animal for food, then the intensely green bile rendered all parts inedible; only one colour was named from the hue of a flower, in spite of the great variety which tropical flowers show. Points of equal interest are the indefinite character of the word used for "blue," this being applied indifferently to blue-green, dirty yellow, grey, &c., and the complete absence of any word for "brown," the language resembling in this respect Homeric Greek. The Murray islander recognised "red" far more distinctly than any other colour; yellow was the next most recognisable hue, "blue" could only be differentiated when in considerable strength, and brown was merely a dull-looking light.

In this connection the simple experiments made upon peripheral colour vision were extremely suggestive. It is well known that in the European the red-green visual field is the smallest, whilst the blue and yellow fields are far larger, but in the Murray islander the green field was distinctly the smallest, and the red field extended widely into the peripheral regions; the largest field of all was, however, the blue one, these colours being far better recognised with peripheral vision than in vision involving the central macula. Probably, as Dr. Rivers suggests, the defective stimulation of the macula by blue light may be related to the excess of yellow pigment present in the Papuan race, and would not be in itself a sign of defective retinal capacity for excitation by these rays.

Many other points of great interest are detailed in this part of the reports, colour contrast, after-images, visual perception of distance, binocular vision, capacity to bisect lines, capacity to compare the length of vertical with that of horizontal lines, susceptibility to such well-known visual illusions as those of Müller-Lyer, Zöllner's line displacements, &c. In regard to all these points there appears to be little, if any, difference between the Murray islander and the average European; the details of these experiments will well repay the reader, particularly as Dr. Rivers has presented the results and described the methods in such a manner that his account can interest those who have not especially devoted themselves to this kind of work.

The second part of the present volume of reports deals with other sensory phenomena. The investigation of hearing was undertaken by Dr. C. S. Myers; it was rendered difficult by the not infrequent presence of defects in the ears due to the now prohibited practice of deep diving for pearls. The experiments on the younger inhabitants were free from such hampering circumstances, and the results showed that, as compared with Europeans, both the acuity of hearing and the capacity to distinguish differences of tone were distinctly inferior in the case of the islanders; on the other hand, it is remarkable that the range, as estimated by modified Galton whistles, was at least as extensive in the islander as in the European. The investigation of the sensations of smell by Dr. Myers was also extremely difficult, owing to the great objections entertained by the islanders for this class of experiment, but it seems from such observations as could be made that there is no marked hyper-sensitiveness to olfactory stimulation in this primitive race as compared with Europeans.

Dr. Myers also made some limited experiments on tastes; a specially interesting feature brought out by these observations is the complete absence of any word to describe the extremely conspicuous gustatory sensation which we denote as "bitter," although it is certain that the sensation was experienced. In connection with this remarkable omission is the circumstance that, even in Europeans, there is considerable confusion as to the sensory significance of the qualities connoted by the word "bitter." Cutaneous sensations, muscular sense, &c., were undertaken by Dr. McDougall, and here there are some striking, but not unexpected, differences between the Murray islander and the European. In the former the sense of pure contact was twice as delicate as in the average Englishman, whilst the susceptibility to pain through pressure, &c., was far less pronounced. It is somewhat surprising, considering how unfamiliar the islanders were with the necessary procedure, to find that, as regards the estimation of different weights, the average least recognisable weight increment was actually smaller in their case than in the corresponding average of thirty Englishmen, being 3·2 per cent. as compared with 3·9 per cent.

Finally, the very important subject of reaction-time was undertaken by Dr. Myers, who gives most valuable details of the results of his observations. It appears that, as regards auditory reaction-time, the younger Murray islanders give results identical with the average young English townsmen, but that, as regards visual reaction-time, the Murray islanders give distinctly longer results. This lag becomes more perceptible when the attention is definitely fixed on the visual stimulus rather than the preconcerted movement, a procedure which always lengthens the reaction-time of Europeans, but which lengthened that of the islander comparatively more. Further, when the method of choice visual signal was used, involving a complexity of psychical conditions, then the increased lag became still more apparent. The reader is referred to the original for the very instructive and, from a psychological standpoint, most suggestive details of these observations.

In conclusion, the authors are to be heartily congratulated on the appearance of this work, which is a very important contribution to both physiology and psychology. The reports form a lasting memorial both of the activity of Cambridge anthropology and of the genuine character of the scientific spirit which now actuates those who study the various aspects of ethnography; the appearance of the remaining volumes promised by Dr. Haddon will be looked forward to with the greatest interest by a wide circle of biological students.

F. G.

A REVISION OF PRINCIPLES.

The Principles of Mathematics. By Bertrand Russell, M.A. Vol. i. Pp. xxviii + 534. (Cambridge: University Press, 1903.) Price 12*s*. 6*d*. net.

