



XXII. Researches in Acoustics.—No. X

Alfred M. Mayer

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XXII. *Researches in Acoustics*.—No. X.

By ALFRED M. MAYER*.

[CONTENTS.—The Variation of the Modulus of Elasticity with Change of Temperature, as determined by the Transverse Vibration of Bars at Various Temperatures. The Acoustical Properties of Aluminium.]

Summary of the Research.

POISSON, in his *Traité de Mécanique* (Paris, 1833, t. ii. pp. 368–392) † discusses the laws of the transverse vibrations of a bar free at its ends and supported under its two nodes. He shows that the frequency of the vibrations of the bar is given by an equation, which, reduced to its simplest expression, is $N = V \times 1.0279 \frac{t}{l^2}$; in which N is the number of vibrations per second of the bar, t its thickness, l its length, and V the velocity of sound in the direction of the length of the bar.

To ascertain how nearly the frequency of the transverse vibrations of a bar, computed by Poisson's formula, agrees with the result obtained by experiment, the following method of experimenting was used.

Rods of steel, aluminium, brass, glass, and of American white-pine (*Pinus Strobus*)—substances differing greatly in their moduli of elasticity, densities, and physical structures—were carefully wrought so as to have the length of $1.5 \pm$ metre, the thickness of 0.5 cm., the width of 2 cms., and a uniform section throughout their lengths. The velocity of sound in these rods was determined by vibrating them longitudinally at a temperature of 20° , while held between the thumb and forefinger, and their frequencies of vibration ascertained by the standard forks of Dr. R. Koenig's tonometer.

Out of each of these long rods were cut three bars of the length of 20 cms., and these bars, also at 20° , were supported on threads at their nodes, vibrated transversely by striking them at their centre with a rubber hammer, and their frequencies of vibration determined by the forks of the tonometer.

The mean departure of the observed from the computed numbers of transverse vibrations (see Table I.) is $\frac{1}{328}$; the computed frequency being always in excess of the observed,

* From an advance proof from the American Journal of Science for February 1896 communicated by the Author, having been read before the British Association at Oxford, August 1894.

† See also 'The Theory of Sound,' by Lord Rayleigh, 1894, vol. i. chap. 8.

except in the case of glass, where the computed is $\frac{1}{275}$ below the observed frequency.

In Table I. l =length and t =thickness of bar in centimetres at 20° ; V =velocity of sound in centimetres in bar at 20° ; N =number of vibrations per second at 20° .

The close agreement of the computed and observed values shows that, by vibrating a bar at various temperatures, the variation of its modulus of elasticity with change in its temperature can be obtained. We observe N at various

temperatures of the bar; then $V = \frac{N}{1.0279 \frac{t}{l^2}}$ is computed,

and the modulus $M = \frac{V^2 d}{g}$. As t , l , and d (the density of the bar) vary with the temperature, the coefficient of expansion of each bar and its density at 4° were determined, so that the dimensions and density of the bar could be computed for each of the temperatures at which it was vibrated.

Experiments were made on five bars of different steels, on two of aluminium, on one of St. Gobain glass, one of brass, one of bell-metal, one of zinc, and one of silver. The results of these experiments may be summed up as follows:—

The modulus of elasticity of St. Gobain glass is 1.16 per cent. less at 100° than at 0° .			
"	"	the five steels	" 2.24-3.09 " " " "
"	"	brass	" 3.73 " " " "
"	"	bell-metal	" 4.3 " " " "
"	"	aluminium	" 5.5 " " " "
"	"	silver	" 2.47 " " 60 "
"	"	zinc	" 6.04 " " 62 "

The decrease of the modulus of elasticity of glass, aluminium, and brass is proportional to the increase of temperature; straight lines referred to coordinates giving the results of experiments on these substances. The five steels, silver, and zinc give curves, convex upwards, showing that the modulus decreases more rapidly than the increment of temperature; while bell-metal alone gives a curve which is concave upwards, its modulus decreasing less than the increment of temperature. (See Curves, fig. 5, p. 185.)

The more carbon a steel contains, the less is the fall of its modulus of elasticity on elevating the temperature of the steel. Thus, the modulus of the steel with 1.286 per cent. of carbon is 2.24 per cent. less at 100° than at 0° , while the steel containing 0.15 per cent. of carbon has a modulus at 100° which is 3.09 per cent. lower than its modulus at 0° .

So far as experiments on a single steel containing nickel
Phil. Mag. S. 5. Vol. 41. No. 250. March 1896. N

permit of any general deductions, it appears that the presence of nickel in a low carbon steel lowers its modulus of elasticity. Thus, steels No. 3 and 4, having respectively $\cdot 47$ and $\cdot 51$ per cent. of carbon, have a modulus of 2130×10^6 ; while steel No. 5, containing $\cdot 27$ per cent. of carbon and 3 per cent. of nickel, has a modulus of 2080×10^6 , which is $2\cdot 35$ per cent. lower than that of steels Nos. 3 and 4.

The presence of nickel in a steel may, in a diminished degree, have the effect of carbon in lessening the lowering of the modulus when the temperature of the steel is increased. Thus the percentage of the lowering of the modulus, by heating from 0° to 100° , of steel No. 5 containing $0\cdot 27$ of carbon and 3 per cent. of nickel, is the same as that of steel No. 3 with $0\cdot 47$ per cent. of carbon.

If a bar of any one of the substances experimented on is struck with the same energy of blow, by letting fall on the centre of the bar a rather hard rubber-ball from a fixed height, the sound emitted by the bar diminishes in intensity and in duration as the temperature of the bar is raised. Thus :

Brass	at 0° vibrates during 75 secs. ; at 100° it vibrates during 45 secs.			
Bell-metal	"	"	55	" 15
Aluminium	"	"	40	" 12
J. & C. Cast Steel	"	"	80	" 5
Bessemer Steel	"	"	45	" $1\cdot 5$
St. Gobain Glass	"	"	6	" $3\cdot 5$

Zinc at 0° vibrated during 5 secs. ; at 20° only during $1\cdot 5$ sec. At 62° it vibrated for so short a time that it only gave three beats with forks of 1090 and 1082 v. s. At 80° it was not possible to determine the pitch of the bar, and at 100° the bar when struck gave the sound of a thud. The bar of silver acted in a similar manner to the bar of zinc—it was even less sonorous than zinc,—thus flatly denying the “silvery tones” attributed to it.

These phenomena do not depend on the fall of modulus, but on changes in the structure of the metal on heating it, which cause the blow to heat the bar and not to vibrate it.

Bell-metal was found to be an alloy peculiarly well suited for bells, as the intensity and duration of its vibrations were the same at 50° as at 0° , all other substances showing a marked diminution of intensity and duration of sound at 50° .

A bar of unannealed drawn brass, after it has been heated to 100° , has its modulus at 20° increased $\frac{36}{100}$ per cent. (See Table III. and fig. 11, p. 188.)

