potential between the metals; it can only give us the value of the temperature coefficient, which is equal to the Peltier effect divided by the absolute temperature. Then, again, the pyro-electricity of tourmaline is explained by the unilateral conductivity of a tourmaline crystal whose temperature is changing, discovered by the author and Prof. Silvanus Thompson. If this unilateral conductivity is regarded as proving the existence of an electromotive force in a crystal which is increasing or decreasing in temperature, the explanation is valid, but in the text nothing is said about an electromotive force, and the student might be led to infer that a mere difference in resistance could explain pyro-electricity. The way in which a current flows past an insulating obstacle, the lines of flow closing in on the obstacle, and leaving nothing corresponding to "dead water" behind it, is given as a proof that the electric current has no mechanical momentum; but unless the corners of the obstacle were infinitely sharp, a slowly-moving fluid might flow in the same way as electricity, even though it possessed inertia, so that the proof is not conclusive. It is also stated that the effects on light produced by a magnetized body, discovered by Dr. Kerr, of Glasgow, have been deduced by Prof. Fitzgerald from Maxwell's theory of light. As a matter of fact, however, the results deduced from this theory by Fitzgerald do not coincide with those observed by Dr. Kerr and Prof. Kundt. The production in an unequally-heated conductor of an electromotive force is explained by supposing the atoms in such a body to be moving faster in one direction than the opposite, and therefore, since they are supposed to drag the ether with them, producing a flow of ether in the direction in which they are moving fastest; but, on the dualistic theory of electricity adopted in this book, this ether stream would consist of equal quantities of positive and negative electricity moving in the *same* direction, and this would not produce any electrical effect.

At the end of the book are three popular lectures delivered by Prof. Lodge, the first on the relation between electricity and light, the second on the ether and its functions, and the third his admirable one at the Royal Institution, on the discharge of a Leyden jar, which is a model of what such a lecture ought to be.

Taken as a whole, we think that the book is one which ought to be read by all advanced students of electricity; they will get from it many of the views which are guiding those who are endeavouring to advance that science, and it is so stimulating that no one can read it without being inspired with a desire to work at the subject to which it is devoted.

THE CALCULUS OF PROBABILITIES.

Calcul des Probabilités. Par J. Bertrand. (Paris: Gauthier-Villars, 1889.)

"EVERYBODY makes errors in Probabilities at times, and big ones," writes De Morgan to Sir William Hamilton. M. Bertrand appears to form an exception to this dictum, or at least to its severer clause. He avoids those slips in the philosophical part of the subject into which the greatest of his mathematical predecessors have fallen. Thus he points out that, in investigating the "causes" of an observed event, or the ways in which it might have happened, by means of the calculus of probabilities, it is usual to make certain unwarranted assumptions concerning the so-called *a priori* probability of those causes. Suppose that a number of black and white balls have been drawn at random from an urn, and from this datum let us seek to determine the proportion of black and white balls in the urn. It is usual to assume, without sufficient grounds, that *a priori* one proportion of balls, one constitution of the urn, is as likely as another. Or suppose a coin has been tossed up a number of times, and from the observed proportion of heads and tails let it be required to determine whether and in what degree the coin is loaded. Some assumption must be made as to the probability which, prior to, or abstracting from, our observations, attaches to different degrees of loading. The assumptions which are usually made have a fallacious character of precision.

Again, M. Bertrand points out that the analogy of urns and dice has been employed somewhat recklessly by Laplace and Poisson. It is true that the ratio of male to female births has a constancy such as the statistics of games of chance present. But, before we compare boys and girls to black and white balls taken out at random from an urn, we must attend not only to the average proportion of male to female births, but also to the deviations from that average which from time to time or from place to place may be observed. The analogy of urns and balls is more decidedly inappropriate when it is applied to determine the probable correctness of judicial decisions. The independence of the judges or jurymen which the theory supposes does not exist.

"Quand un juge se trompe il y a pour cela des raisons: il n'a pas réellement mis la main dans une urne où le hazard l'a mal servi. Il a ajouté foi à une faux témoignage, le concours fortuit de plusieurs circonstances a éveillé à tort sa défiance, un avocat trop habile l'a ému, de hautes influences peutêtre l'ont ébranlé. Ses collègues ont entendu les mêmes témoins, on les a instruits des mêmes circonstances, le même avocat a plaidé devant eux, on a tenté sur eux la même pression."

With equal force does M. Bertrand expose the futility of the received reasoning by which it is pretended to determine the probability that the sun will rise to-morrow from the fact that it has risen so many days in the past.

These reflections are just and important; but their value is somewhat diminished by the fact that they have been, for the most part, made by previous writers with whom our author seems unacquainted. Thus Prof. Lexis has more carefully considered the extent of the error committed by Laplace and Poisson in applying to male and female births and other statistics rules derived from games of chance. The fundamental principles of Probabilities have been more fully explored by Dr. Venn. M. Bertrand, like Laplace, starts by defining the probability of an event as the ratio of the number of favourable cases to the number of possible cases. He does not explain what constitutes a "favourable case"—that, when a die is thrown, the probability of obtaining the 3 or 4 is one-sixth, because as a matter of fact each side in the long run turns up once out of six times. Accordingly, when he argues that in a great number of trials each event is most likely to occur with a frequency correspond-

ing to its probability, he lays himself open to the charge of circularity which Dr. Venn has brought against Bernouilli's theorem. Without pronouncing on this delicate question, we may safely say, with respect to the first principles of the subject, that no point which has been left obscure by Dr. Venn has been cleared up by M. Bertrand.

