	.24
	2	1.88	1.74	1.29	1.06	.77
Pp.	1	2.02	1.81	1.53	.95	.74
	2	1.97	1.64	1.16	.95	.71
Hn.	1	1.98	1.88	1.50	1.30	.89
	2	2.03	1.55	1.43	1.31	.56
Average.		1.74	1.41	1.18	.80	.44
$E \times D^2$		157.03	141.00	130.09	96.80	74.36

The first three mentioned had some knowledge of optical illusions and the first two were acquainted in part with the hypothesis upon which I was working. The others had no knowledge of the nature or object of the experiment except that which was gained as a result of their own observation in the progress of the same.

Turning to an examination of the results shown in the tables we find that, with a very few exceptions, there is a uniform decrease in the influence of the secondary stimuli corresponding to an increase in the distance between the primary and secondary stimuli. The majority of the exceptions to be noted will be found in Table II., in which are shown the results for primary stimulus 17.0 cm.

TABLE II.

INFLUENCE OF TWO SECONDARY STIMULI, EACH 2.0 CM. LONG, UPON A
PRIMARY STIMULUS 17.0 CM. LONG AT DISTANCES 10.0,
10.5, 11.0, 11.5 AND 13.5 CM.

Distances.		10.0 cm.	10.5 cm	11.0 cm.	11.5 cm	13.5 cm
Subject.	Series	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.
E.	1	1.15	.75	.93	.25	.44
	2	1.42	1.11	.53	.15	.00
Pa.	1	.73	.97	.57	.20	.19
	2	.96	.68	.72	.40	— .16
B.	1	1.06	.95	.46	.05	— .18
	2	1.13	.93	.61	.28	.27
C.	1	1.74	1.45	.64	.23	— .60
	2	1.54	1.31	1.42	1.12	.50
H.	1	1.70	1.60	1.18	1.04	.18
	2	1.68	1.49	1.60	1.28	.89
Pi.	1	1.05	.57	.56	.37	.18
	2	1.41	1.01	.44	.07	— .09
Pr.	1	1.75	1.61	1.49	1.00	.66
	2	1.67	1.08	1.11	.41	.46
G.	1	1.94	1.27	.90	.71	.20
	2	1.41	1.22	1.16	.81	.98
Pp.	1	1.83	1.58	1.06	.83	.46
	2	1.62	1.39	1.17	1.02	.40
Hn.	1	1.79	1.41	1.45	.99	.47
	2	1.76	1.26	1.17	.62	.45
Average.		1.47	1.18	.91	.59	.29
$E \times D^2$		147.00	130.09	110.11	78.03	52.85

The only explanation which I can offer for the greater irregularities manifest in Table II. is the fact that a stimulus 17.0 cm. cannot be distinguished with certainty from either 18.0 cm. or 16.0 cm. and inasmuch as the 17.0 cm. stimulus in the order of the experiment follows sometimes the 16.0 cm. and sometimes 18.0 cm. stimulus, the judgment when it related to the 17.0 cm. stimulus was unequally influenced by the preceding

TABLE III.

INFLUENCE OF TWO SECONDARY STIMULI, EACH 2.0 CM. LONG, UPON A
PRIMARY STIMULUS 18.0 CM. LONG AT DISTANCES 10.5,
11.0, 11.5, 12.0 AND 14.0 CM.

Distances		10.5 cm.	11.0 cm.	11.5 cm.	12.0 cm.	14.0 cm.
Subject.	Series.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.
E.	1	1.43	.71	.28	.19	— .17
	2	1.53	1.23	.77	.27	— .23
Pa.	1	.64	.45	.29	.41	.16
	2	.76	.62	.41	.29	— .28
B.	1	1.32	1.12	1.25	.60	.58
	2	1.50	1.19	.67	.50	.01
C.	1	1.76	1.58	1.27	.86	.01
	2	1.33	1.17	.91	.90	.45
H.	1	2.18	2.11	1.60	1.16	.18
	2	2.55	2.40	1.80	1.37	.98
Pi.	1	.82	.66	.35	.26	— .15
	2	1.99	1.78	1.19	.78	.02
Pr.	1	3.15	3.02	1.61	1.11	1.34
	2	1.23	.92	.49	.59	— .07
G.	1	2.40	1.96	1.66	1.37	.16
	2	2.06	1.41	.89	.51	.53
Pp.	1	2.30	2.46	1.98	2.01	1.60
	2	1.49	1.13	1.08	.46	.25
Hn.	1	1.38	1.19	1.51	.57	.63
	2	1.42	1.02	.68	.52	.30
Average.		1.66	1.41	1.07	.75	.32
$E \times D^2$		183.01	170.61	141.51	108.00	62.72

judgments relating to the 18.0 cm. and the 16.0 cm. stimuli. The subject was particularly liable to such confusion, because she was not informed as to the number of primary stimuli which were used, nor that a primary stimulus of different length was always introduced when the cards were changed.

Considering the general averages of all results for each distance, we find that the irregularities referred to have been eliminated and a consistent decrease in influence corresponding to each increase in distance is manifest. For example, the influence of two secondary stimuli, each 2.0 cm. in length, acting upon a primary stimulus 18.0 cm. in length at a distance of 10.5 cm. is found to be 1.66 cm.; at distance 11.0 cm. the influence of the same secondary stimuli is 1.41 cm.; at distance 11.5 this influence has diminished to 1.07 cm.; at 12.0 cm. distance the influence is 0.75 cm. and at 14.0 cm. it is 0.32 cm.

influence. This same degree of regularity is manifest in each of the other tables.

An attempt to establish anything like an exact proportion between the decrease in influence and the increase in the square of the distance was a failure. It became at once apparent that the decrease in influence was far more rapid than the increase in the square of the distance.

The foregoing fact directed attention to another principle, viz., the intensity of visual stimuli decreases as the stimulus is moved toward the periphery of the retina. There are not, within my knowledge, any recorded experimental data which directly confirm this last statement, and, indeed, the well known device of the astronomer of using the periphery of the retina in order to bring to view an otherwise indiscernible star seemed at first thought in direct contradiction to such a statement. This astronomical device, however, only shows really that the periphery of the retina may under favorable conditions, be more sensitive to very *faint* stimuli than the fovea centralis and can be explained by the fact that the fovea centralis being constantly bombarded by intense stimuli becomes insensitive to very weak ones.

On the other hand, the common facts of every day experience that we see most distinctly when the stimulus falls upon the center and less distinctly when it is moved toward the periphery, together with the well known facts of nerve distribution upon the retina afford sufficient confirmation of the statement that the intensity of the same objective stimulus decreases as the stimulus is moved toward the periphery.

Applying this principle to the case under consideration, we see that when the secondary stimuli are removed to a greater distance from the primary stimulus, they are removed at the same time towards the periphery of the retina, inasmuch as the eye maintains the same position relative to the primary stimulus. We have, therefore, at the same time, an increase in distance and a decrease in intensity of the secondary stimulus, although it remains objectively the same length. Both of these factors thus entering into the conditions of our experiments call for a decrease in the influence of the secondary stimulus, according

to our hypotheses, and the very rapid decrease to which attention was called is, so far, in confirmatory of rather than contradictory to this hypothesis.

If our hypothesis is valid, it follows that the influence exerted by a stimulus A multiplied by the square of its distance would equal the influence of a stimulus B multiplied by the square of its distance. In other words, $E \times D^2 = C$, in which E represents the influence of any secondary stimulus, D is the distance of that stimulus and C is a constant.