In this research I had the good fortune to have the assistance of Dr. Rudolph Koenig, of Paris. He not only placed at my service the resources of his laboratory and

workshop, but generously gave me constant assistance during the experiments—making the determinations of the numbers of vibration of the rods and bars with the standard forks of his tonometer. Without his aid this work could not have been done. For instance, in the cases of the bars of silver and zinc the beats they give with a fork are so few that they cannot be compared with a chronometer; but Dr. Kœnig, from his long experience in the estimation of beats, was enabled to form an accurate judgment of their number per second from the *rhythm* of the beats. The determination of pitches extending through such a range of vibrations as occurs in this research can only be made with Dr. Kœnig's "grand tonomètre"—a unique apparatus of precision, giving the frequency of vibrations from 32 to 43690 v. s., and really indispensable to the physicist who would engage in precise quantitative work in Acoustics.

We now proceed to give accounts of the several operations performed in the progress of this research.

Determination of the Velocity of Sound in Rods.

In the determinations of the velocity of sound in the rods of 1·5 m. in length, I used the method of Chladni*. Kundt's method of obtaining nodal lines of fine powders in a tube, by vibrating a rod whose end carries a cork which fits loosely the end of the tube, is not accurate. The weight and friction of the cork, the necessity of firmly clamping the rod at a node, and, above all, the want of knowledge of the velocity of sound in the air of glass tubes of different diameters, renders this method, so beautiful and ingenious, worthless for accurate measures of the velocity of sound in solids.

The curves in fig. 1 show the very diverse determinations of the velocity of sound in the air in tubes of different diameters by the physicists Kundt†, Schneebeli‡, Seebeck§, and Kayser||. The velocity of sound in metres is given on the axis of Y; the diameter of the tube in centimetres on the axis of X. Ku stands for Kundt, Sch for Schneebeli, Se for Seebeck, and Ke for Kayser. The most precise measures of velocities are those of Kayser, who closed the end of the tube with a cork attached to the end of a steel bar, while the other end of the bar was securely clamped. The frequency of the transverse vibrations of the bar was registered by a style

* *Traité d'Acoustique*, Paris, 1809, p. 318 *et seq.*

† *Bericht. der Akad. der Wiss. zu Berlin*, 1867.

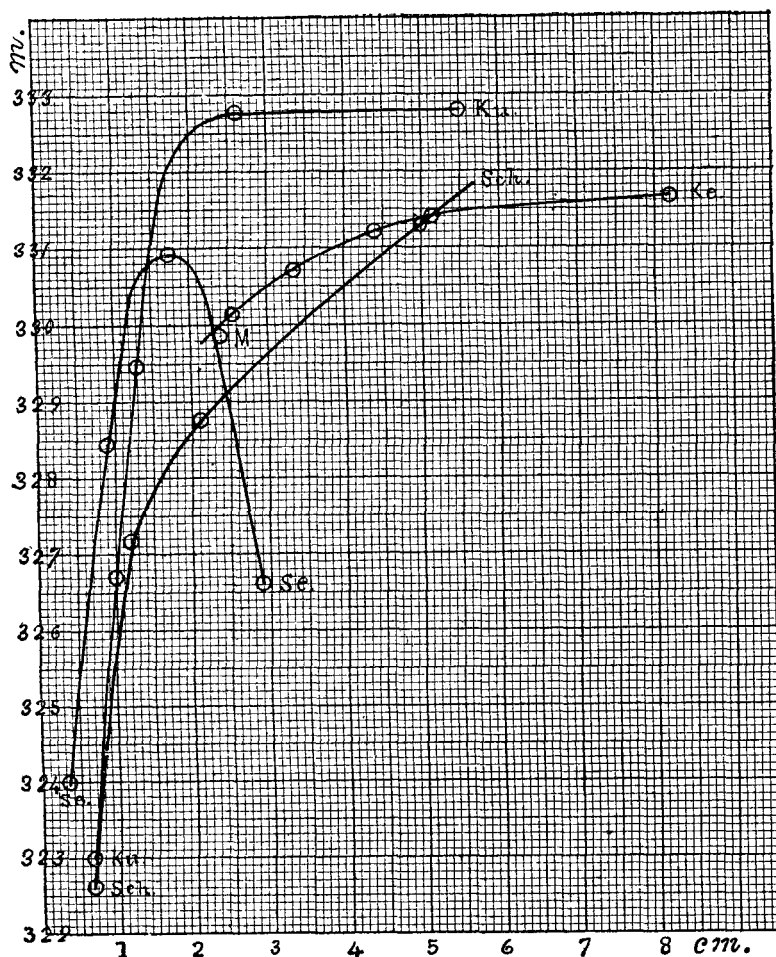
‡ *Pogg. Ann.* 1869, t. 136.

§ *Pogg. Ann.* 1870, t. 139.

|| *Pogg. Ann.* 1877, t. 2. p. 218.

describing the sinusoids of the vibrating bar. Thus the weight and friction of the cork introduced no error. In a similar manner I obtained the velocity, marked M in fig. 1, by vibrating a rod of aluminium. The frequency of the

Fig. 1.



longitudinal vibrations of the rod was measured while the cork at the end of the rod was vibrating in the mouth of the tube. The result agrees closely with Kayser's. It is needless to discuss the curves of fig. 1.

The method of Chladni, used exactly as that eminent man used it, remains the best we have. It is important, however, to note that the rod must be held between the thumb and forefinger when it is vibrated and not *clamped* when vibrated. When clamped it always gives a higher frequency, as shown by the following experiments :—

Steel rod clamped	3429.2
Steel rod held between fingers . .	3428.4
Aluminium rod clamped	3377.0
Aluminium rod held between fingers.	3376.4

The frequency of the vibrations of the rods of steel, brass, aluminium, glass, and pine wood, when held at the middle of their lengths and vibrated so as to give their fundamental tones, gave exactly the octaves of these fundamental tones when held at one-quarter of their lengths and vibrated.

Determination of the Lengths of the Long Rods and of the Lengths and Thicknesses of the Bars.

The lengths of the rods of $1.5 \pm$ metres were ascertained by comparison with the rod of steel whose length was measured at the Bureau International des Poids et Mesures. The lengths and thicknesses of the bars which were vibrated transversely were measured with micrometer calipers. The readings of these calipers were tested by comparison at 20° with a series of standards of various lengths of inches and fractions of inches, made for me with great care by Mr. George M. Bond, who has charge of the gauge department of the Pratt, Whitney Co. In reducing the comparisons to centimetres, I adopted the value of the inch as equal to 25.4 millimetres. In obtaining the length of a bar, the mean of several measures in the axis of the bar and in directions parallel to the axis and at various distances from it was adopted. The thickness of a bar was taken as the mean of measures taken throughout the length of the bar at points designated by the intersections of lines drawn parallel and at right angles to the axis of the bar.

The dimensions of the bars were measured at 20° , except those of steels Nos. 3, 4, 5, which were measured at $18^{\circ}25'$.

Determinations of the Coefficients of Expansion of the Bars.