THE appearance of a book addressed equally to mathematicians and to philosophers, setting forth all the assistance which philosophy can afford in the shape of material for mathematics to work with, is a remarkable event, and the fact that the criticism, pertinent and lucid as it is, of the work of the great Continental thinkers is adverse on many fundamental points should claim for it the patient consideration of both classes of students. We quote :—

"The distinction of philosophy and mathematics is broadly one of point of view: mathematics is constructive and deductive, philosophy is critical, and in a certain impersonal sense controversial. Wherever we have deductive reasoning, we have mathematics; but the principles of deduction, the recognition of indefinable entities, and the distinguishing between such entities, are the business of philosophy."

In answer to the question, "what is mathematics?" we are told that

"Pure Mathematics is the class of all propositions of the form 'p implies q' where p and q are propositions containing one or more variables, the same in the two propositions, and neither p nor q contains any constants except logical constants."

These logical constants are defined in terms of the fundamental concepts which mathematics accepts as indefinable; the philosophical discussion of the latter occupies part i. of this volume. The remaining six parts are devoted to the establishment of the main thesis, that what is ordinarily known as mathematics is deducible from these fundamental concepts by purely logical processes. This, of course, necessitates a philosophical account of the processes which are admissible; the carrying out of the deductions in their most abstract and rigorous form lies in the province of symbolic logic, and is reserved for the second volume.

The mathematical reader is recommended in the preface to pass over some of the more philosophical portions and begin at part iv., on "Order." We do not endorse this recommendation, for the exact establishment of the notion of order is one of the most tedious pieces of work that the mathematical philosopher has to do; besides, many of the preceding chapters are not only extremely interesting in themselves, but absolutely essential to a correct appreciation of the science of arithmetic subsequently developed. For example, a *number* will be found to be defined as a *class*.

Concerning the notion of class, some slight criticism may not be inappropriate. The distinction between class, class-concept, and concept of class, which is of fundamental importance to exact thinking, is made admirably clear, but the same cannot be said of what is necessary to constitute a class. A class may be defined either extensionally, by an enumeration of its terms, or intensionally, by the concept which denotes its terms. The former method seems applicable only to finite classes; we cannot agree with the author that it is logically, though not practically, applicable to infinite classes, unless some meaning is attached to the word "enumeration" different from what is ordinarily understood. On the other hand, the latter method implies that a class is defined by a predicate, and contains those terms of which the predicate is predicable; but this leads to an apparent contradiction which Mr. Russell has discovered; for consider the predicates which are not predicable of themselves, for example, humanity, which is not human; "not predicable of itself" seems to be a predicate defining a class of predicates, yet to suppose that this defining predicate either is, or is not, contained in that class, leads to a contradiction. A similar contradiction is reached when we consider the class whose terms are all the classes, each of which does not constitute as *one* a term of itself as *many*; for in attempting to form this class, at any stage the terms already obtained constitute a class which must be included as a new term, and so on. This may be compared with the attempt to sum a numerical series each of whose terms is the sum of all the preceding terms; the comparison does not completely explain the paradox, but suggests that a distinction should be made among infinite classes somewhat like that between convergence and divergence.

Leaving the logical side of the subject, we come to the first mathematical idea to be defined, that of number. It was formerly supposed that the notions of "1" and "+1" were fundamental, and that from them all other numbers could be defined. In the present work the number of terms in a class is defined, in a manner slightly differing from Peano's, as the class of all classes similar to the given class. Similarity depends on a one-one relation, which can be defined without reference to number, and indicates by Mr. Russell's "principle of abstraction" the possession of a common property which may be called the number. Various reasons are given for preferring this definition, one of the chief being the inclusion of the infinite numbers introduced by Cantor.

Part iii. deals with quantity and magnitude, between which a subtle distinction is drawn, and contains an introduction to the problems of infinity and continuity, which are to be more fully discussed in part v. Part iv. develops the difficult theory of order and Dedekind's theory of integers. The next part is necessarily based largely on the work of Cantor. To readers unacquainted with the Mengenlehre," the introduction of transfinite numbers must appear rather startling, but this is perhaps partly due to an unusual weakness in the English language. It must be remembered that by a transfinite cardinal number is meant a certain kind of infiniteness of aggregate, the same number belonging to different aggregates which are similar in the preceding sense; and a transfinite ordinal number is another name for a type of infinite series, or of generating relation.