As has already been shown, the value of C in the results previously recorded is not constant. For example, in Table I. the value for the five distances decreases from 183.01 to 62.72. This rapid decrease was due to the very rapid diminution of the value of E , and this last we have attributed to the decreased intensity of the 2.0 cm. stimulus occasioned by its removal towards the periphery, in addition to the increased distance.

In order to compensate for this decrease in intensity of the secondary stimulus, I prepared a new series of figures in which the same primary stimulus and the same distances were employed as in the former experiments, but the length of the secondary stimulus was altered. The amount by which the secondary stimulus should be altered in length was determined as follows: I selected arbitrarily one value of C , viz., that shown in the second column of each table of results. I then determined for each distance what the value of E should be, using the value of C selected as a constant. I was thus enabled to determine what effect a secondary stimulus of the same subjective intensity should have at different distances.

Now at a given distance, we know by experiment the effect of a secondary stimulus 2.0 cm. in length; we also know for the same distance, by computation as above shown, what the effect of a secondary stimulus of a certain standard intensity ought to be. The problem is to determine how much the secondary stimulus shall be lengthened or shortened in order that it may have the same subjective intensity as the standard.¹

For lack of a better, I adopted the purely objective method of solving this problem, using increase in objective length as

¹ Compare Weber's Law.

equivalent to increase in subjective intensity. For example, referring to Table I., we select the influence of a secondary stimulus of 2.0 cm. at a distance of 11.0 cm. as the standard. The value of C (see second column) in this case is 170.61. We have assumed that this value should be a constant, if the intensity of the secondary stimulus remained constant. But we find that the value of C when the distance of the 2.0 cm. stimulus is 11.5 cm. (see third column) is only 141.51, the actual influence of secondary stimulus being only 1.07 cm. Now if the value of C were constant the actual influence of secondary stimulus ought to be 1.29 cm., provided the intensity of our 2.0 cm. stimulus had remained the same. This conclusion is reached as follows: The value of C should be 170.61, but as a matter of fact it is only 141.51. This indicates that the influence of secondary stimulus (1.07 cm.) is less than is to be expected of a secondary stimulus equal in intensity to that one which we have selected as the standard (second column) and, indeed, 1.07 cm. is as much less than the influence of a secondary stimulus of standard intensity ought to have been, as 141.51 is less than 170.61. In other words $(170.61 \times 1.07 \text{ cm.}) \div 141.51 = 1.29 \text{ cm.}$ which is what the influence of a secondary stimulus of standard intensity ought to be at the distance 11.5 cm. Further, if a secondary stimulus 2.0 cm. in length has produced an effect of 1.07 cm., how long must the secondary stimulus be in order that it may produce an effect of 1.29 cm.? Proceeding according to the objective method, this question is answered by the following arithmetical operation: $(1.29 \times 2.0 \text{ cm.}) \div 1.07 \text{ cm.} = 2.41 \text{ cm.}$, which last is the length which our secondary stimulus must have at distance 11.5 cm., in order to be equal in intensity to the standard, which is a 2.0 cm. stimulus at distance of 11.0 cm. Proceeding according to this method, I calculated, upon the basis of results given in the three preceding tables, what the length of the secondary stimulus should be in our new series of figures, in order that a standard intensity might be maintained throughout. The method of procedure is unquestionably crude, and is justified only on the ground that it was used merely as an empirical device. It is doubtless possible to determine definitely the relation between increase in subjective inten-

sity and increase in objective length of visual stimuli. When this is done it will doubtless be possible to construct a series of figures, in which the secondary stimulus at different distances remains of the same subjective intensity. The time at my disposal did not admit of such a determination.

The validity of the objection just raised against the method of constructing the new series of figures was fully justified by the results of the experiments made with these figures. These results are shown in Tables IV., V. and VI. In the case of Table VI., primary stimulus 18.0 cm., the addition to length of secondary stimulus has produced a result which gives to C a practically constant value. But in Tables IV. and V. the value of C appears in a constantly diminishing ratio, showing that

TABLE IV.

INFLUENCE OF SECONDARY STIMULI OF VARIOUS LENGTHS, BUT ESTIMATED TO BE OF THE SAME SUBJECTIVE INTENSITY OR VALUE, UPON A LINE 16.0 CM. IN LENGTH AT THE SAME DISTANCES SHOWN IN TABLE I.

Lengths of the Secondary Stimuli		1.79 cm.	2.00cm.	2.15 cm.	2.90 cm.	3.70 cm
Distances.		9.5 cm	10.0 cm	10.5 cm.	11.0 cm.	13.0 cm
Subject.	Series.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.
E.	1	1.10	.71	.69	.68	— .06
	2	1.36	1.17	1.08	.95	.70
Pa.	1	1.15	.93	.88	.70	.36
	2	.88	.82	.48	.54	.07
B.	1	1.28	.93	.75	.67	.48
	2	1.21	.91	.62	.63	.35
C.	1	1.37	1.31	1.05	.77	.60
	2	1.36	1.14	.72	.69	.48
H.	1	1.27	1.04	.80	.67	— .09
	2	1.30	.96	.73	.43	.21
Pi.	1	1.40	1.34	1.20	1.06	.89
	2	1.60	1.61	1.42	1.22	1.08
Pr.	1	1.92	1.66	1.22	1.11	.50
	2	1.72	1.39	1.25	.99	.80
G.	1	2.00	1.64	1.55	1.43	.51
	2	1.29	.56	.69	.34	.39
Pp.	1	1.63	1.44	1.25	.88	.63
	2	1.40	1.34	1.20	1.06	.89
Hn.	1	1.55	1.36	1.05	1.01	.89
	2	1.79	1.80	1.67	1.34	.96
Average.		1.43	1.20	1.01	.86	.53
$E \times D^2$		129.06	120.00	113.52	104.06	89.57

the addition in length was not sufficiently large. In other words we have a somewhat new verification of Weber's Law, viz., equal increments in objective length of visual linear stimuli do not imply equal increase in subjective intensity of the visual stimulus.

Comparing Tables IV., V. and VI. with I., II. and III. respectively, we find that the increase in the length of the

TABLE V.

INFLUENCE OF SECONDARY STIMULI OF VARIOUS LENGTHS, BUT ESTIMATED TO BE OF THE SAME SUBJECTIVE INTENSITY OR VALUE UPON A LINE 17.0 CM. IN LENGTH, AT THE SAME DISTANCES SHOWN IN TABLE II.

Lengths of the Secondary Stimuli.		1.77 cm.	2.00 cm.	2.35 cm.	3.32 cm.	4.90 cm.
Distances.		1.00 cm.	10.5 cm.	11.0 cm.	11.5 cm.	13.5 cm.
Subject.	Series.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.
E.	1	1.22	1.12	1.03	.97	.42
	2	1.23	.90	.81	.72	.43
Pa.	1	.80	.84	.41	.23	.02
	2	.89	.65	.40	.25	.15
B.	1	1.20	.85	.86	.58	.48
	2	1.24	1.08	.77	.57	.33
C.	1	1.47	1.28	1.02	.93	.30
	2	.77	.87	.62	.64	.20
H.	1	1.70	1.54	1.50	1.50	1.16
	2	.88	.75	.72	.71	.51
Pl	1	1.14	1.07	1.03	.97	.95
	2	1.00	1.01	.80	.45	.27
Pr.	1	1.61	1.39	1.35	1.04	.96
	2	1.00	.75	.86	.58	.50
G.	1	1.76	1.49	1.29	1.38	.84
	2	1.10	.79	.77	.48	.35
Pp.	1	1.77	1.46	1.28	1.35	.71
	2	1.53	1.02	1.06	.91	.34
Hu.	1	1.98	1.63	.93	.98	.68
	2	1.20	1.19	.94	.69	.58
Average.		1.28	1.08	.92	.80	.51
$E \times D^2$		128.00	119.07	111.32	105.80	92.95

secondary stimulus has greatly increased the constancy of C , in other words the effect of the secondary stimulus of increased length has been uniformly greater. It appears, therefore, more than probable that if the length of the secondary stimulus were increased according to subjective rather than objective standards,

the value of C would become really constant as is demanded by our hypothesis.