To determine the coefficients of expansion of the bars, I devised the apparatus shown in fig. 2. In a brass tube, T,

The diagram illustrates two electrostatic machines, M and M', used in an experiment. Machine M consists of a vertical plate A, a horizontal plate B, and a curved plate C. Machine M' consists of a vertical plate A', a horizontal plate B', and a curved plate C'. Both machines are connected to a common circuit. The circuit includes a battery F, a resistor R, and a galvanometer G. The connections are as follows: the bottom of plate A is connected to the positive terminal of the battery F; the bottom of plate A' is connected to the negative terminal of the battery F; the top of plate B is connected to the positive terminal of the battery F; the top of plate B' is connected to the negative terminal of the battery F; the top of plate C is connected to the positive terminal of the battery F; the top of plate C' is connected to the negative terminal of the battery F. The galvanometer G is connected in series with the battery F and the resistor R.

pressure is further increased by flat rings which surround the holes in the washers, and are pressed against these washers by means of the springs, D, D'. By this arrangement the surfaces of the ends of the bars are exposed, while the contact of the washers on the bars makes a water- and steam-tight joint. Thus the bar may be surrounded with ice, or with steam, or with a current of water of different temperatures, and be cooled or heated up to its terminal planes, while the holes in the washers allow the micrometer-screws, M, M', to be brought to contact with the terminal planes of the bar. Two helical springs are attached to the column A. The other ends of these springs are fastened to rods projecting from the tube T. Thus the same pressure of contact is always made between the bar and the end of the micrometer-screw M. The tube T is supported in Vs, V, V', and the greater part of

the weight of the tube is taken off the Vs by helical springs fastened to a frame above the apparatus. The tension of these springs can be so regulated that the tube rests on the Vs with the same pressure when it has steam passing through it and when it is filled with ice. The column, A, and the Vs, V, V', are insulated from the base of the apparatus by thin plates of ebonite, *e*. Between the binding-screws, E and E', and connected by wires, are the voltaic cell F, the galvanometer G, and a box of resistance-coils, R. The micrometer-screw, M', with which the variations in length of a bar are measured, is mounted as follows:—The screw passes through its nut in a massive brass plate which rotates around nicely fitted centres at H. These centres are supported by two side plates not shown in the figure. A spring, K, is fastened to the lower part of the swinging nut-plate and brings this plate against the plate, L, firmly fastened to the base of the apparatus. When the swinging plate is vertical and the axis of the screw horizontal, the swinging plate fits accurately the surface of the fixed plate, L. By turning the rod, N, the swinging plate and its screw can be rotated away from the bar. This arrangement allows the screw to be swung out of the way while the tube, T, is being placed in the Vs. Also, it prevents any strain between the micrometer-screw, M', and the column, A; which would take place if M' were fixed and it should be brought in contact with a hot bar in the tube, T.

With careful manipulation, successive electric-contacts can be made on a bar in the tube, T, surrounded by ice, so that the variations in a series of measures will not exceed $\frac{1}{2000}$ mm., with a resistance of about 200 ohms placed in the circuit.

It may be reasonably objected to this apparatus that when the micrometer-screw touches the bar at 0° it is cooled and shortened, and that when it touches the bar at 100°, or at temperatures higher than that of the screw, the latter is heated and elongated. This error, however, is quite small, and may be neglected in our work. If we assume that one centimetre of the screw is heated 10°, which is a large estimate, considering the duration of contact of screw and bar during a measure, the shortening or elongation of 1 cm. of the screw by cooling or heating it 10° amounts to only $\cdot 0012$ mm., or $\frac{1}{166666}$ of the length of the bar. This change in the length of the screw will affect the coefficient of expansion of the bars only $\cdot 00000006$.

Determination of the Densities of the Bars at 4°.

The bar whose density was to be determined was immersed in water at 4° for a couple of hours. The bar was then suspended by a platinum wire in water at 4° and weighed. The bar was then removed from the wire and a quantity of water equal in volume to the volume of the bar was added to the water in the vessel, and the platinum wire, now immersed exactly as it was when the bar was attached to it, was weighed. This weight, subtracted from the previous weighing, gave the weight of the bar in water. Every precaution was taken to prevent, by means of screens, the action on the balance of the currents of cold air in the balance-case, which are produced by the constant descent of air from the sides of the cool vessel.

The Apparatus in which the Bars were Heated and Cooled. On the precautions used so that one is sure of having the real temperature of the bar when it is vibrated.

The apparatus used to heat and cool the bar is shown in fig. 3. In a brass box, C, is inclosed a box, C', containing the bar, B, supported on its nodes, N, N, by threads held by upright rods. From this central box two tubes, T, P, pass through the outer box C. The inner box is made water-tight and steam-tight by a rubber washer which is pressed between the top of the box and its cover by means of screws. Through the tube, T, the bar is vibrated by letting fall upon its centre a rubber ball fastened to a light wooden rod. On the blow of the ball it rebounds, and the rod is caught by the fingers in its upward motion. The cork is then at once replaced in the tube, T. The sound from the bar is conveyed to the ear, at E, by means of a tube (fig. 4). One branch of this bifurcated tube leads through a rubber tube to the pipe, P, of the box, fig. 3. The other branch leads to the fork, F, the number of whose beats per second made with the vibrating bar is measured by a chronometer. The pipe, S, allows the steam to issue when water is boiled in the box, C', by a gas lamp. The flow of gas through this lamp was neatly regulated by a stop-cock turned by a long lever. The box, C, is covered, except at the bottom, with thick felt.

To determine the frequencies of vibration of a bar through a range of temperature from 0° to 100°, the following method was used. The box, C, was filled with ice, surrounding the inner box, C'. It thus remained for an hour so that the boxes were cooled down to 0°, and the moisture in the inner

Fig. 3.

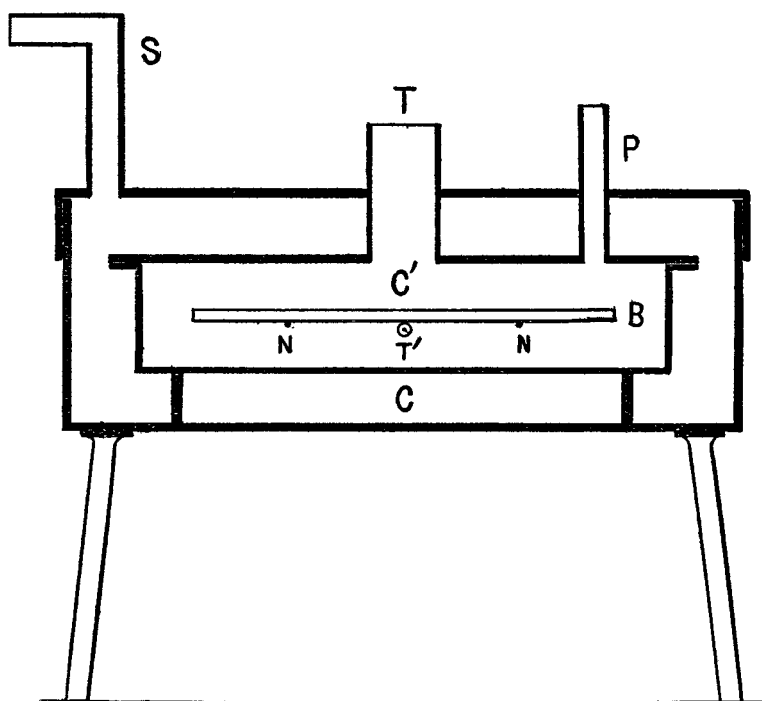
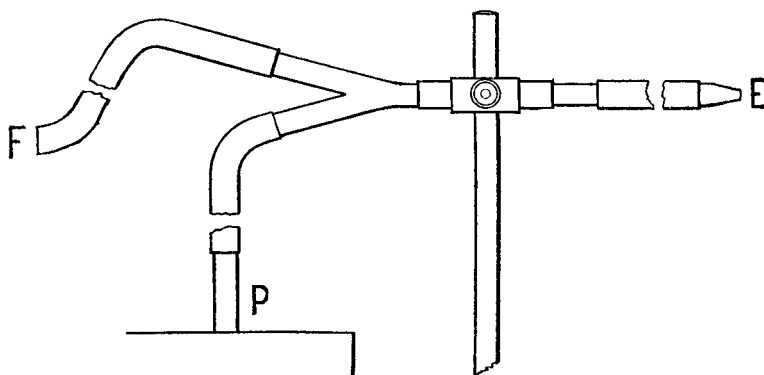


Fig. 4.



box had been condensed so far as it could be at 0° . The bar, which had been in ice for two hours, was wiped dry and quickly introduced into the inner box. A thermometer, T (made by Baudin and corrected), which entered the boxes through stuffing-boxes, and whose bulb touched the under surface of the bar, was read till it became stationary. The bar was now vibrated, and its frequency of vibration determined for the temperature given by the thermometer.

