



# XLVII. The striated electrical discharge

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XLVII. *The Striated Electrical Discharge.* By J. H. JEANS,  
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[Continued from Ser. 5, vol. xlix. p. 262.]

PART II.

§ 12. **T**HE first part of this paper contained an examination of the differential equation which, upon Prof. Thomson's theory of conduction by ions, is satisfied by the intensity of electric force at any point of an electrical discharge. With a view to simplifying the discussion of this equation, the assumption has been made that the quantities  $q$  and  $\alpha$  depend only upon the electric intensity at the point at which they are measured.

But a simple calculation will show that this electric intensity is not, in itself, sufficiently strong to effect ionization †, and the causes which seem most likely to account for ionization ‡ are not such that  $q$  will satisfy the condition which has been imposed upon it. We must, therefore, examine to what extent the results which were obtained only upon this assumption will remain valid if the assumption is not complied with.

At the outset it may be noticed that differential coefficients of  $q$  do not occur in the equation for the intensity. Hence

\* Communicated by Prof. J. J. Thomson, F.R.S.

† J. J. Thomson, 'Recent Researches,' § 213, p. 192.

‡ The causes of ionization have recently been discussed by Prof. Thomson, *Phil. Mag.* l. p. 278.

this equation will, at any specified point of the discharge, remain unaltered if we suppose that  $q$  has the same value at every point of the discharge as it has at the point in question. Using this value for  $q$ , we can draw a diagram similar to fig. 1 (Part I. p. 251), and at the point in this diagram corresponding to that point of the discharge which we are considering, the value of  $\frac{dp}{dy}$  will have the same sign as has the value of  $\frac{dp}{dy}$  for a point moving with the shading at this point.

But  $q$  varies from point to point of the discharge, and as  $q$  varies the vertex of the parabola in fig. 1 will move up and down the axis of symmetry, while the parabola will always pass through the two points A and B. The vertex can never pass to infinity along this axis, since  $q$  can never actually vanish\* (equation 10, Part I.), and the vertex can never pass on to the axis  $y=0$ , since  $q$  can never become infinite.

Hence the points at which  $\frac{dp}{dy}$  vanishes will no longer (as in § 5) lie on a single parabola, but they will all lie within a certain region which is bounded by two parabolas, both of which pass through the two points A, B, and are concave to the line joining them. From this it follows that there must be curves of the four types shown in fig. 2, which will satisfy the differential equation, and therefore there must be two curves similar to those shown in fig. 5, which will satisfy the differential equation together with the boundary conditions.

Under the present conditions it would be useless to attempt to discover under what circumstances these two forms of solution will be the only possible forms. In what follows it will be assumed that we are dealing with a solution of the type which is represented by the discontinuous line in fig. 5.

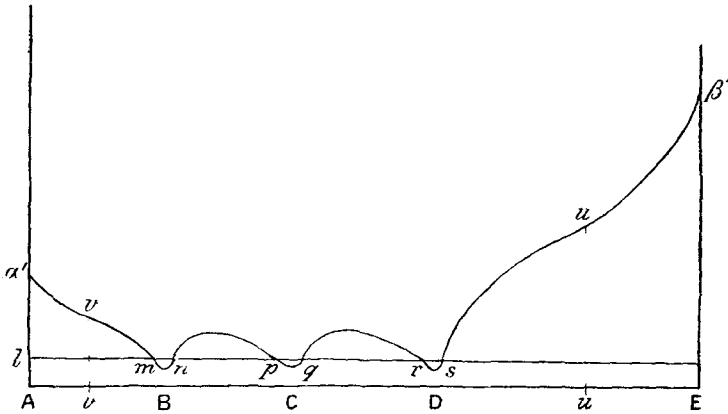
§ 13. Reference has already been made to the variety which is observed in the appearance of the discharge near to the anode. The theory has been found to be capable of accounting for the phenomena observed near the cathode, and also for the striated column in the middle of the tube, but it has not, so far, accounted for the apparent difference in the behaviour of the two electrodes. The considerations put forward in the next two sections are meant to suggest a way in which this difference may arise, although no attempt

\* For instance, if we regard dissociation as the result of collision, we must remember that all velocities are *possible* at every point. Hence however small the chances of dissociation may be, the "expectation" of the number of dissociations (and therefore the value of  $q$ ) will never absolutely vanish.

has been made to supply a formal proof of the principle upon which the argument rests.