TABLE VI.

INFLUENCE OF SECONDARY STIMULI OF VARIOUS LENGTHS, BUT ESTIMATED TO BE OF THE SAME SUBJECTIVE INTENSITY OR VALUE, UPON A LINE 18.0 CM. IN LENGTH, AT THE SAME DISTANCES SHOWN IN TABLE III.

Lengths of the Secondary Stimuli		1 85 cm.	2 00 cm.	2 41 cm.	3.54 cm	5.43 cm.
Distances.		10 5 cm.	11.0 cm	11.5 cm.	12 0 cm.	14.0 cm.
Subject.	Series.	Influence of Secondary Stimulus in cm	Influence of Secondary Stimulus in cm	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.	Influence of Secondary Stimulus in cm.
E.	1	1.41	1.35	1.19	1.29	.48
	2	1.39	1.25	1.22	1.30	1.18
Pa.	1	1.52	1.40	1.25	1.15	1.08
	2	1.34	1.23	1.06	.97	.72
B.	1	1.61	1.36	1.26	1.10	.95
	2	1.40	1.24	1.59	1.08	.98
C.	1	2.02	1.92	1.76	1.57	1.35
	2	1.39	1.30	1.14	.83	.84
H.	1	1.32	1.36	1.30	1.18	1.10
	2	.81	.64	.48	.40	.13
Pi.	1	1.28	1.34	1.27	1.18	1.02
	2	1.66	1.47	1.37	1.25	1.28
Pr.	1	1.93	1.64	1.72	1.65	1.47
	2	.99	.80	.58	.57	.64
G.	1	1.48	1.07	1.35	1.06	1.02
	2	1.11	1.18	.84	.68	.43
Pp.	1	1.28	1.40	1.36	1.34	1.07
	2	1.91	1.87	1.59	1.45	1.18
Hn.	1	2.02	1.55	1.21	1.02	1.18
	2	1.37	1.26	.94	1.12	.63
Average.		1.46	1.32	1.22	1.11	.94
$E \times D^2$		160.96	159.72	161.34	159.84	184.24

For purposes of comparison, I present three tables, VII., VIII. and IX., showing the results of some experiments conducted by Misses E. Dickson and B. Brock, students in my laboratory course. In these experiments only four subjects were used and ten tests of each was made with each figure. Thus each of the general averages represents forty judgments.

The primary stimulus in these experiments was 24.0 cm., 25.0 cm. and 26.0 cm., respectively. The secondary stimulus was objectively 2.0 cm. in length, the same as in the experiments previously recorded, but inasmuch as the increased length of the primary stimulus makes it necessary to remove the

secondary stimulus towards the periphery, the subjective intensity of the secondary stimulus is materially decreased as compared with the secondary stimuli of the experiments previously discussed.

Comparing these results with those of Tables I., II. and III., we find the same general features, viz., decrease in inverse proportion to distance, but a decrease more rapid than is demanded by increase in square of the distance.

Detailed comparison of results in the several tables, bring out some interesting relations. For example, in Table I., where we have primary stimulus 18.0 cm., secondary stimulus 2.0 cm. and distance 14.0 cm., the influence of secondary stimulus is 0.32 cm. In Table VII. for the same distance, but a primary

TABLE VII.

INFLUENCE OF TWO SECONDARY STIMULI, EACH 2.0 CM. LONG UPON A PRIMARY STIMULUS 24.0 CM. LONG AT DISTANCES 13.5, 14.0, 14.5, 15.0 AND 16.0 CM.

Distances.	13.5 cm.	14.0 cm	14.5 cm.	15.0 cm.	16.0 cm.
Subject.	Influence of Secondary Stimuli in cm	Influence of Secondary Stimuli in cm	Influence of Secondary Stimuli in cm.	Influence of Secondary Stimuli in cm.	Influence of Secondary Stimuli in cm.
A.	1.4	.80	1.0	.9	.7
Kg.	2.2	1.2	.8	— .2	— .7
Ch.	1.2	1.1	.9	.8	.6
Cl.	1.1	.9	.8	.6	.0
Average.	1.5	1.0	.9	.5	.15

stimulus of 24.0 cm. the influence is 1.0 cm. Similarly, comparing the effect when primary stimulus is 25.0 cm., and the same distance, 14.0 cm., we find, in Table VIII. an effect of

TABLE VIII.

INFLUENCE OF TWO SECONDARY STIMULI, EACH 2.0 CM. LONG UPON A PRIMARY STIMULUS 25.0 CM. LONG AT DISTANCES 14.0, 14.5, 15.0 AND 15.5 CM.

Distances.	14.0 cm.	14.5 cm.	15.0 cm	15.5 cm
Subjects.	Influence of Secondary Stimuli in cm.	Influence of Secondary Stimuli in cm.	Influence of Secondary Stimuli in cm.	Influence of Secondary Stimuli in cm.
A.	.7	.8	— .2	— .7
Kg.	1.2	.8	.5	— .4
Ch.	1.6	1.3	1.0	.8
Cl.	1.7	1.4	1.0	.5
Average.	1.3	1.1	.7	.05

1.3 cm. Similar relations appear throughout, when the influence in the case of different primary stimuli with secondary stimuli at the same distances is observed. The effect of the secondary stimulus increases not only in proportion to its own intensity, but also in proportion to the intensity, or length, of the primary stimulus.

THE EFFECT OF SECONDARY STIMULI WHEN THE PRIMARY STIMULUS IS VARIED.

This relation to which reference was made in the preceding paragraph, was also brought out by a series of experiments especially designed for the purpose. A series of eight cards were prepared as follows: The primary stimuli were 8.0, 9.0, 10.0, 11.0, 12.0, 13.0, 14.0 and 15.0 cm. long; the secondary stimuli were in each case 2.0 cm., in length and the distance between primary and secondary stimuli was in each case 9.0 cm. The only variable factor, therefore in the conditions was the length of the primary stimulus.

TABLE IX.

INFLUENCE OF TWO SECONDARY STIMULI, EACH 2.0 CM. LONG UPON A PRIMARY STIMULUS 26.0 CM. LONG AT DISTANCES 14.5, 15.0, 15.5 AND 17.0 CM.

Distances	14.5 cm.	15.0 cm.	15.5 cm.	17.0 cm.
Subjects.	Influence of Secondary Stimuli in cm	Influence of Secondary Stimuli in cm	Influence of Secondary Stimuli in cm	Influence of Secondary Stimuli in cm
A.	1.8	1.8	.6	.7
Kg.	1.7	1.6	1.2	.1
Ch.	1.5	1.3	1.1	.8
Cl.	1.5	1.3	.5	.4
Average.	1.6	1.5	.9	.5

The method of conducting the experiments was in general similar to that already detailed. The cards were presented in the order given above for the first five tests of each series and in the reverse order for the last five tests of each series — ten tests constituting a series. The experiments were conducted by Misses Newton, McConnell and Pauline Smith, three students in the department of psychology. The results are shown in Table X.

TABLE X.