The lamp was now placed under the box, and the water in it boiled till the thermometer reached its maximum reading and the reading remained stationary during a half-hour. The vibration frequency at this temperature was taken. The flame of the lamp was now lowered and the box allowed to cool very slowly, at the rate of 1° fall of temperature in about eight minutes. When the thermometer read 80° , 60° , 40° , the flame of the lamp was carefully adjusted, so that these successive temperatures were maintained during 15 minutes. We then took the frequency of vibration of the bar.

The numbers of vibrations of the forks used in the determinations of the pitches of the bars were corrected for temperature by the coefficient $\cdot 0001118$, determined by Dr. Kœnig in 1880 (*Quelques Expériences d'Acoustique*, Paris, 1882, p. 172 *et seq.*).

The subsequent tables show the results of the experiments and give the computations of velocities and moduli founded on them. The curves express graphically the effect of change of temperature on the modulus of elasticity of all the bars experimented on. The circles, on or near the curves, give the data as determined by the experiments.

In Table III., T =temperature of bars, l =the length, t =the thickness, and V =the velocity of sound through the bars, in centimetres. M =the modulus in grammes per square centimetre section of the bar. g , at Paris, equals $980\cdot 96$. D =the density, and N =the number of vibrations of bar per second at temperature, T .

All of the bars were annealed, except those of Jonas and Colver steel, of the French aluminium, and of brass; these were experimented on just as they came from the draw-bench.

For the analyses of the substances of the bars experimented on, I am indebted to my colleagues, Professors Stillman and Leeds.

TABLE I.

Bar.	l .	t .	V at $20^{\circ} =$	N computed by $N = V \sqrt{1.0279 \frac{t}{l^2}}$	N observed at 20° .	Diff.
Steel No. 1.....	20.022	.5025	150.02×3427.4 v. s. $= 514178$ cms.	662.49 v. d.	660.8	$+1.69 = \frac{1}{390}$
" No. 2.....	20.0246	.5037		663.91 "	661.0	$+2.91 = \frac{1}{227}$
" No. 3.....	20.0225	.5022		662.07 "	660.3	$+1.77 = \frac{1}{379}$
Aluminium.						
No. 1	20.0253	.4993	150.05×3377 v. s. $= 506719$ cms.	648.51 "	646.6	$+1.91 = \frac{1}{133}$
No. 2	20.0296	.4991		647.97 "	647.0	$+0.97 = \frac{1}{656}$
No. 3	20.0233	.4998		649.80 "	648.0	$+1.8 = \frac{1}{360}$
Brass.						
No. 1	20.02	.50116	150.05×2386.4 v. s. $= 358079$ cms.	460.23 "	459.0	$+1.23 = \frac{1}{378}$
No. 2	20.02	.50147		460.53 "	458.95	$+1.58 = \frac{1}{290}$
No. 3	20.02	.50108		460.16 "	458.36	$+1.81 = \frac{1}{253}$
St. Gobain Glass	23.516	.747	152.2×3582 v. s. $= 538016$ cms.	747.03 "	749.75	$-2.72 = \frac{1}{275}$
White Pine .. Density = .365.	41.15	.803	171.18×3072.75 v. s. $= 525993$ cms.	256.38 "	256.0	$+0.38 = \frac{1}{673}$

Mean departure of computed from observed value = $\frac{1}{323}$ of observed value.

TABLE II.

Tables of Analyses, of Densities at 4°, and of Coefficients of Expansion of Bars.

	Iron.	Carbon.	Silicon.	Phos.	Sulph.	Mang.	Nickel.	Density at 4°.	Coeff't. of Expans.
J. & C. Steel.	98·259	1·286	0·015	0·059	0·031	0·350	7·827	·0000110
No. 3	98·738	0·47	0·15	0·022	0·62	7·848	·0000118
No. 4	98·628	0·51	0·158	0·024	0·68	7·845	·0000120
No. 5	95·719	0·27	0·101	0·031	0·69	3·189	7·851	·0000119
Bess'r.	99·03	0·15	0·02	0·09	0·06	0·65	7·841	·0000122

Brass.

Copper	64·34
Zinc	34·97
Lead	·58
Iron	·11
Density.....	8·476
Coefft. expan.....	·0000185

Bell Metal.

Copper	80·08
Tin	18·97
Lead	·12
Zinc	·49
Density.....	8·347
Coefft. expan.....	·0000187

Aluminium (Amer.).

Aluminium	98·99
Free Carbon (graphite)	·19
Combined Carbon	·16
Tin	·21
Silicon	·32
Iron ..	·15
Density.....	2·702
Coefft. expan.....	·0000232

Aluminium (French).

Aluminium	97·80
Carbon with Si ..	·14
„ free.....	·04
„ with Copper	·09
Copper ..	1·29
Silicon	·64
Density.....	2·730
Coefft. expan.	·000022

Silver, Pure.

Density	10·512
Coefft. expan.....	·0000184

Zinc.

Zinc	99·75
Iron	·10
Lead	·04
Density	6·8107
Coefft. expan.....	0000296

St. Gobain Glass.

Silicon	72·3
Alumina	·8
Lime.....	15·3
Soda	11·8
Density.....	2·545
Coefft. expan.....	·00000777 (Fizeau).

TABLE III.