The curved line in fig. 7 is the graph which was found to represent the distribution of intensity along the line of discharge which is predicted by theory. The anode is represented by A, the cathode by E, and the ordinate at any

Fig. 7.



intermediate point is equal to  $y$ , the semi-square of the electric intensity at that point. The line  $l m n \dots$  is the line ( $y = \eta$ ) below which the differential equation with which we have been working ceases to hold. The value of  $p$  ( $p = \frac{dy}{dx}$ ) at a point  $m$  at which the graph meets this line will be slightly different from the value of  $p$  at the adjacent point B in which the ideal mathematical graph meets the axis  $y = 0$ . Let this small difference be denoted by  $\varpi_m$ , the difference being taken so that  $\varpi_m$  is positive. Thus we shall have

$$\varpi_m = \frac{4\pi i}{k_2} + p_m,$$

$$\varpi_n = \frac{4\pi i}{k_1} - p_n,$$

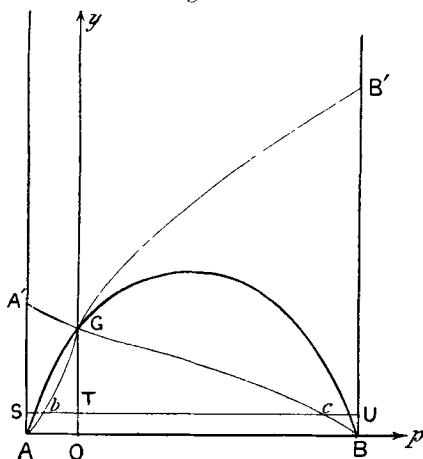
$$\varpi_p = \frac{4\pi i}{k_2} + p_p,$$

and so on, the suffixes  $m, n, p \dots$  referring to the points  $m, n, p \dots$  in fig. 7.

Fig. 8 represents the plane of  $p, y$  (cf. fig. 2), and STU is the line  $y = \eta$ . In this plane the graph for  $y$  will be represented by a single curve, and each striation will correspond

to a single convolution of this curve about the point T, the curve being confined within the area  $AbGcBOA$ . The range of possible values for  $\varpi_n$ , it is easy to see, will be the line  $Tc$ , the limiting values being  $Uc$  and  $UT$ . Similarly the

Fig. 8.



range of possible values for  $\varpi_p$  will be  $bT$ , and so on. Now the ratio of  $Tc$  to  $bT$  depends on, and is roughly of the same order as, the ratio of  $TU$  to  $TS$ , and therefore of  $k_2$  to  $k_1$ ; and direct experiment shows that this ratio is large.

As we pass along the graph away from the anode, the curve in fig. 8 will revolve about T in a left-handed direction. All the curves which meet  $STU$  within the range  $Tc$  must, by the time they reach  $TS$ , be compressed into a smaller range  $Tb$ . Hence if  $\delta\varpi_n, \delta\varpi_p, \dots$  are the variations which occur in passing from any one curve to an adjacent curve, we may reasonably expect that  $\frac{\delta\varpi_n}{\delta\varpi_p}$  will be large.

Passing beyond  $p$  the curve enters the region  $y < \eta$ . In this region the form of the curves is entirely unknown, but there are no grounds for supposing that the curves which have cut  $ST$  within a range  $bT$  will spread out so as to cut  $TU$  in a range comparable with  $TC$ . The supposition upon which we shall proceed is, that if we for the moment regard  $k_2/k_1$  as very great, then  $\frac{\delta\varpi_p}{\delta\varpi_q}$  will either be finite or will be of a lower order of small quantities than  $\frac{\delta\varpi_p}{\delta\varpi_n}$ .

Hence there are reasons for supposing that  $\frac{\delta\varpi_n}{\delta\varpi_q}$  will be

large. In other words, as we pass over the striations, moving towards the cathode, two adjacent curves will tend to approximate more and more closely with every stria passed over, until they may, by the time the cathode is reached, be indistinguishable.

The argument is reversible; two groups which are extremely close to one another at the cathode may diverge as we pass away from the cathode (the divergence increasing approximately in geometrical progression with every stria) until by the time the anodes of the two graphs are reached we have a divergence sufficient to give materially different values for the distance between the electrodes, the total fall of potential, &c.

§ 14. Hence, if the distance between the electrodes or the potential-difference between them be altered, the graph will tend to adjust itself at the anode rather than at the cathode. If there is, for any reason, a tendency for the discharge to take a certain form at the cathode, it will be possible for the discharge to approximate very closely to this form without using up any of the disposable constants of the graph, while the same is not true of the anode. The nature of the discharge near the cathode may therefore reasonably be expected to depend principally upon the physical conditions which obtain at the cathode, while the discharge at the anode will depend less upon the state of the anode than upon the distance and potential-difference between the two electrodes.