INFLUENCE OF TWO SECONDARY STIMULI, 2.0 CM. LONG, UPON PRIMARY STIMULI OF VARIED LENGTHS, BUT SAME RELATIVE POSITION AND DISTANCE 9.0 CM.

Lengths of Primary Stimuli		8.0 cm.	9.0 cm.	10.0 cm.	11.0 cm.	12.0 cm.	13.0 cm.	14.0 cm.	15.0 cm.
Subjects	No. of Experiments.	Influence	Influence	Influence	Influence	Influence	Influence	Influence	Influence
Mc.	100	.24	.38	.40	.37	.51	.52	.61	.94
N.	100	.06	.17	.11	.24	.46	.03	.53	.28
K.	100	.28	.31	.06	.04	.43	.25	.36	.22
Average.		.15	.29	.19	.22	.47	.27	.50	.48

For purposes of comparison, I present also Table XI., which records the results of experiments conducted under exactly similar conditions by Misses Canning and Blalock. In this group, however, the lengths of the several primary stimuli were

TABLE XI.

INFLUENCE OF TWO SECONDARY STIMULI, 2.0 CM. LONG, UPON PRIMARY STIMULI OF VARIED LENGTHS, BUT SAME RELATIVE POSITION AND DISTANCE 14.0 CM.

Lengths of Primary Stimuli		18.0 cm.	19.0 cm.	20.0 cm.	21.0 cm.	22.0 cm.	23.0 cm.	24.0 cm.	25.0 cm.
Subjects	No. of Experiments.	Influence	Influence	Influence	Influence	Influence	Influence	Influence	Influence
Bl.	100	.00	.07	.01	.04	.21	.29	.64	.32
Cu.	100	.03	.12	.12	.16	.63	.36	.35	.78
Average.		.015	.10	.07	.10	.42	.33	.50	.55

18.0, 19.0, 20.0, 21.0, 22.0, 23.0, 24.0 and 25.0 cm., and the distance between primary and secondary stimuli was 14.0 cm.

Referring to the summary of results in both tables, it will be observed that in general the influence of secondary stimuli at the same distances increases as the length of the primary stimulus increases. Irregularities, however, in the rate of increase and actual exceptions to the rule are particularly noticeable. The fact that relatively speaking the same inconsistencies appear in both tables would seem to indicate that the order in which the cards were presented was a factor which

affected the result and that the natural ebb and flow of attentive processes was involved. In addition, it should be observed that linear stimuli of the lengths here given cannot be distinguished from one another with any considerable degree of certainty, unless they differ by more than one cm. in length.

We may eliminate these inconsistencies in the two tables by taking an average of three different lengths of primary stimulus. For example: The results in the case of primary stimuli 8.0, 9.0, and 10.0 cm. were 0.15, 0.29, and 0.19 cm., respectively. The mean of 8, 9, and 10 is 9; the average of 15, 29, and 19, is 21. Therefore the corrected result for primary stimulus, 9.0 cm., would show an influence of secondary stimulus equal to 0.21 cm. Similarly the corrected result for primary stimulus 10.0 cm., shows an influence of 0.23 cm. When Tables X. and XI., are corrected according to the method just outlined the average appears as shown Table XII. Here we see a consistent increase in influence of secondary stimulus corresponding to increase in length of primary stimulus.

TABLE XII.

THE RESULTS SHOWN IN TABLES X. AND XI. WHEN REARRANGED AS DESCRIBED IN THE TEXT.

Lengths of Primary Stimuli	9.0 cm.	10.0 cm.	11.0 cm.	12.0 cm.	13.0 cm.	14.0 cm.	15.0 cm.
Average. Table X.	.21	.23	.29	.32	.41	.42	

Lengths of Primary Stimuli	19.0 cm.	20.0 cm.	21.0 cm.	22.0 cm.	23.0 cm.	24.0 cm.
Average. Table XI.	.06	.09	.19	.28	.42	.46

A closer examination of the two summaries of results shown in Table XII. reveals another fact which has entered as a disturbing element into my experiments and which I have not succeeded in satisfactorily isolating. I refer to the fact that the stretch of open space between the primary and secondary stimuli enters as an element in determining the influence of the secondary stimuli. When variations in the size of this open space are small its influence upon the result may perhaps be disregarded. But in cases of the sort now under discussion, these open spaces play a considerable part. For example, in

Table XII. we see that when length of primary stimulus is increased from 9.0 cm. to 14.0 cm. the influence is increased from 0.21 cm. to 0.42 cm. Now if we accept the hypothesis that this influence varies directly as the product of the intensities of the two sensations,¹ and inversely as the square of the distance between them, then the proportion of increase ought to be readily determined by use of the well known formula for the law of gravity. Using this formula ($f = C[(m \times m') \div D^2]$) as a basis of calculation, it will be found that the relation between the influence in the case of the 9.0 cm. and 14.0 cm. primary stimuli, will be as 0.22 is to 0.35. The relation as determined empirically was 0.21 to 0.42. The entire series as determined empirically is 0.21, 0.23, 0.29, 0.32, 0.41, 0.42. The entire series as determined by calculation based upon the formula is 0.22, 0.24, 0.27, 0.30, 0.32, 0.35.

It will be observed that in the latter half of the series there is an increase in the influence of the secondary stimuli, which is in excess of that which is warranted by our hypothesis as represented by the formula in question. It should also be noted that this unexpected increase in influence is coincident with a gradual lessening of the open spaces which separate the primary from the secondary stimuli. In the case of the 9.0 cm. primary stimulus this open space was 3.5 cm., whereas in the case of the 14.0 cm. primary stimulus this space was only 1.0 cm. This would seem to indicate that close proximity of the ends of the primary and secondary stimuli, increases the effect of the secondary stimulus.

The indication just referred to is further emphasized by reference to the other half of Table XII. Here we have primary stimuli increasing in length from 19.0 cm. to 24.0 cm. The series of figures showing the influence of secondary stimulus as determined empirically is 0.06, 0.09, 0.19, 0.28, 0.42, 0.46. A corresponding series calculated upon the basis of the gravity formula would be 0.19, 0.20, 0.21, 0.22, 0.23, 0.24. Here too there is a corresponding decrease in the size of the space which separates the two stimuli.

It is further evident from the foregoing that the disturbing effect of too close proximity of the ends of the two stimuli is in

¹Intensity of sensation is equivalent to 'sensation mass.'

proportion to the length of the primary stimuli. This relation is manifest in a series of experiments recorded in Table XIII. The experiments were conducted by myself, the subjects being students of psychology. Three subjects were used and results for three series of ten judgments each are shown. In the first column are recorded the judgments of the length of a single line (4.0 cm.) without secondary stimuli; in the second column are shown judgments of the length of the same line when secondary stimuli (2.0 cm.) have been introduced at distance 3.5 cm. The distance between the end of the primary stimulus and the end of the secondary stimulus was only 0.5 cm. The third column shows judgments of a single line 6.0 cm. in length and the fourth column shows judgments of the same line when secondary stimuli have been added at a distance of 4.5 cm. The distance between end points is again only 0.5 cm. And so with each primary stimulus, viz., 8.0 cm., 10.0 cm. 12.0 cm., and 14.0 cm., the distance between primary and secondary stimuli is respectively 5.5 cm., 6.5 cm., 7.5 cm. and 8.5 cm., but the distance between end points of primary and secondary stimuli is in every case only 0.5. cm.

