

point out to the charitably disposed that there are a number of desiderata: there are, for example, no specimens of either the African or the American "Fin-foots."

LETTERS TO THE EDITOR.

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The Line Spectra of the Elements.

THE proper replies to Prof. Runge's letter in last week's issue of NATURE are three in number: viz. (1) that, as I pointed out in my former letter (NATURE of May 12, p. 29), the reasoning in my paper is valid if, as I there proved and as Prof. Runge now admits in the first sentence of his letter, Fourier's theorem can be applied to motions which approximate to non-periodic motions in any assigned degree and for any assigned time; (2) that I am not aware of anything I have written which countenances Prof. Runge's supposition that "Prof. Stoney has not noticed that a distinct property of the function is wanted in order to get a proper" [rather, a mathematically accurate] "resolution into a sum of circular functions"; and (3) that Prof. Runge is mistaken when he supposes that "the amplitudes and periods" [or frequencies] "of the single terms . . . do not approach definite values when the interval" [i.e. the periodic time of the recurrence required by Fourier's theorem] "increases indefinitely."

What the true state of the case is, is most easily shown as regards the frequencies of the lines; and as the proof is, I believe, new, and leads to a result of importance in the interpretation of spectra, I subjoin it.

Take a motion of the electron—

$x =$ The sum of partials such as $\left(a \sin \frac{2\pi kt}{j} + b \cos \frac{2\pi kt}{j} \right)$; (1)

with similar expressions for the other two co-ordinates; in which the oscillation-frequencies, the k 's, may be commensurable with one another, or incommensurable. If incommensurable, the motion is non-recurrent. Let this motion be arrested at intervals of T , and immediately started afresh as at the beginning. We thus obtain a recurrent motion consisting of a certain section of the motion (1) repeated over and over again. This new motion can be analyzed by Fourier's theorem, and we have to inquire what we thus obtain. Without losing anything in generality, we may confine our attention to the motion parallel to the axis of x , and to the single partial of that motion which is written out above, as all the partials lead to similar results.

Let us then examine by Fourier's method the motion which is represented by the equation—

$$x_k = a \sin \frac{2\pi kt}{j} + b \cos \frac{2\pi kt}{j} \dots \dots (2, a)$$

from $t = 0$ till $t = T$, and which is repeated from that on at intervals of T . If T is a multiple of j/k , Fourier's theorem simply furnishes equation (2, a) as the complete expression for all time of the motion; so that in this case it indicates the same definite line in the spectrum as is furnished by the original partial of equation (1).

If T is not a multiple of j/k ,

$$T \text{ will be } (m + \alpha) \frac{j}{k},$$

where m is a whole number and α a proper fraction. Equation (2, a) then becomes

$$x_k = a \sin \frac{2\pi(m + \alpha)t}{T} + b \cos \frac{2\pi(m + \alpha)t}{T} \dots \dots (2, b)$$

which is true from $t = 0$ till $t = T$, after which the motion is to be repeated. Then, by Fourier's theorem—

$$\left. \begin{aligned} x_k = & A_0 + A_1 \sin \frac{2\pi t}{T} + A_2 \sin \frac{4\pi t}{T} + \dots \\ & + B_1 \sin \frac{2\pi t}{T} + B_2 \sin \frac{4\pi t}{T} + \dots \end{aligned} \right\} (3)$$

NO. 1180, VOL. 46]

is true of this motion for all time, in which

$$\begin{aligned} A_n \cdot \int_0^T \sin^2 \frac{2\pi nt}{T} \cdot dt &= a \int_0^T \sin \frac{2\pi(m + \alpha)t}{T} \cdot \sin \frac{2\pi nt}{T} \cdot dt \\ &+ b \int_0^T \cos \frac{2\pi(m + \alpha)t}{T} \cdot \sin \frac{2\pi nt}{T} \cdot dt; \\ B_n \cdot \int_0^T \cos^2 \frac{2\pi nt}{T} \cdot dt &= a \int_0^T \sin \frac{2\pi(m + \alpha)t}{T} \cdot \cos \frac{2\pi nt}{T} \cdot dt \\ &+ b \int_0^T \cos \frac{2\pi(m + \alpha)t}{T} \cdot \cos \frac{2\pi nt}{T} \cdot dt; \end{aligned}$$

which, when integrated, give the following values for A_n and B_n —

$$\begin{aligned} A_n &= \frac{a \sin 2\pi\alpha - b(1 - \cos 2\pi\alpha)}{2\pi} \left(\frac{1}{d} - \frac{1}{s} \right) \\ B_n &= \frac{a(1 - \cos 2\pi\alpha) + b \sin 2\pi\alpha}{2\pi} \left(\frac{1}{d} + \frac{1}{s} \right) \dots \dots (4) \end{aligned}$$

where d stands for $(m - n + \alpha)$, and s for $(m + n + \alpha)$.