§ 15. These considerations seem to suggest an explanation of the observed constancy in the appearance at the cathode, as well as of certain other experimental results. For instance, Sir W. Crookes, using as cathode a metal electrode one-half of which was coated with lampblack, found that the dark space opposite the lampblack was longer than that opposite the metal, showing the dependence of the negative discharge upon the state of the cathode\*. At the same time he found that the appearance of the negative end of the discharge was independent of the position of the anode†.

Goldstein, using two movable electrodes, observed that whichever electrode was moved, the striæ and the negative dark space behaved as though they were rigidly connected with the cathode‡.

§ 16. Let us now try to form some conception of the state of things which must, upon our theory, occur in a vacuum-

\* Crookes, Phil. Trans. 1879, p. 137, § 491.

† *Loc. cit.* § 489.

‡ Wied. Ann. xii. p. 273. In this connexion see also Graham, Wied. Ann. lxiv. p. 71.

tube. Consider for the moment an ideal discharge in which the velocity of every individual positive ion is  $k_1\bar{X}$ , and the velocity of every negative ion is  $k_2\bar{X}$ . Let us further suppose that the equations of § 3 are absolutely true at every point, and that the distribution of electric force is that given by the ideal mathematical graph of fig. 5, these suppositions not being inconsistent with one another.

The volume-densities of ions are now given at every point of the tube by the equations of § 3—

$$n_1 = \frac{1}{(k_1 + k_2)e\bar{X}} \left\{ i + \frac{k_2}{4\pi} \frac{dy}{dx} \right\},$$

$$n_2 = \frac{1}{(k_1 + k_2)e\bar{X}} \left\{ i - \frac{k_1}{4\pi} \frac{dy}{dx} \right\}.$$

We shall accordingly meet the following distribution of densities as we pass along the tube, starting from the cathode E (fig. 7). The volume-density of positive ions  $n_1$  will be finite at E, that of negative ions  $n_2$  will be zero. Passing along ED both  $n_1$  and  $n_2$  increase, until finally  $n_1$  becomes infinite at D, the first discontinuity, while  $n_2$  has a finite value. In passing D an abrupt change of densities occurs, and on the further side of D and close to D,  $n_1$  is finite while  $n_2$  is infinite. As we pass along DC,  $n_1$  increases and  $n_2$  decreases, so that as we approach C the same state is reached as occurred at D,  $n_1$  becomes infinite and  $n_2$  finite; while the same abrupt change occurs in passing through C as occurred at D, and so on.

The velocity of ions of both sorts becomes zero at D, C . . . , so that at these points we get a wall of positive ions in contact with a wall of negative ions, both faces being at rest. In the ideal case there is a succession of such "slabs" of ions. The slabs are at rest, and ions pass from slab to slab with finite velocities. The discharge may therefore be regarded as a series of small discharges between consecutive slabs.

In nature the conditions are different. The velocities  $k_1\bar{X}$ ,  $k_2\bar{X}$  are not velocities of individual ions, but measure the mean forward velocity of a swarm of ions, the velocities of individual ions varying both in amount and direction. Hence it is impossible that a "slab" of ions such as we have been considering should exist in nature, and even if this were not so, such a slab would be immediately broken up by the bombardment of undissociated molecules.

Hence it becomes clear that the accumulation of ions which occurs in the neighbourhood of a point of minimum

electric force must consist of a great number of ions which are not at rest, but are moving equally in all directions. This accumulation of ions will be more accurately described as a "nucleus" than as a "slab." There is no longer a sudden discontinuity of densities in the middle of such a structure; this discontinuity must be replaced by a rapid but continuous change. The number of ions of either kind in a nucleus is being continually increased by arrivals from the adjacent nuclei, and also by the dissociation of some of the molecules which pass into it in the course of their motion; the number is at the same time decreased by the ions which are carried away from the nucleus, and also by the recombination of ions inside the nucleus. When the steady state is reached the losses and gains arising from these four causes exactly balance.

It will be noticed that the view which we have arrived at, and which regards the discharge as a series of smaller discharges, is almost identical with that put forward by Spottiswoode and Moulton, and others\*.

*Comparison of Theory with Experiment.*

§ 17. The theory which has been developed attempts to predict the arrangement of light and darkness in a discharge-tube, so that a comparison of these predictions with the arrangement actually observed will, to a certain extent, supply a test of the truth of the theory. A second and more searching test is afforded by a comparison of a theoretical graph with graphs plotting out the result of actual measurement. A series of measurements of the intensity of electric force at different points of a discharge-tube has been made by Graham †, and there is a second and more recent series by Mr. H. A. Wilson ‡. Both experimenters have given their results in the forms of graphs for  $X$ , so that comparison will be facilitated by imagining the theoretical graph for  $y$  changed into a graph for  $X$ .