Referring now to the average of results for all subjects, there is shown a marked increase in influence for each primary stimulus, despite the fact that the secondary stimulus was further removed and probably less intense. It is at once manifest that the formula under consideration cannot be used to determine the relative effect of secondary stimuli in the case of such a series as that which is represented in this table, unless another element can be introduced into the formula. It is not yet clear what this element should be. We can only say that close proximity of the ends of the two stimuli increases the effect of the secondary stimuli and that this increase in effect is itself increased in proportion to length of primary stimulus.

INFLUENCE OF SECONDARY STIMULI OF VARYING LENGTH
UPON A PRIMARY STIMULUS OF A CONSTANT LENGTH
AND AT A CONSTANT DISTANCE.

For showing this relation I have not conducted a separate series of experiments, but have rearranged the results shown in Tables I., II. and III. and represent them in Table XIV.

TABLE XIII.
INFLUENCE OF SECONDARY STIMULI 2.0 CM. LONG UPON PRIMARY STIMULI OF DIFFERENT LENGTHS AND AT
DIFFERENT DISTANCES.

Lengths of Primary Stimuli		4.0 cm.		6.0 cm.		8.0 cm.		10.0 cm.		12.0 cm.		14.0 cm.	
Distances.		3.5 cm.		4.5 cm.		5.5 cm.		6.5 cm.		7.5 cm.		8.5 cm.	
Subjects.	Series	Single Stimulus	Secondary Stimuli	Single Stimulus.	Secondary Stimuli.	Single Stimulus.	Secondary Stimuli.	Single Stimulus.	Secondary Stimuli.	Single Stimulus	Secondary Stimuli.	Single Stimulus.	Secondary Stimuli.
E. B.	1	4.04	4.46	5.98	6.52	7.84	8.70	9.85	10.85	12.05	13.16	13.89	15.33
	2	3.89	4.09	5.91	6.12	7.74	8.12	9.74	10.43	11.78	12.53	13.72	14.56
	3	3.86	4.04	5.76	6.35	7.62	8.27	9.57	10.36	11.46	12.47	13.43	14.42
N. B.	1	3.82	3.96	5.84	6.05	7.75	8.24	9.67	10.34	11.68	12.47	13.51	14.25
	2	3.88	4.01	5.92	6.26	7.88	8.36	10.13	10.73	12.24	12.84	14.08	14.82
	3	3.87	4.01	5.84	6.27	7.73	8.28	9.70	10.55	11.68	12.53	13.51	14.65
A. Mc.	1	3.95	4.24	5.85	6.35	7.79	8.44	9.70	10.57	11.67	12.61	13.86	14.50
	2	3.88	4.44	5.87	6.40	7.70	8.69	9.96	10.76	11.93	12.78	13.77	15.30
	3	3.73	3.85	5.61	5.96	7.70	8.30	9.65	10.30	11.75	12.41	13.76	14.69
Average.		3.88	4.12	5.84	6.25	7.75	8.38	9.77	10.54	11.80	12.64	13.72	14.72
Infl. of Sec. Stimuli.			.24		.41		.63		.77		.84		1.00

Unfortunately, the results which are comparable were not obtained in the same series of experiments, but all the results for constant secondary stimulus (2.0 cm.) were obtained first and then results for variable secondary stimulus were obtained in a subsequent series of experiments with the same subjects. Inasmuch as the magnitude of an illusion decreases with practice on the part of the subject¹, we find that in the second series of experiments the influence of the same secondary stimulus under similar conditions is less than in the first series. This is seen by comparing the instances in which the length of the secondary stimulus was the same. For example, when primary stimulus was 18.0 cm., distance 11.0 cm. and length of secondary stimulus 2.0 cm. in both series, the influence in Series 1 was 1.41 cm. and in Series 2 the influence was 1.32 cm., showing a decrease in influence due to practice of 0.09 cm.; similarly, when primary stimulus was 17.0 cm. and secondary stimuli 2.0 cm., influence in Series 1 was 1.18 cm. and in Series 2 it was 1.08 cm., showing a decrease in influence of 0.80 cm.; further, when primary stimulus is 16.0 cm., and secondary stimulus 2.0 cm., influence in first series was 1.41 and in second series 1.20 cm., showing a decrease of 0.21 cm. In comparing the results for the two series therefore we must either subtract these values from the first or add them to the second or, perhaps more accurately, subtract one half from the first and add one half to the second.

Comparisons of individual results of the two series are not satisfactory because of irregularities and we must resort to a comparison of averages in order to discover any consistent relations.

Taking first the results for primary stimulus 18.0 cm. we find the mean of all the distances used is 11.8 cm.; the length of the secondary stimulus used throughout the first series is 2.0 cm.; the mean length of secondary stimulus in the second series is 3.04 cm., the average influence in first series is 1.04 cm. and in second series is 1.21 cm., or if corrected as above suggested the influence in first series is 0.99 cm., and in second series is 1.26 cm. There is thus apparent a more or less direct ratio between the length of the secondary stimulus and its influence.

¹ Cf. Judd, *Genetic Psychology for Teachers*, p. 26.

The results shown in Table XIV. lend themselves, however, to a more comprehensive treatment and enable us to apply directly the formula implied by our hypothesis. By this hypothesis, $f = C(m \times m') \div D^2$ in which f is the force of attraction existing between primary and secondary stimuli, m is the mass or intensity of the primary stimulus, m' is the mass or intensity of the secondary stimulus, D is the distance between primary and secondary stimuli, and C is a constant which must be empirically determined.

From the results of Table XIV. this constant appears to be 0.339, determined as follows: In case of primary stimulus 18.0 cm. $(m \times m') \div D = 0.258$ and the influence as shown above was 0.99 cm. Hence we have $0.99 = C \times 0.258$, or $C = (0.99) \div 0.258 = 0.383$. Determining C for the six possible instances, I found the average to be 0.339 with a mean variation of 0.038.

Using the constant thus determined, it will be found by making proper substitutions that the formula given is an approximate expression for each of the results obtained by experiment, when the conditions are comparable with the foregoing and that the consolidation of individual results increases the perfection of such an approximation.

It should be expressly remarked, however, that the formula with constant above given, cannot be applied indiscriminately to all results in which widely varying distances involving varying intensities of secondary stimuli are included; nor can it be applied successfully to cases in which the ends of primary and secondary stimuli are less far removed from one another than 1.0 cm.

MEAN VARIATIONS.

It will be noted that the mean variation does not appear in the tables. This is because the results shown in the tables always represent a calculated effect and not a judgment. This effect was determined by subtracting one series of judgments from another. A mean variation parallel to the results shown in the table would have no definite meaning. As regards the judgments made by the subjects I may make the following general statements: For all judgments the mean variation ranged from 0.2 to 0.8 cm. As a rule the mean variation is somewhat larger

TABLE XIV.

COMPARISON OF INFLUENCE OF SECONDARY STIMULI OF DIFFERENT LENGTHS UPON PRIMARY STIMULI OF THE SAME LENGTH AND AT THE SAME DISTANCE.