In the chapters on real numbers and irrationals, we approach controversial ground. The particular object which the arithmetisers of mathematics have here in view is to complete the series of rational numbers by the introduction, without any appeal to intuition, of other numbers, so as to satisfy the abstract definition of continuity. One consequence of this will be that it will then be possible to assign a real number to every point on a straight line. Three great thinkers —Dedekind, Weierstrass and Cantor—have done this, making their definitions of an irrational number depend upon the theory of *limits*. Their methods are explained and criticised, the chief objection being that there is no adequate ground for assuming that a limit such as that of the series of rationals whose squares are less than 2 does really exist. Instead of this Mr. Russell defines a *segment* as a class of rationals less than a variable term of itself, and shows that segments possess all the usual properties of real numbers. This theory agrees very closely with Cantor's, the point of divergence being where Cantor appears to regard the rational number a as identical with the real number defined by the series $(a, a, a, \ldots)$ whereas Mr. Russell will not admit this. On the one hand it is obvious that the two concepts are as distinct as "man" and "featherless biped," and therefore cannot be identical; but, on the other hand, it seems unnecessary to insist too much on the distinction, because no confusion need arise from using the expression "a" in two different senses. Thus, if b is the irrational number defined as the series $(\ldots a_n, a_{n+1}, \ldots)$ we may write $b-a=(\ldots a_n-a, a_{n+1}-a, \ldots)$ and in this equation a is a series or so-called real number on the left and a rational number on the right. The conclusion is that the series of rational numbers cannot be completed exactly as it stands, but the rationals must first be replaced by series, or, if preferred, by segments, and then by means of other series the continuum of real numbers can be constructed.

Limitations of space forbid detailed comment on part vi., in which, incidentally, Euclid gets some rather hard knocks; and in the matter and motion of part vii. Newton's laws are condemned as confused, worthless, and wholly lacking in self-evidence, while we are told that force is a mathematical fiction, and velocity and acceleration must not be regarded as physical facts.

On the whole the book is very interesting, although somewhat too long. The presentation is admirably clear, and the seriousness of the style is relieved here and there by neatly turned bits of humour. It does not pretend to say the last word on any subject, and, indeed, bristles with unsolved difficulties, towards the correct solution of which a great step is undoubtedly made by its publication. R. W. H. T. H.

ELECTROCHEMICAL ANALYSIS.

Quantitative Chemical Analysis by Electrolysis. By Prof. Classen. Translated by Bertram B. Boltwood. Pp. vii+315. (New York: John Wiley and Sons; London: Chapman and Hall, Ltd., 1903.) Price 12*s.* 6*d.* net.

ELECTROCHEMICAL methods of analysis are now coming into such general use on the Continent and in America, and to a smaller extent in this country, that chemists will be prepared to welcome the latest translation of Prof. Classen's "Quantitative Analyse durch Elektrolyse."

The translation is made from the fourth German edition published in 1897, but, as the translator has been allowed wide latitude by the author, he has brought the book well up to date, and we find several features in this book which are not in the German original.

In chapters xiii. and xiv., for example, which deal respectively with "measurements of current strength" and "sources of current," there are quite a number of new blocks, as, for example, Bredig's amperemanometer and the Weston ammeters and voltmeters. We also find several new diagrams in chapter xvi., which deals with the accessory apparatus employed in analysis. As a matter of fact, we think, considering that the book is devoted to electro-analysis, some of the apparatus described is rather superfluous. A quadrant electrometer is not usually to be found in a laboratory devoted to electro- or any other analysis, the description of such apparatus appertaining more to works on physics or perhaps on general electrochemistry. In chapter xviii. the author gives details as to "arrangements for analysis." The details which are given refer mainly to the very thorough installations at Aachen, and two photo-plates of the laboratories, as they are at present, also one showing the former equipment of the private laboratory, are given. One cannot learn very much from these photographs, but they improve the appearance of the book, and incidentally give an idea of the large number of platinum basins which Prof. Classen possesses.

On p. 153 we come to the analytical portion of the book, the first metal dealt with being iron. For the analysis of iron there is no doubt that Classen's oxalate method is extremely satisfactory, and the analytical results obtained are generally very accurate. At the same time, as Kohn and others have shown, this is really due to a balancing of errors. The iron deposited always contains traces of carbon, but, on the other hand, there is usually a trace of iron left in the solution, and these two errors balance. Classen states that iron, when deposited from solutions containing citrates and tartrates, always contains carbon, but leaves it to be inferred that when oxalates are employed, the metal is deposited free from carbon. Prof. Classen employs the oxalate method not only for iron, but he uses it for almost every metal, very often, too, when other ways are vastly superior, and he seems very much afraid that someone else will take credit for the method, because in almost every case we find a bracket in which it is stated that this is the "method of the author." As a matter of fact, there are only a few cases in which the employment of oxalates has any real advantage, as *e.g.* with iron and zinc. There is certainly nothing to be gained by using it when depositing copper, nickel, or mercury, where there are many much more satisfactory methods. Cobalt, according to the author, when deposited, shows its characteristic metallic properties. Generally speaking, when electrically deposited, cobalt is brownish or smoky in appearance—are these its characteristic metallic properties?

Section ii. of the analytical portion deals with the analysis of nitrates, and section iii. with the determination of the halogens.

Section iv., on the separation of the metals, is perhaps one of the best parts of the book. It may be very easy, and generally is, provided one employs the correct conditions, to analyse from pure salts of the metals, but the electrolytic separation of metals is