We are only concerned with the main features of the theoretical graph for  $y$ , and it is obvious that these will be reproduced in the graph for  $X$ . In particular, the maxima and minima of the two graphs will correspond, although the same is not true of points of inflexion. Unlike the graph for  $y$ , the graph for  $X$  will always meet the axis of  $x$  at right

\* Spottiswoode and Moulton, *Phil. Trans.* 1880, p. 201. See also J. J. Thomson, *Recent Researches*, § 217; E. Goldstein, *Phil. Mag.* x. pp. 178-190; A. Schuster, *Phil. Mag.* xlvii. p. 554.

† W. P. Graham, *Wied. Ann.* lxiv. p. 49.

‡ H. A. Wilson, *Phil. Mag.* xlix. p. 505.



angle In the neighbourhood of a point at which  $X$  vanishes (a nucleus), the graph for  $X$  will approximate to the upper halves of two parabolas, the axes of which coincide with the axis of the graph. The latera recta of the parabolas will be in the ratio of  $k_1$  to  $k_2$ , the concavity of the larger being turned towards the cathode.

§ 18. Let  $\xi$  be the value of  $X$  which corresponds to  $\eta$ , the critical value of  $y$  below which the differential equation (8) ceases to hold. If we can find out in what way  $\xi$  depends upon the pressure and current, we shall have obtained some indication of the closeness of approximation which is to be looked for.

Let  $N_1$  be the large value of  $n_1$  corresponding to the value  $\xi$  of  $X$ . At the point at which  $n_1$  attains this value,  $dy/dx$  will be very nearly equal to  $4\pi i/k_1$ , so that we have approximately, by equation (5)

$$N_1 e = \frac{1}{(k_1 + k_2)\xi} \left\{ i + \frac{k_2}{4\pi} \cdot \frac{4\pi i}{k_1} \right\} \\ = \frac{i}{k_1 \xi}.$$

The value of  $\xi$  is therefore seen to be  $\frac{i}{k_1 N_1 e}$ , and since  $k_1$  may be supposed to vary inversely as  $p$ , the pressure of the gas, this will be proportional to  $\frac{pi}{N_1}$ . If we suppose that the critical value  $\xi$  always corresponds to the same volume density of ions, we shall have for the relation between  $\xi$ ,  $p$ , and  $i$ ,

$$\xi = Cpi,$$

where  $C$  is a constant so long as we are dealing with the same gas.

We must therefore expect that the value of  $X$  below which the experimental graph begins to appreciably differ from the ideal mathematical graph, will decrease as the product  $pi$  decreases. The graphs which approximate most closely to the ideal form demanded by theory, ought to be those for which this product has the smallest value, and an inspection of the graphs of Graham and Wilson seems to show that this is the case.

§ 19. A further cause of disagreement between the graphs of theory and observation may perhaps be found in the intermittence of the current. The theoretical graph gives the distribution of potential-gradient on the assumption that the current is steady throughout. If the current is in any way

variable or intermittent, the density of ions will not have sufficient time to take up its equilibrium distribution for each small flow of current, and will therefore oscillate about a mean which is intermediate between the theoretical distribution for the maximum current and the distribution for zero-current. Hence the graph for  $X$  or  $X^2$ , if plotted from the readings of an electrometer, will be intermediate between the graphs found in this paper and a straight line.

Thus intermittence in the current effects a smoothing out of the graphs, and in particular, the minima of the graphs will be much less marked. In connexion with this result, it ought to be noticed that perfect steadiness of the current in the part of the circuit outside the tube will not necessarily involve the steadiness of the current at all points inside the tube.

§ 20. The results which have been reached have been entirely qualitative. It would, however, be natural to inquire within what limits of pressure, current, &c., a striated discharge is possible, and what will be the length and number of the striæ corresponding to specified external conditions. To these and all similar questions our theory has supplied no answer.

If we refer back to the mathematical graph of fig. 5, we see that given the pressure, the current, and the distance between the electrodes, there are still an infinite number of solutions mathematically possible, so that the original mathematical solution can throw no light on the arrangement of striæ: all arrangements are equally possible.

When we turn to the physical graph of fig. 6, this is no longer the case. The three elements specified above will completely determine the graph; there are no points in the physical graph corresponding to the discontinuities in the mathematical graph, at which a choice of ways is open to us. This definiteness has been effected by the introduction of the new intermolecular forces which are supposed to come into play when the density of ions reaches a certain amount. Until, therefore, we have a definite knowledge of the nature of these forces, it will be impossible to arrive at an exact knowledge of the relations between quantities which, except for these forces, would be meaningless.

In conclusion, I wish to express my deep indebtedness to Prof. Thomson, not only for the continual use which I have made of his published papers, but also for the assistance I have received from him personally.