Primary Stimulus 16.0 cm.				Primary Stimulus 17.0 cm.				Primary Stimulus 18.0 cm.			
Series.	Distance in cm. between Primary and Secondary Stimuli.	Length of Secondary Stimuli in cm.	Influence of Secondary Stimuli in cm.	Series.	Distance in cm. between Primary and Secondary Stimuli.	Length of Secondary Stimuli in cm.	Influence of Secondary Stimuli in cm.	Series.	Distance in cm. between Primary and Secondary Stimuli.	Length of Secondary Stimuli in cm.	Influence of Secondary Stimuli in cm.
1	9.5	2.00	1.74	1	10.0	2.00	1.47	1	10.5	2.00	
2	"	1.79	1.43	2	"	1.77	1.28	2	"	1.85	
1	10.0	2.00	1.41	1	10.5	2.00	1.18	1	11.0	2.00	
2	"	2.00	1.20	2	"	2.00	1.08	2	"	2.00	
1	10.5	2.00	1.18	1	11.0	2.00	0.91	1	11.5	2.00	
2	"	2.15	1.01	2	"	2.35	0.92	2	"	2.41	
1	11.0	2.00	0.80	1	11.5	2.00	0.59	1	12.0	2.00	
2	"	2.90	0.86	2	"	3.32	0.80	2	"	3.58	
1	13.0	2.00	0.44	1	13.5	2.00	0.29	1	14.0	2.00	
2	"	3.70	0.53	2	"	4.90	0.51	2	"	5.43	
Average Distance 10.8 cm.				Average Distance 11.3 cm.				Average Distance 11.8 cm.			
Average Length of Secondary Stimulus in Series 1 = 2.00 cm.				Average Length of Secondary Stimulus in Series 1 = 2.00 cm.				Average Length of Secondary Stimulus in Series 1 = 2.00 cm.			
Average Length of Secondary Stimulus in Series 2 = 2.31 cm.				Average Length of Secondary Stimulus in Series 2 = 2.87 cm.				Average Length of Secondary Stimulus in Series 2 = 3.04 cm.			
In Series 1 $(m \times m') \div D^2 = 0.253$				In Series 1 $(m \times m') \div D^2 = 0.266$				In Series 1 $(m \times m') \div D^2 = 0.258$			
" " 1 Average Influence of Secondary Stimuli = 1.01 cm.				" " 1 Average Influence of Secondary Stimuli = 0.84 cm.				" " 1 Average Influence of Secondary Stimuli = 0.99 cm.			
In Series 2 $(m \times m') \div D^2 = 0.314$				In Series 2 $(m \times m') \div D^2 = 0.382$				In Series 2 $(m \times m') \div D^2 = 0.393$			
" " 2 Average Influence of Secondary Stimuli = 1.10 cm.				" " 2 Average Influence of Secondary Stimuli = 0.97 cm.				" " 2 Average Influence of Secondary Stimuli = 1.26 cm.			

LAW OF ATTRACTION.

when secondary stimuli are introduced and the variation is larger when the primary stimulus is increased in length.

TACTUAL ILLUSIONS.

In order to compare the visual and tactual illusions and to show the law of attraction as applied to the latter, I reproduce from the article 'Ueber den Einfluss von Nebenreizen'¹ to which reference has been made, the results of some experiments with a tactual illusion similar to the Müller-Lyer visual illusion. The line was produced by pressure of a thin strip of brass upon the skin of the forearm. The projections or arms were produced by pressure of short brass rods drawn to a point. An apparatus was so constructed that the pressure from line and all points could be given at the same time. For a more detailed description of the method and nature of these experiments, the original article must be consulted.

So far as these results are comparable with results of visual experiments previously detailed, they appear in Tables XV. and

TABLE XV.

TACTUAL ILLUSION. FIGURE SIMILAR TO THE MÜLLER-LYER FIGURE WITH PROJECTING ARMS EXTENDING OUTWARD.

Length of Primary Stimulus in cm.	Length of Secondary Stimuli in cm.	Angle Formed by Secondary Stimuli.	Subject Ke.		Subject W		Subject M.		Average.		$\frac{m \times m'}{D^2}$
			Influence in cm.	M. V.	Influence in cm.	M. V.	Influence in cm.	M. V.	Influence in cm.	M. V.	
6.0	2.9	20°	2.9	0.9	2.5	0.5	3.4	0.4	2.9	0.6	0.90
6.0	5.0	20	3.1	1.0	2.9	0.7	4.3	0.4	3.4	0.7	0.99
7.0	2.9	20			2.5	0.4			2.5	0.4	0.84
7.0	5.0	20			3.4	1.0			3.4	1.0	0.97
8.0	2.9	20	2.2	0.7	2.0	0.2	3.4	0.5	2.5	0.5	0.76
8.0	5.0	20	2.4	0.6	2.5	0.4	4.0	0.3	3.0	0.4	0.94
6.0	2.9	30	2.8	0.5	1.7	0.4	2.7	0.3	2.3	0.4	0.90
6.0	5.0	30	2.3	1.0	1.8	0.2	2.8	0.7	2.3	0.6	0.99
7.0	2.9	30			2.2	0.3			2.2	0.3	0.84
7.0	5.0	30			1.5	0.3			1.5	0.3	0.97
8.0	2.9	30	2.1	0.2	1.2	0.6	3.1	0.9	2.1	0.6	0.76
8.0	5.0	30	2.1	0.7	1.8	0.5	3.4	0.7	2.4	0.6	0.94

XVI. The last column of each of these tables contains the value of $(m \times m') \div D^2$ arithmetically expressed. Here m equals

¹ *Archiv f. d. Gesamte Psychologie*, Vol. I., pp. 31-109.

TABLE XVI.

TACTUAL ILLUSION. FIGURE SIMILAR TO THE MULLER-LYER FIGURE WITH PROJECTING ARMS EXTENDING TOWARD THE CENTER.

Length of Primary Stimulus in cm.	Length of Secondary Stimuli in cm.	Angle Formed by Secondary Stimuli.	Subject Kc		Subject W.		Subject M		Average.		$\frac{m \times m'}{D^2}$
			Influence in cm.	M. V.	Influence in cm.	M. V.	Influence in cm.	M. V.	Influence in cm.	M. V.	
10.0	2.9	25°	1.9	0.7	1.4	0.2	1.0	0.6	1.4	0.5	0.71
10.0	5.0	25	2.2	0.3	1.5	0.3	1.6	0.3	1.8	0.3	0.89
12.0	2.9	25	1.8	0.4	1.9	0.7	1.3	0.4	1.7	0.5	0.62
12.0	5.0	25	2.5	0.2	1.5	0.0	1.6	0.4	1.9	0.2	0.83
10.0	2.9	45	1.8	0.3	0.9	0.3	0.5	0.3	1.1	0.3	0.71
10.0	5.0	45	1.6	0.3	1.1	0.5	0.6	0.2	1.1	0.3	0.89
12.0	2.9	45	1.6	0.5	1.2	0.2	0.6	0.3	1.1	0.3	0.62
12.0	5.0	45	2.0	0.3	1.3	0.2	0.5	0.2	1.3	0.2	0.83

the length in centimeters of the line or primary stimulus; m' equals the distance of the end point of the projecting arm from the end of the line or primary stimulus. The assumption that this last is the secondary stimulus is somewhat questionable. Its justification, so far as there is any, is based upon the following facts: (1) The introspective evidence of the subjects showed that the end points of the line were most prominent in consciousness, and consciousness of stimulation of the skin between the two end points of the line was very vague and sometimes altogether absent. Consequently the judgment really concerned a distance between two points (corresponding to the line) influenced by a consciousness of a *distance* between these two points and four other points. I have therefore considered these distances the secondary stimulus rather than the actual points stimulated.

D in the formula above given is the distance from center of primary to center of secondary stimulus, measured along the line which consciousness must inevitably follow. In other words D is here one half the primary plus one half the secondary stimulus.

A comparison of the average E (which here represents the average influence of the secondary stimulus or, more properly, the effect of the force of attraction between the primary and secondary stimuli) with the numerical equivalent of $(m \times m') \div D^2$

shows a fairly consistent proportional relation. The accuracy of the proportion is increased when averages of all comparable groups is taken.

It must be observed that comparisons of instances in which the angles are different cannot be made. The intensity of the secondary stimulus is decreased as its angle increases. We cannot compare satisfactorily the results of the two tables for a similar reason.