This furnishes a very remarkable spectrum, a spectrum of lines that are equidistant on a map of oscillation-frequencies, and that extend over the whole spectrum. But they are of very unequal intensities. If T is a long period, m is a high number. The lines are then ruled close to one another, and their intensities are insensible except when n is nearly equal to m , the two brightest lines being the next to the position of the original line of equation (1), one on either side of it, and the others falling off rapidly in brightness in both directions.

If we take a longer period for T , m becomes a still higher number; the lines are more closely ruled and are more suddenly bright up to those on either side of the position of the original line of equation (1), to which also they are now closer; so that, at the limit, when T increases indefinitely, equation (3) becomes a mathematical representation of the original line of equation (1).

This interesting investigation is all the more important as it gives a clue to how rulings of lines which are equidistant and brighter up to the middle may arise; and I feel sure that Prof. Runge will join me in not regretting that he expressed the doubts which led to its solution.

G. JOHNSTONE STONEY.

9 Palmerston Park, Dublin, June 3.

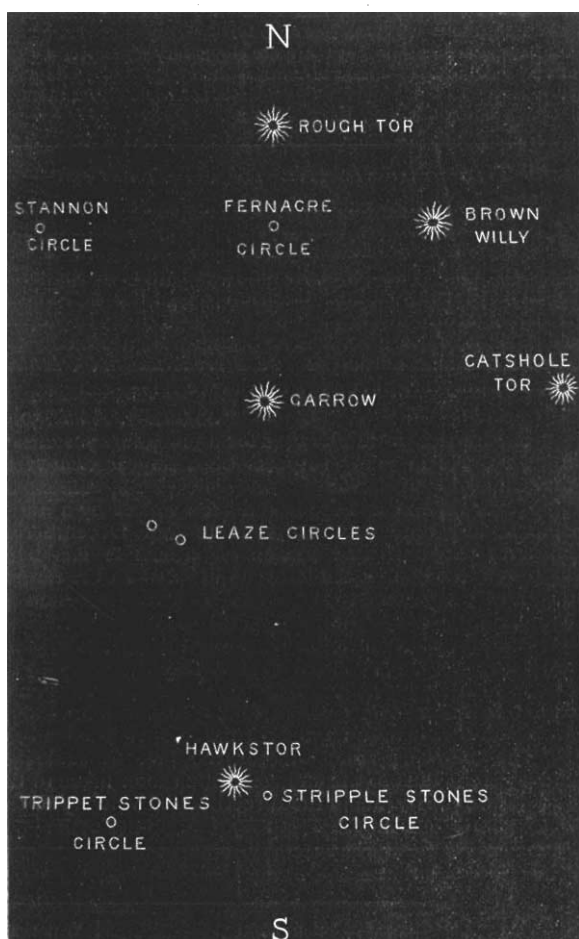
Stone Circles, the Sun, and the Stars.

ARTICLES by Mr. Norman Lockyer and Mr. Penrose, recently published in NATURE, have dealt with the positions of ancient Egyptian and Greek temples with relation to the rising sun, and to the pole star, or some star or stars in its vicinity. For some years past I have endeavoured to show, in papers read before the British Association and other Societies, that our stone circles had a relation to the rising sun, indicated usually by an outlying stone or by a notable hill-top in the direction in which the sunrise would be seen from the circle, and I have in some cases found similar indications towards the north, which may have referred to the pole or other northern star or stars. A paper containing many details as to these cases will shortly appear in the Journal of the Royal Archaeological Institute.

There are six circles on Bodmin Moors, which at first sight appear to have no relation to each other, but which, if the 6-inch Ordnance map is to be relied upon, would seem to have been arranged on a definite plan (see accompanying plan).

The Stannon and Fernacre Circles are in line 1° (true) north of east with the highest point of Brown Willy, the highest hill in Cornwall; and the Stripples Stones and Fernacre Circles are in line with the summits of Garrow and Rough Tor, at right angles with the other line—namely, 1° west of (true) north. A line from the Trippet Stones Circle to the summit of Rough Tor would also pass through the centre of one of the Leaze Circles (about 12° east from true north). Other hills are in the direction of the rising sun. The Trippet Stones are $11\frac{1}{2}^\circ$ south of west from the Stripples Stones, 10° east of south from the Stannon Circle, and about 13° west of south from the Fernacre Circle. The respective bearings of the other circles have already been given, and all are true (not magnetic) according to the 6-inch Ordnance map.