It is with respect to the purely mathematical portion of the calculus, or that part of its metaphysics which is inextricably mixed with mathematics, that we expected and have found most assistance from M. Bertrand. Hitherto the study of Probabilities has been barred by the dilemma which M. Bertrand thus states:—

"On ne peut bien connaître le calcul des probabilités sans avoir lu le livre de Laplace; on ne peut lire le livre de Laplace sans s'y préparer par les études mathématiques les plus profondes."

Much of Laplace's analysis which must have affected many eager students like stickjaw has been simplified by M. Bertrand. He is in general more readable than Poisson. Several of the theorems which he gives seem to be new. His methods of determining from a given set of observations the characteristic, or *modulus*, appertaining to the source of error are specially interesting.

M. Bertrand's mathematical power enables him to carry the torch of common-sense to those perplexed parts of the subject where less qualified critics, awed by the imposing mass of symbols, have hesitated to differ from Laplace or Poisson. Of this kind is the simultaneous determination of several quantities from a great number of equations. When Laplace computes that the odds are a million to one against the occurrence of an error of assigned magnitude in the determination of Jupiter's mass, M. Bertrand shows reasons for suspecting the accuracy of such computations. In fact, he carries out Poinsot's witty direction:

"Après avoir calculé la probabilité d'une erreur il faudrait calculer la probabilité d'une erreur dans le calcul."

The true import and proper application of the theory of errors of observation are thus well expressed:—

"On peut accepter sans crainte le résultat, mais il est téméraire d'évaluer en chiffres la confiance qu'il doit inspirer."

M. Bertrand teaches with authority—and not like those who have not followed the higher mathematical reasonings of the calculus—in what spirit its conclusions should be accepted.

Still, even with regard to those parts of the subject where a first-rate mathematician has so great an advantage, we venture to think that the work would have been much more valuable if the writer had taken the trouble to acquaint himself more fully with what his predecessors had done. For example, in discussing the reasons for taking the arithmetic mean of a set of observations (presumed to be equally good) relating to a single quantity, M. Bertrand does not dwell on the argument that the probability-curve—with which the arithmetic mean is specially correlated—is apt to represent the grouping of errors for this reason, that an error may be regarded as a function of a great number of elements each obeying some definite law of facility, and that the values of such a function conform to the probability-curve. It is true that Laplace, from whom this argument may be derived, has not himself used it very directly. But in a writer on the method of least squares we may expect some conversance with more recent works, in particular with Mr. Glaisher's classical paper in the Memoirs of the Astronomical Society (London). Moreover, Laplace does employ the mathematical theorem which we have indicated, not indeed to prove that the law of facility for errors of observation in general is the probability-curve, but that, whatever that law of facility be, the most advantageous combination is a certain linear function. A treatise in which this celebrated argument is not discussed cannot be regarded as exhaustive. But it is remarkable that with respect to the combination of observations, M. Bertrand seems to defer more to Gauss than to his own eminent countryman.

M. Bertrand has indeed slipped in a doctrine for which the authority of Laplace may be quoted, that in choosing the best combination of a set of observations "there is an essential difference between the most probable value of a quantity and the value which it is best to adopt" (Bertrand, Art. 138); the latter being the mean (first power) of the observations (Art. 155)—which M. Bertrand rather awkwardly terms "la valeur probable." M. Bertrand does not seem to realize the gravity of the assumption which is contained in the latter clause. Later on he employs Gauss's criterion of erroneousness—namely, the mean square of error. But the ground, nature, and relation of these two principles are not very clearly explained by the writer. With respect to the philosophical foundation of the method of least squares he has not superseded the necessity of studying Laplace.

With these reservations, M. Bertrand's work may be regarded as one of the most complete treatises on the subject. Nowhere else are the two elements so peculiarly combined in the science of Probabilities—common-sense and mathematical reasoning—to be found existing together in such abundance. F. Y. E.

ARGENTINE ORNITHOLOGY.

Argentine Ornithology. By P. L. Sclater, Ph.D., F.R.S., and W. H. Hudson, C.M.Z.S. Vol. II. (London: W. H. Porter, 1889.)

THE completion of this important work is an event of considerable importance to every lover of neotropical zoology, and the authors have both performed their parts well, while the ten plates by Mr. Keulemans are beautifully drawn and admirably coloured. Among the increasing number of Englishmen who settle in the Argentine Republic, there are sure to be many who will pursue natural history studies, and to all such a well-executed book like the present will be invaluable. The joint authors of the work are happy in their association, for while Dr. Sclater brings to the work a vast experience, and a sound scientific knowledge of his subject, it is certain that never was there a better describer of the habits of birds than Mr. Hudson. Although of English parentage, he is a native-born Argentine, and he has grown up among the birds whose life and history he so well knows how to portray. In turning over the pages of this volume, we have found many interesting extracts which we should have liked to present to our readers,