The figures which are used to represent the intensity of the secondary stimulus are at best only relative, not absolute, and hence the reason that we cannot at present complete the formula and assign a definite numerical value to C .

If, however, comparisons are made of instances in which the conditions upon which the intensity of m' depends are constant, the accuracy of the proportion existing between the attractive forces and the values of $(m \times m') \div D^2$ is very striking. For example, taking data from the first two lines of Table XV., we have the proportion $2.9 : 3.4 :: 0.90 : 0.99$ or $3.060 : 2.871$, there being a difference in the proportion of 0.189 cm. But inasmuch as there were four secondary stimuli in the experiments recorded, the actual difference in proportion for a single secondary stimulus would be only 0.047 cm.

The inaccuracy of the proportion in the second half of each table is largely increased. Here the results are from experiments in which the angle of the secondary stimulus was quite large. When this angle was large the magnitude of the illusion was considerably diminished. As a consequence the observer was more liable to be misled by other influences than the immediate objects of perception. Moreover the difficulties of accurately determining the numerical value of the illusion were increased for the experimenter. Hence the values given in the latter half of each table are less trustworthy than the corresponding values in the first half.

INDIVIDUAL DIFFERENCES.

It is a notable fact that some individuals are more susceptible than others to an illusion of the kind under discussion. Binet has remarked¹ that young children are more susceptible than

¹ *L'Année Psychologique*, 1894, 'L'illusion d'optique de Mueller-Lyer,' p. 330.

older persons. This fact is also very manifest in the results which are here reported. For example, referring to Tables I., II. and III. a very casual examination is sufficient to reveal the fact that the illusion values for the first four subjects are less than the corresponding values for the remaining six subjects. The first four subjects were adults, while the remaining six were children varying in age from twelve to fourteen years.

This difference may be accounted for partly on physiological and partly on psychological grounds. In the case of children, the nervous organism is not so firmly 'fixed'; alterations among its parts may be more easily effected. Attraction between the elements of the organism, therefore, has a greater effect.

On the other hand, psychologically speaking, the judgment of the other person is more evenly balanced, which is perhaps equivalent to saying that experience furnishes to the older person a larger supply of data upon which a judgment may be based. A high degree of susceptibility to illusion, therefore, may indicate on the one hand, especially in children, a nervous organism which is plastic and impressible and therefore highly educable, and on the other hand, especially in adults, a weakness of judgment.

In the article 'Ueber den Einfluss von Nebenreizen' to which reference has already been made, I reported experiments by myself upon several groups of children taken from different classes of two elementary schools in Würzburg. The object of these experiments was to determine quantitatively the effect of a secondary stimulus upon the localization of a point stimulated upon the skin of the fore arm. It was found that a fairly consistent parallel existed between the amount of influence exerted by the secondary stimulus and the degree of mental ability attributed to the pupil by his teacher. This parallel was more striking when groups of dull children were compared with groups of bright children. The children used in these experiments varied in age from six to fourteen years. It is very questionable if the same relation would hold for older individuals.

The individual variations in the case of subjects of the same age are marked also in the case of the visual illusion, but the

number of subjects for which results are reported is so small that comparison would be valueless. Future experiment must determine whether or not a relation, such as I have indicated, exists.

GEOMETRIC-OPTICAL ILLUSIONS.

The literature of this subject is peculiarly rich and not unprofitable. I shall attempt to touch briefly upon such salient features only as are directly related to the phenomena which have been under my observation.

Heymans has shown¹ that the Müller-Lyer illusion (*a*) increases with the length of the projecting arms, is then (*b*) stationary, and finally (*c*) decreases as the length of the arms increases.

These facts which seem to me fatal to most of the theories which have been advanced to explain the illusion, are perfectly in accord with the law of attraction as developed in the foregoing pages. For, increase in length of projecting arm means, (*a*) increase in intensity of the secondary stimulus and (*b*) increase in the distance of the secondary from the primary stimulus. In the former case, we have *increase* in influence and in the latter case *decrease* in influence of the secondary stimulus. If we begin to increase the length of the arms when they are very short, each increment in length corresponds to a relatively large increase in intensity, but as a result of the operation of Weber's law, there comes a time when a very large increase in length of arm (or secondary stimulus) results in only a relatively small increase in intensity. On the other hand, each increase in square of the distance has diminished the influence of the secondary stimulus in proportion.

In the beginning therefore, (*a*) the increase in influence due to increase in intensity is greater than the decrease in influence due to increase in distance, later (*b*) the effect of the two factors is equal, the one counterbalancing the other, and finally (*c*) the decrease in influence due to increase in distance is greater than the increase in influence due to increase in intensity.

Heymans further shows in the same connection (p. 227), that there is a consistent proportional relation between the size

¹ *Zeitschrift für Psychologie und Physiologie*, Vol. IX., p. 236.

of the illusion and the cosine of the angle formed by the projecting arm and the central line (Schenkelwinkel). When the angle increases the illusion becomes less pronounced. This fact harmonizes with our law of attraction, inasmuch as it is to be expected that an attractive force will have greater effect when acting in a straight line than when acting at an angle upon a given object.

The application of the law of attraction to the other geometric-optical illusions with any degree of accuracy is difficult, if at all possible. In general, we may observe, however, that displacements take place in the direction of greater 'sensation masses.' In the Poggendorf figure, for example, the points where the diagonal joins the parallels are drawn, the one upward and the other downward, by the relatively large sensation masses represented by the sides of the two angles formed. The same may be said of the Zoellner figure.

In all such cases, where there is a displacement of a line from its true objective position, it will be found that certain points which mark the direction of said line are acted upon unequally by neighboring 'sensation masses.'

The most difficult factor to determine in all of these phenomena is the value of a particular 'sensation mass,' or, as previously termed, the intensity of a particular visual stimulus. Spatial measurement is a very inadequate expression of this intensity, as we had reason to observe in our discussion of the results of Tables IV., V. and VI. There are evidently several elements which go to determine the intensity of a given visual stimulus. The first of these is undoubtedly spatial size. The second is the proportionate part of all active sensory processes which the sensation in question represents. When an experience is already crowded with sensory elements, the addition of a new element has comparatively little sensory value. A third element is position in the visual field — the same stimulus being more intense upon the fovea centralis than on the periphery. A fourth element is the amount of central reinforcement which may be given the stimulus. Mach says,¹ for example: 'Der

¹ *Pfüger's Archiv*, Vol. 60, p. 509. Also *Zeitschrift für Psychologie und Physiologie*, Vol. 16, p. 298.

blosse Wille rechts zu blicken gibt den Netzhautbildern an bestimmten Netzhautstellen einen grösseren Rechtswerth.' An illustration of both the third and fourth elements just mentioned is furnished by a phenomenon which I have observed in making some experiments with the illusion to which Professor Loeb first called attention. One of the illustrations which Professor Loeb offered was as follows: If one places two pieces of money on a table so that they seem equally far removed to one's right and then places a third piece further towards the right so that the three pieces form a right angle triangle it will be found that the relative position of the first two has been so altered that the lower one which is on a horizontal line with the third, now appears further to the left than the upper one. In my own experience the phenomenon to which Professor Loeb calls attention does not always appear and in fact the reverse phenomenon sometimes appears, *i. e.*, the lower one of the two vertically arranged pieces appears further toward the right. On giving the matter closer attention I found that the change in result was brought about by a difference in the direction of attention. If, for example, the attention is directed to the two lower pieces the third is attracted by both and the phenomenon mentioned by Professor Loeb may be observed. On the other hand if attention is directed toward the one above and the one to the extreme right, or the two forming the hypoteneuse of the triangle, it will be found that the reverse phenomenon takes place, *viz.*, the third is again attracted by the two to which attention is being given, the angle opposite becomes obtuse and the upper of the two pieces which were arranged vertically now appears to lie too far to the left. Here attention, or central reinforcement, and bringing of the two images nearer the fovea, both operate to increase the intensity, or sensation mass, of the two sensations, diminishing in a corresponding degree the intensity of the third sensation. The consequence is that the two stronger attract the third with a greater force than it attracts them and it is therefore displaced from its true relative position.