More remarkable, perhaps, than the position of these circles are their distances from each other, which, on the level map, are almost exactly as 3, 7½, 2, and 8, for the sides of the irregular four-sided figure, of which four of the circles form the corners, while the diagonals are of the same length within a hundred feet, the differences being much less than the 1 per cent. which Mr. Flinders Petrie has found to be the average error of ancient British and even Assyrian workmanship. The builders of these



circles may be supposed to have aimed in their measurements at even numbers of some unit, and the unit which gives the best results appears to be a Royal Persian or Egyptian cubit of 25.1 inches (not at all the unit one would expect). The actual measurements, as nearly as I can get them from the 6-inch Ordnance map, are:—

	Feet.	Cubits of 25.1 inches.
Stannon Circle to Fernacre Circle ...	6275	= 3000
Fernacre Circle to Stripples Stones ...	15730	= 7520
(Practically 7500 cubits)		
Stripples Stones to Tripple Stones ...	4180	= 1998.4
(Practically 2000 cubits)		
Tripple Stones to Stannon Circle ...	16575	= 7924
(Probably meant for 8000 cubits)		

Diagonals.

Fernacre Circle to Tripple Stones ...	16950	= 8103
Stannon Circle to Stripples Stones ...	16350	= 8055
(Perhaps meant for 8125 cubits)		

It must not be forgotten that these measurements are taken from the level map, while the ground between the circles is very irregular, but it seems more probable that the builders of these circles made allowance for the irregularities of the ground than

that the distances, as shown by the map, are merely the result of accident.

If, however, the 25.1 inch cubit were the unit of measurement for the distances between the circles, it ought to appear in the measurements of the circles themselves—and it does; for the diameter of the Trippet Stones Circle is exactly fifty of such cubits, and the diameters of the Fernacre and Stripples Stones Circles are (as nearly as I can judge in their ruinous condition) seventy of such cubits.

The Egyptians appear to have constructed separate buildings for the observation of the sun and of the stars, but if the circle builders used the same circles for both purposes, placing them so that when standing in them they could see the sunrise over a fixed point on one hill, and a certain star rise over a fixed point on another hill in another direction, their system was much more economical, though perhaps less exact than that of the more civilized Egyptians.

A. L. LEWIS.

The Height of the Nacreous Cloud of January 30.

THE cloud referred to by Mr. T. W. Backhouse, in your issue of February 18 (p. 365), attracted universal attention over Eastern Yorkshire and even in Lincolnshire, so that numerous letters were sent to the *Leeds Mercury*, *Yorkshire Post*, &c.

Its general appearance in these parts is being dealt with by Mr. H. Bendelack Hewetson, of Leeds. I will therefore merely state that the intensity of the fringes surpassed anything in my previous experience. Even those observed in 1884-85, in connection with the Krakatō glows, did not approach it in this respect.

Here, as it happened, Venus lay just upon its lower edge. As this was fairly horizontal, save for a break not far from the middle, I was very pleased to get from Mr. C. J. Evans, Ackworth, near Pontefract, a second observation. In response to further appeals, observations, of varying accuracy, from the places in the subjoined table, were available for calculating the height.

Leeds (Prof. A. Lupton saw the cloud "nearly overhead," "if not, a little S.S.W.," "within 20° of the vertical"), Ackworth, and York enable us to determine the direction and position of the central part of the lower edge of the cloud as seen from the north and east. They give a point just above Mirfield Junction, 9 miles S.S.W. from Leeds, 32 miles S.W. from York and 13½ miles W. of Ackworth. The Driffield direction passes the same spot. The only other record, Market Rasen, is very divergent, the observer there putting the centre point south of west, whereas Mirfield is north of west.

The altitude of this point was capable of closer determination, thanks to references to Venus and Jupiter. The results of careful reductions are given in degrees in the following table, accompanied by the corresponding heights in miles, and distances and directions from Mirfield.

Place.	Miles from Mirfield.	Direction from Mirfield.	Altitude in degrees.	Resulting height above Mirfield.	Observer.
Ackworth ...	13½	E.	40° (245°)	11 (213½)	C. J. Evans, &c.
Tadcaster ...	22	N.E.	35°-40°	15	C. Rawson.
York ...	32	N.E.	22½°	13½	J. E. Clark.
Wetwang ...	50	E. N.E.	16½°	15½	E. M. Cole.
Driffield ...	56	E. N.E.	10°-15°	16	J. Lovell.
Hull ...	54½	E. by N.	12°	13	S. Bowen.
Market Rasen ...	58	E.S.E.	16°	15	W. Whiteman.
Sunderland ...	88	N. by E.	7½°	14	T.W. Backhouse.

The adopted height from the above eight records is 14 miles, or 75,000 feet.

The Leeds record gives 25 miles, but Prof. Lupton wrote only from memory, in response to inquiry after some interval. It may, however, indicate that the south-west edge of the cloud was more nearly above Leeds. By shifting it 4 miles in that direction the Leeds height becomes 16 miles, and that for Ackworth 11½ miles, Tadcaster 12½, York 12, and the rest, except Market Kasen (unchanged), become half a mile less. The mean result, taken as before, but now including Leeds, is 13.6 miles, or substantially the same.

1 As the records at these stations appeared to be much the more trustworthy, double weight is given them in the reduced value. In the Ackworth value, four times the height is assigned to the 40° as to the 45°, the latter, on inquiry, being stated to be less probable.