Bar.	T.	l .	t .	$\frac{t}{\frac{1}{2}}$.	D.	N.	$V = \frac{N}{1.0279 \frac{t}{\frac{1}{2}}}$.	$M = \frac{V^2 d}{g}$.	M.
Jonas & Colver Cast Steel.	0.2	20.0207	.5036	.0012564	7.828	662.0	512602	20975855.40	2097×10^6
	20	20.0246	.5037	.0012562	7.823	661.0	511908	20980648.6	2090×10^6
	40	20.0286	.5038	.0012561	7.818	659.6	511012	2080863837	2081×10^6
	61	20.0328	.5039	.0012560	7.814	658.6	509389	2072936852	2073×10^6
	80	20.0366	.5040	.0012554	7.809	657.1	508470	2064153151	2064×10^6
Steel No. 3	99.8	20.0406	.5041	.0012551	7.804	655.14	507065	2060537870	2050×10^6
	0	20.3513	.64049	.0015464	7.849	820.38	516124	2131480318	2131×10^6
	18.25	20.3558	.64053	.0015460	7.844	818.91	515229	2123516662	2123×10^6
	34	20.3594	.64074	.0015457	7.8395	817.50	514539	2115797406	2116×10^6
	60	20.3657	.64094	.0015453	7.832	814.70	512907	2100333667	2100×10^6
Steel No. 4	80	20.3705	.64109	.0015449	7.8265	812.40	511586	2084081273	2088×10^6
	99.5	20.3752	.64124	.0015446	7.821	810.00	510175	2075047288	2075×10^6
	0	20.3517	.64295	.0015523	7.846	824.71	516238	2131556968	2131×10^6
	18.25	20.3562	.64312	.0015520	7.841	822.71	515708	2125821869	2126×10^6
	40	20.3614	.64328	.0015517	7.8345	821.25	514893	2117356581	2117×10^6
Steel No. 5	51.5	20.3643	.64337	.0015514	7.831	819.67	514002	2109091886	2109×10^6
	81	20.3715	.64360	.0015508	7.823	816.30	512088	2091259452	2091×10^6
	97.6	20.3755	.64373	.0015505	7.819	814.00	510745	2079256951	2079×10^6
	0	20.3513	.64188	.0015498	7.852	813.29	510527	2086250350	2086×10^6
	18.25	20.3559	.64202	.0015494	7.847	811.84	509757	2078637463	2078×10^6
	40	20.3609	.64218	.0015490	7.841	809.68	508929	2067052961	2067×10^6
	60	20.3657	.64233	.0015486	7.835	807.31	507168	2054360194	2054×10^6
	80	20.3706	.64248	.0015482	7.829	805.02	505856	2042249532	2042×10^6
	99.5	20.3753	.64264	.0015479	7.824	802.71	504506	2030066264	2030×10^6

Table III. (*continued*).

Bar.	T.	l .	t .	$\frac{t}{\bar{p}}$.	D.	N.	$V = \frac{N}{1.0279 \frac{t}{\bar{p}}}$	$M = \frac{V^2 d}{g}$.	M.
Bessemer Steel	0.4	20.451	.60144	.0014380	7.8421	761.87	515093	2121057163	2121×10^6
	20	20.456	.60160	.0014377	7.8364	759.80	514032	2110783287	2111×10^6
	40	20.461	.60176	.0014374	7.8306	757.70	512960	2100440462	2100×10^6
	60	20.466	.60190	.0014370	7.8248	755.41	511409	2086216940	2086×10^6
	80	20.471	.60205	.0014367	7.8192	752.90	509710	2070893085	2071×10^6
	100	20.476	.60220	.0014363	7.8134	749.90	508026	2055702283	2056×10^6
Brass when bar was cooled from 99°-6 to 1°.	1	22.0128	.5009	.0012506	8.4774	460.64	358337	1109672142	1109×10^6
	21.8	22.0208	.5011	.0012501	8.4677	458.95	357307	1102010322	1102×10^6
	40	22.0274	.50126	.0012497	8.4592	457.40	356103	1093479554	1093×10^6
	60	22.0348	.50144	.0012492	8.4498	455.70	355052	1085864045	1086×10^6
	89	22.0422	.50162	.0012488	8.4404	454.00	353727	1076578495	1076×10^6
	99.6	22.0494	.50180	.0012483	8.4312	452.30	352678	1069044150	1069×10^6
Brass before bar was heated to 99°-6.	0.4	20.0127	.5008	.0012504	8.4778	460.04	357928	1107191685	1107×10^6
	20	20.0208	.5011	.0012501	8.4685	458.35	356698	1098388907	1098×10^6
Bell-Metal.....	0	22.2402	.82054	.0016589	8.3490	572.05	335300	956864118	9568×10^6
	21	22.2490	.82114	.0016588	8.3390	569.50	333806	947214927	9472×10^6
	50	22.2568	.82168	.0016587	8.3302	567.12	332411	938324839	9383×10^6
	50	22.2610	.82196	.0016587	8.3256	566.02	331766	934172231	9342×10^6
	60	22.2650	.82224	.0016586	8.3208	564.94	331133	930074354	9301×10^6
	74	22.2780	.82266	.0016586	8.3144	563.61	330354	924989117	9250×10^6
	99.15	22.2818	.82338	.0016584	8.3023	561.21	328946	915914834	9159×10^6

Aluminium (American)	0.4	21.612	.55176	.0011813	2.7027	621.00	511423	720621232	750.6×10^6
	45	21.622	.55200	.0011807	2.6990	618.10	509032	712929083	712.9×10^6
	20	21.6346	.55232	.0011800	2.6943	613.90	505933	703215278	703.2×10^6
	60	21.6422	.55250	.0011796	2.6915	611.60	504516	698390199	698.4×10^6
	82	21.654	.55278	.0011789	2.6874	607.88	501447	688868686	688.8×10^6
	99.5	21.662	.55300	.0011785	2.6840	604.71	499247	681974176	681.9×10^6
Aluminium (French)	0.5	20.0170	.49914	.0012457	2.7306	630.00	507632	717306505	717.3×10^6
	20	20.0253	.49930	.0012451	2.7270	646.60	505375	710002959	710.0×10^6
	40	20.0340	.49950	.0012445	2.7232	642.78	502388	700672761	700.6×10^6
	60.5	20.0428	.49965	.0012438	2.7194	639.19	499977	692975831	692.9×10^6
	81	20.0518	.49980	.0012430	2.7156	635.50	497477	685124447	685.1×10^6
	100	20.0600	.50000	.0012425	2.7120	632.00	494734	676700317	676.7×10^6
Silver	0.3	17.2176	.4614	.0015564	10.5142	437.93	273736	803135399	803.1×10^6
	20	17.2250	.46158	.0015557	10.5022	437.35	273489	800762642	800.7×10^6
	30	17.2284	.46168	.0015554	10.4962	436.80	273201	798729900	798.7×10^6
	40	17.2316	.46176	.0015551	10.4900	435.80	272626	794797589	794.8×10^6
	60	17.2380	.46194	.0015545	10.4778	433.00	270979	783307979	783.3×10^6
Zinc	0.3	18.2094	.44517	.0013426	6.8130	559.84	405663	1142925404	1143×10^6
	20	18.2200	.44534	.0013415	6.8010	557.84	404501	1134426237	1134×10^6
	40	18.2308	.44552	.0013405	6.7890	555.76	401844	1117556599	1117×10^6
	50.5	18.2364	.44560	.0013399	6.7826	551.22	400294	1107903520	1108×10^6
	62	18.2426	.44570	.0013390	6.7758	543.61	394291	1073843790	1074×10^6
St. Gobain Glass	0.3	23.496	.74898	.0013566	2.5452	750.65	538313	751865836	751.8×10^6
	24.5	23.501	.74902	.0013562	2.5436	749.67	537769	749865431	749.8×10^6
	40	23.503	.74910	.0013561	2.5424	749.12	537403	748511034	748.5×10^6
	60	23.507	.74922	.0013558	2.5411	748.35	536909	746742620	746.7×10^6
	80	23.510	.74934	.0013557	2.5397	747.62	536497	745196651	745.2×10^6
	99.5	23.514	.74945	.0013554	2.5384	746.70	535955	743311215	743.3×10^6

TABLE IV.
Variation of Modulus of Elasticity with Change of Temperature.