Finally, a fifth element in determining the intensity of a given sensation is the duration of the stimulus — the intensity of a stimulus diminishing as it grows older. This last element

has perhaps not figured in the case of the illusions which have previously been discussed. But in the case of the so called illusion of reversible perspective it probably plays a large part. Such illusions are usually brought about by staring at a figure. The result of the staring is to diminish gradually the intensity of the sensations occasioned by the points which determine the form of the figure. The intensity of the neighboring points is relatively less affected. Consequently, these neighboring points finally have a larger 'sensation mass' or intensity than the others, and when this happens, they determine the form of the figure according to their own disposition. After this has happened a few times, one can so reinforce the intensity of these latter points from within that he is able to control the phenomenon at will.

THEORIES.

Heymans and Wundt¹ both agree that the Müller-Lyer illusion is due to an almost physical impulse (fast physische Zwang) to follow the direction of the projecting arms with eye movements. One might argue in support of this theory that the law of attraction governs in fixing the strength of the impulse to eye movement. The question still remains unsettled, however, as to whether the judgment is a by-product of this impulse to movement, or whether the impulse to movement is itself a product coördinate with the judgment.

Wundt, in criticism of Heymans' contrast theory, calls attention² to the fact that the illusion takes place when either figure is compared with a straight line and no contrast of eye movement is possible. This criticism is justified by my experiments which were concerned with only one type of figure.

It is manifest that such theories³ as the confluence theory of Mueller-Lyer, the Auerbach indirect vision theory, the Brentano pseudoscopic angle theory, the Thiery perspective theory, the Einthoven dispersion image theory and all others which are based upon phenomena growing out of the extension of arms at an angle, are shown to be inadequate by the fact that the illusion is present when no such angles appear in the figure.

¹ *Physiological Psychology*, Vol. II., p. 149.

² *Die geometrisch-optischen Tauschungen*, p. 47.

³ Titchener, *Experimental Psychology*, pp. 321-328.

GENERAL CONCLUSIONS.

The law of attraction as represented in the present paper is an attempt to state in definite form a principle which has been more or less prominent in the theories of several writers upon this subject.

Jastrow,¹ for example, has attempted to explain optical illusion in general on the principle that all objects are judged relatively to their environment. Our judgment of a thing is modified by the other things which surround it.

A great variety of facts which illustrate the principle may be drawn from every day experience as well as from experimental laboratories. For example, Lipps² calls attention to the fact that cows appear to be larger when they are in narrow, low stalls than they do when outside.

Professor Baldwin in an article upon the 'Effect of Size-contrast upon Judgments of Position in the retinal Field,'³ reports that a point, in the field of vision, midway between two figures of unequal size, as two squares or two circles, will be attracted towards the larger figure. Further, the tendency to error increases with the relative increase of the side of the larger figure and the tendency is about twice as great when the figures are arranged vertically as when they are arranged horizontally.

In an article entitled 'Normal Motor Suggestibility,'⁴ I have reported a series of experiments showing that the localization of a point stimulated upon the skin of the forearm is influenced by the stimulation of a second point, either above or below the one to be localized. It was also shown that the localization of a visual image in the peripheral field was similarly affected by the appearance in the same field of a second visual image, and, similarly, the localization of a sound was affected by a second sound.

The tendency to fuse together of two or more sensations which are simultaneously experienced has been frequently remarked and experimental psychology has shown conclusively

¹ *American Journal of Psychology*, Vol. IV., p. 381.

² *Raumaesthetik*, p. 65.

³ *PSYCHOLOGICAL REVIEW*, Vol. II., p. 244. Cf. also the further figures given in his *Fragments in Philosophy and Science*, pp. 275 ff.

⁴ *PSYCHOLOGICAL REVIEW*, Vol. IX., pp. 329-356.

that one of the chief defects of the older introspective psychology was its failure and inability to recognize in experience the elementary sensations which composed it.

All these and many other similar facts seem to point to a general law of relativity, which may be stated somewhat as follows: *Every sensation is influenced by every other sensation which may be present in any complex experience.* The nature of this influence seems to be a direct interaction of one upon the other, the resulting effect of this interaction being determined by the nature of the interacting sensations.

All of the sensations with which I have dealt experimentally are such as to make up 'extensive ideas';¹ they were either sensations defining position or form and magnitude, and the nature of the interaction seems to have been an attractive force, which I was able to measure. This attractive force is governed in its action by the same general law which governs the action of the attractive forces in nature, with which we are already familiar and which has been given mathematical expression in the well-known formula $f = C(m \times m') \div D^2$. In its application to the phenomena which have been under our observation, f , in the formula, equals the force exerted by two sensations, the one upon the other, m is the intensity or sensation mass represented by a primary stimulus, m' is the intensity or sensation mass represented by a secondary stimulus, and D is the distance between the primary and secondary stimuli, measured from center to center. The constant C must be determined empirically, and is not the same value in the case of visual and tactual sensations.

The apparently physical nature of the law leads to the suggestion that this attractive force operates between the nervous elements, electro-chemical in nature, which mediate sensation.

On the other hand, one might be justified in admiring that universality of the law, manifest in its consistent operation in two such widely separated spheres as the material and the spiritual.

The time is not opportune, I think for a discussion of the vexed question as to the nature of mind which is involved in the two possible theories here suggested.

¹Titchener, *An Outline of Psychology*, p 154.

SUMMARY.

1. When a subject is required to judge the length of a single line, by comparing it with a second line which is variable in length, the single line is always underestimated.

2. Using the same method of comparison, if the subject is required to judge the length of the same line, now accompanied by shorter lines which represent extensions of the line of direction of the original line but which are separated from it by open spaces, the original line is judged to be longer than it was when unaccompanied by the shorter lines, and, generally, it is judged to be longer than objective measurement shows it to be.

3. When the results for several subjects are consolidated it is found that the influence attributable to the addition of the shorter lines, or secondary stimuli, is (*a*) increased when the length of the secondary stimuli is increased, also (*b*) that this influence is increased when the length of the line to be judged is increased, and finally (*c*) an increase in the distance of the short lines from the central lines, or primary stimulus, measured from center to center, is followed by a decrease in the influence of the short lines, or secondary stimuli.

These general relations obtain in the case of individuals as well as for groups of individuals, but the individual variation is somewhat large, and comparisons of individual results are not thoroughly satisfactory.

4. A fourth and a disturbing element in determining the amount of influence of the secondary stimuli was the space between the ends of the primary and the secondary stimuli. When the distance between the ends of the primary and the secondary stimuli was decreased, the amount of influence of the secondary stimuli was correspondingly increased. The exact proportionate relation was not determined.

5. When the disturbing factor just noticed could be disregarded by reason of a favorable arrangement of conditions, it was found that the well known formula, expressing the law of attraction in the material universe, can be applied to the results of the experiments here described.

6. The results of certain experiments in judging the length of lines stimulated by pressure upon the skin of the forearm, also yield to a similar statement.