In this Table the modulus of each substance is taken as 100 at 0°. In computing this table the moduli taken were those obtained from the curves passing through the mean positions of the points determined by the experiments. The results contained in this table are expressed graphically in fig. 5.

T.	J. & C. Steel.	Steel No. 3.	Steel No. 4.	Steel No. 5.	Bessemer Steel.	Brass.	Bell Metal.
0	100.00	100.00	100.00	100.00	100.00	100.00	100.00
20.....	99.61	99.57	99.70	99.58	99.53	99.26	99.03
40.....	99.25	99.10	99.31	99.10	98.99	98.51	98.09
60.....	98.87	98.58	98.8	99.54	98.35	97.76	97.21
80.....	98.42	98.00	98.22	97.98	97.63	97.03	96.38
100.....	97.76	97.34	97.54	97.35	96.91	96.27	95.70
T.	Aluminium (American).	Aluminium (French).	Silver.	Zinc.	St. Gobain Glass.		
0	100.00	100.00	100.00	100.00	100.00		
20.....	98.92	98.86	99.73	99.26	99.76		
40.....	97.83	97.73	98.97	97.78	99.53		
60.....	96.75	96.58	97.53	93.96 (62°)	99.30		
80.....	95.67	95.42	99.07		
100.....	94.59	94.31	98.84		

Fig. 5.

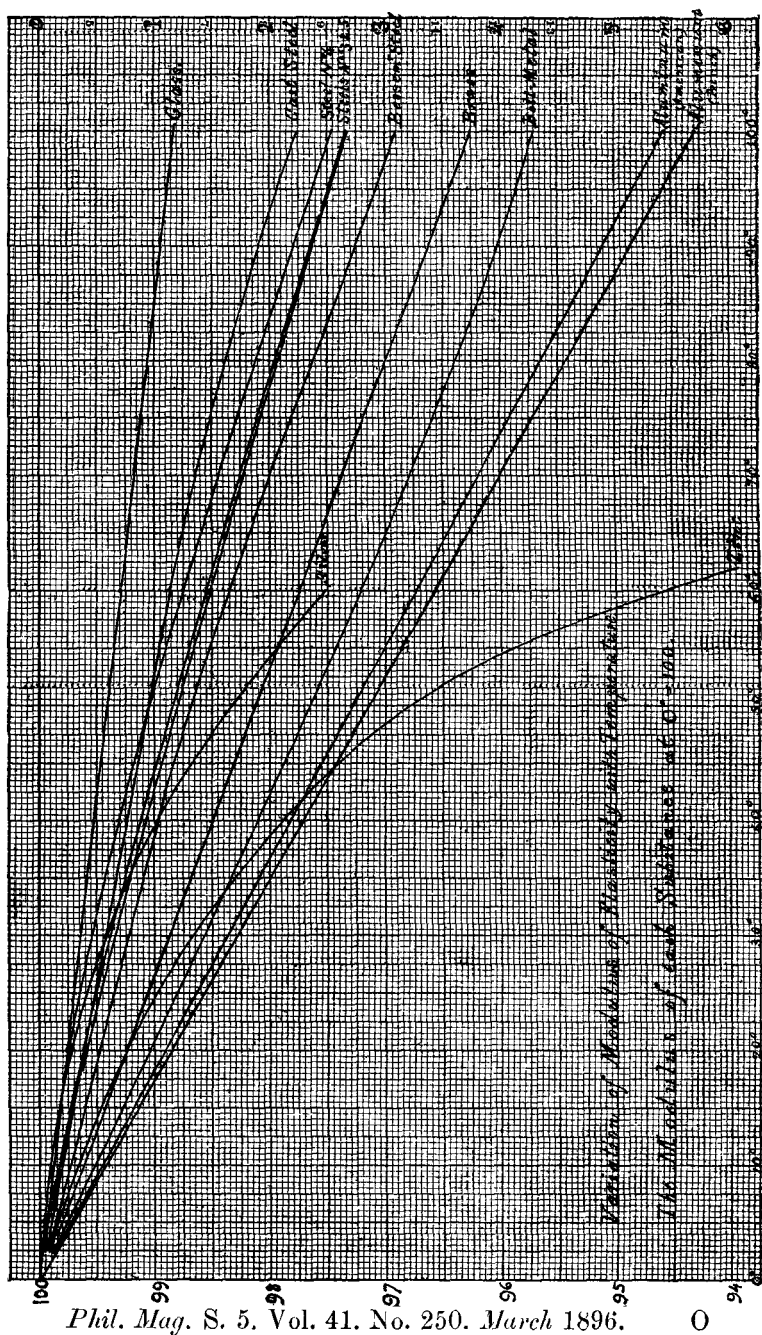


Fig. 6.

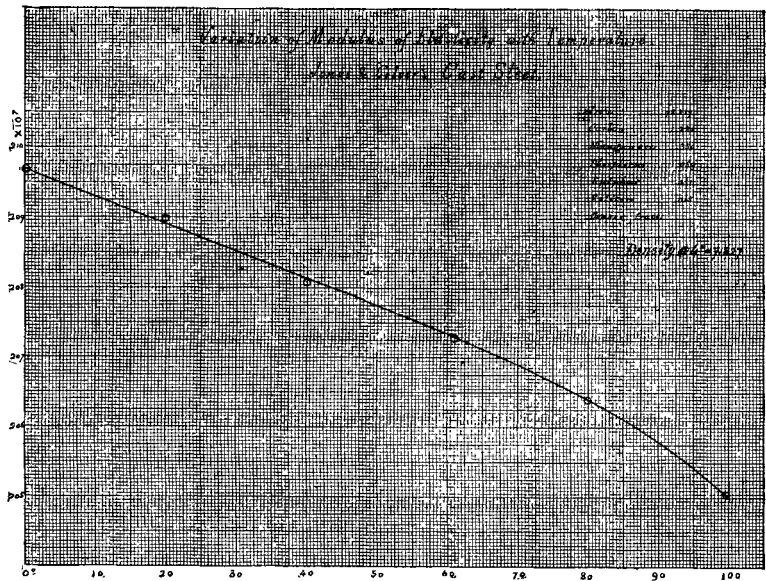


Fig. 7.

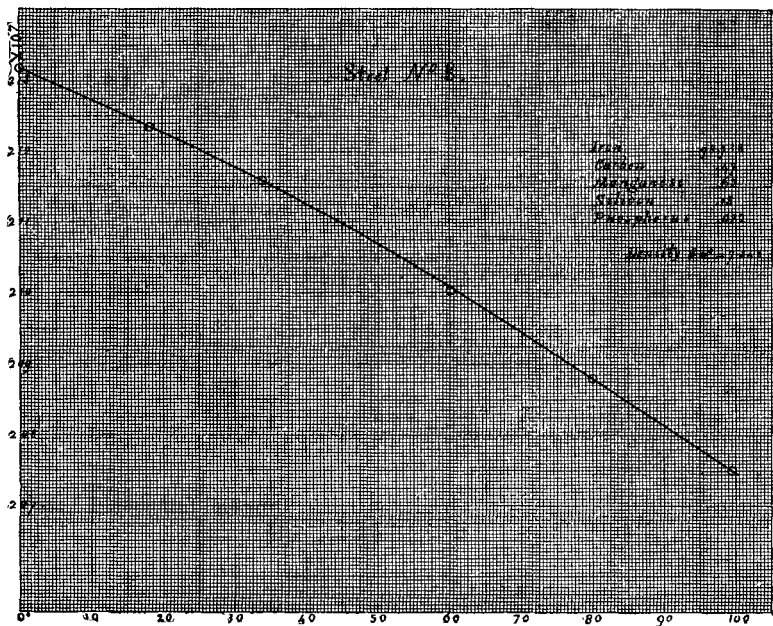


Fig. 8.

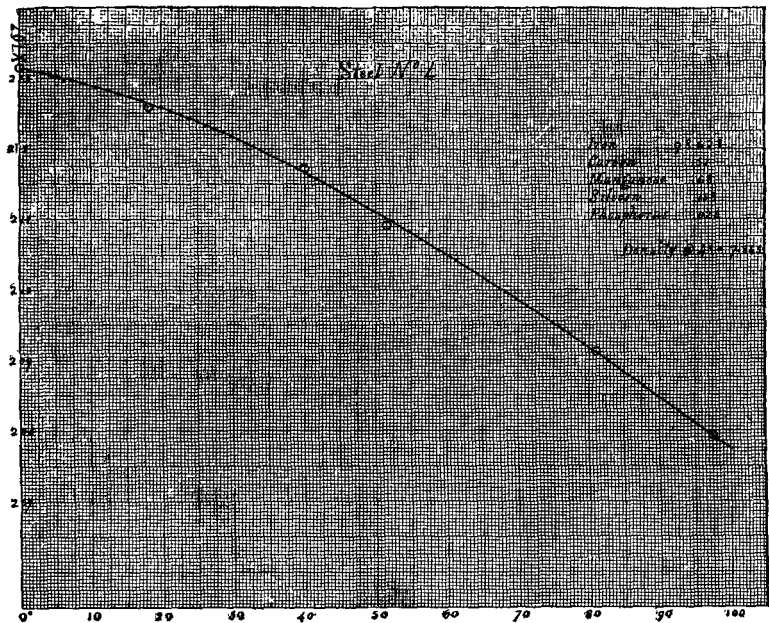


Fig. 9.

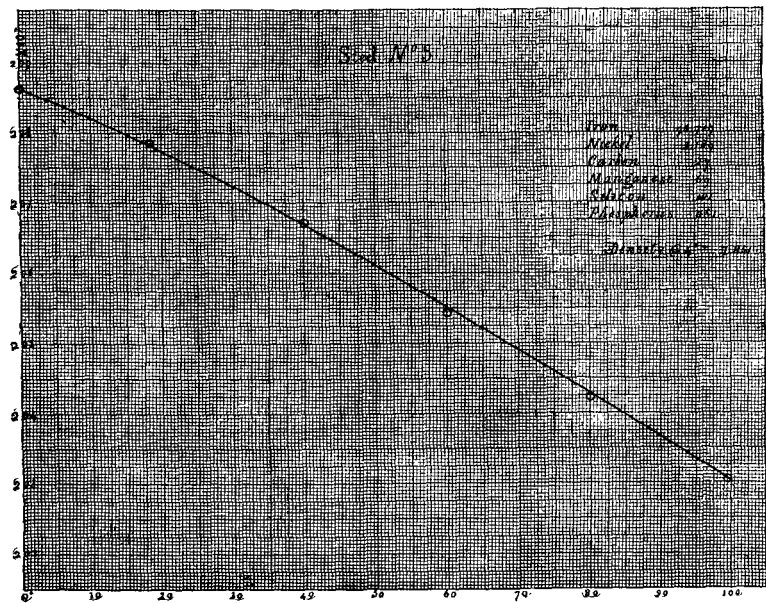


Fig. 10.

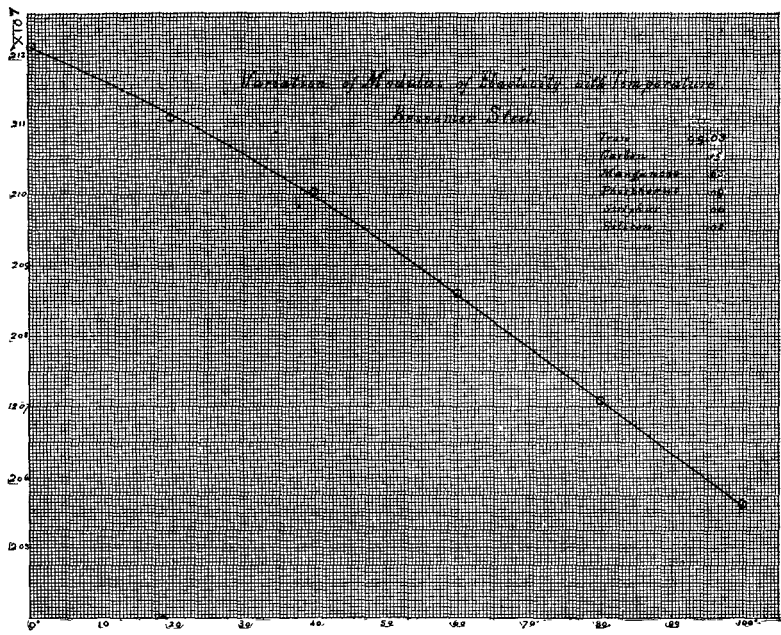


Fig. 11.

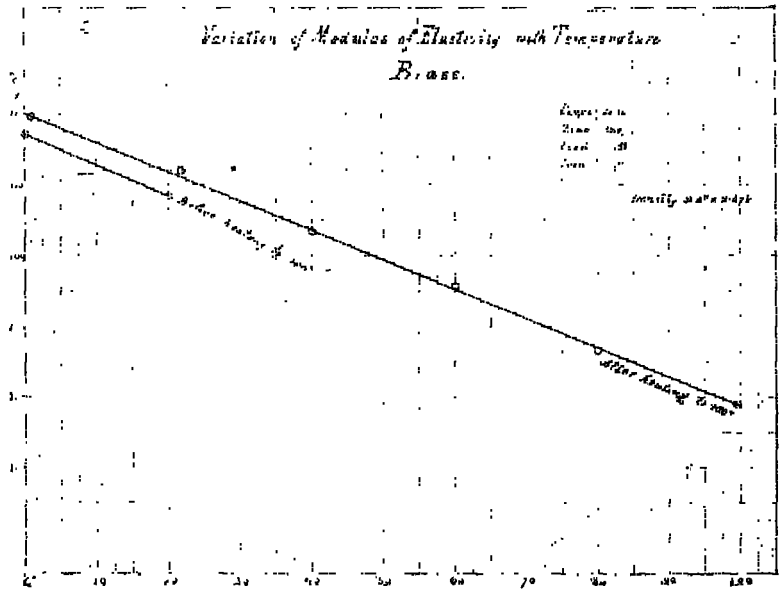


Fig. 12.

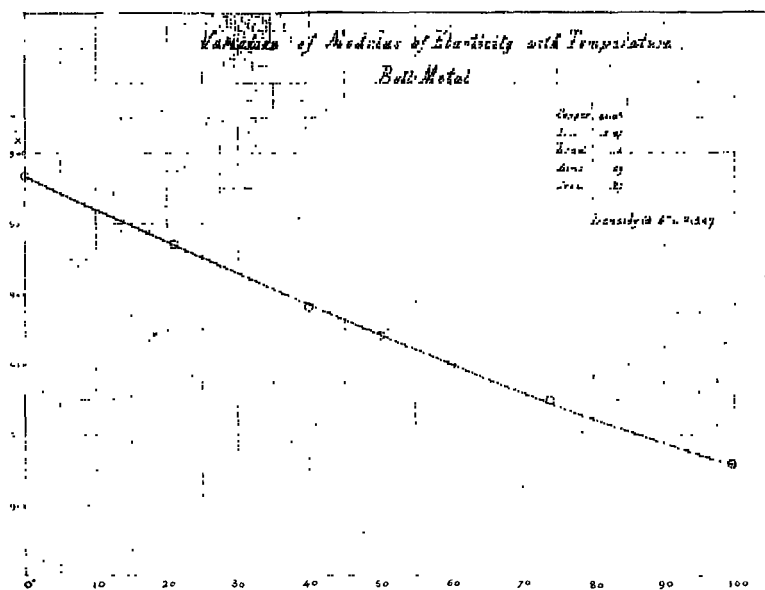


Fig. 13.

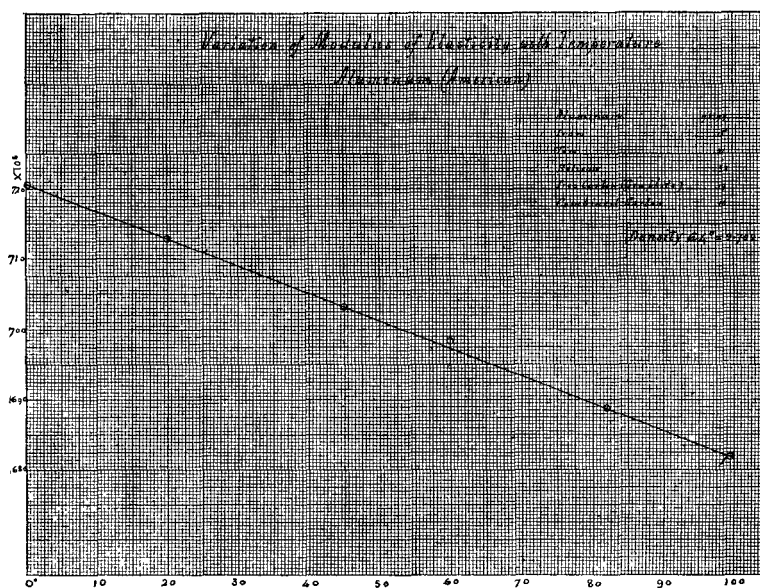


Fig. 14.

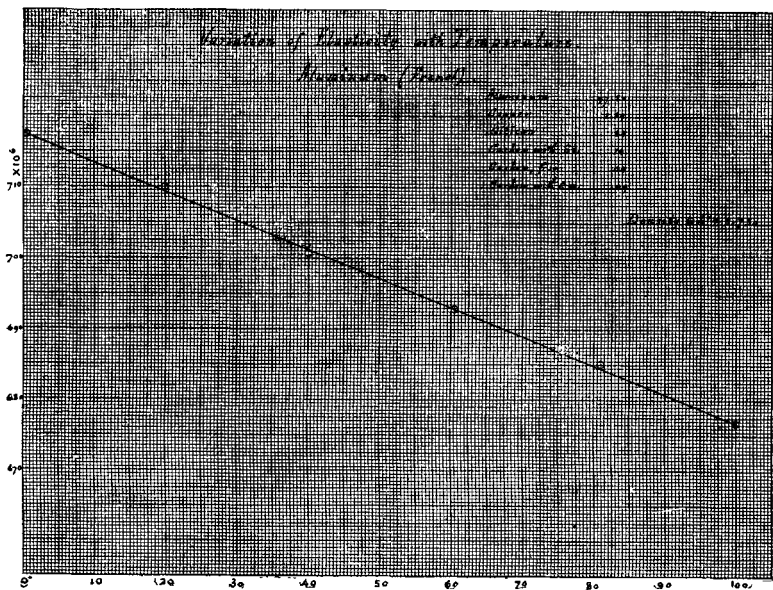


Fig. 15.

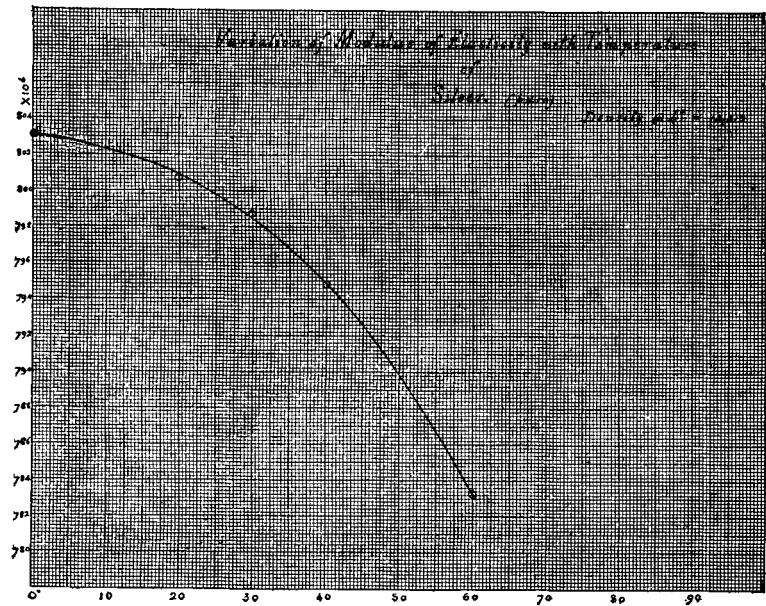


Fig. 16.

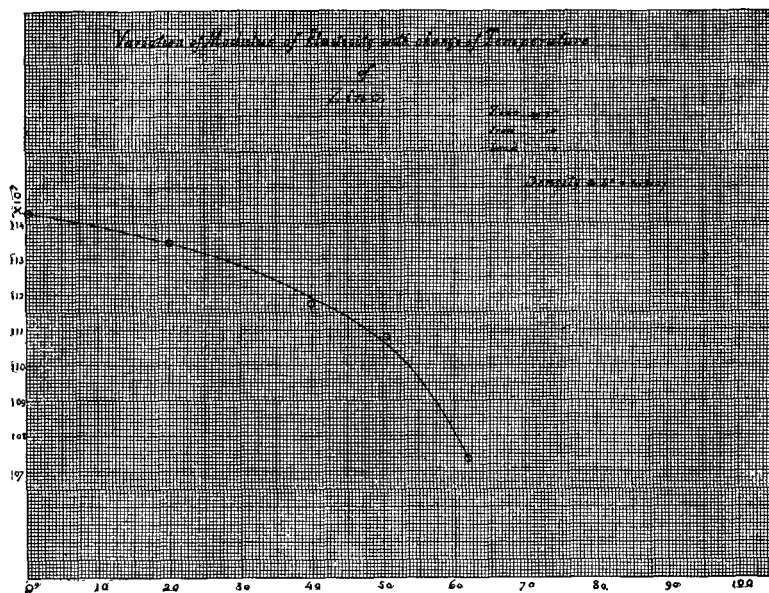
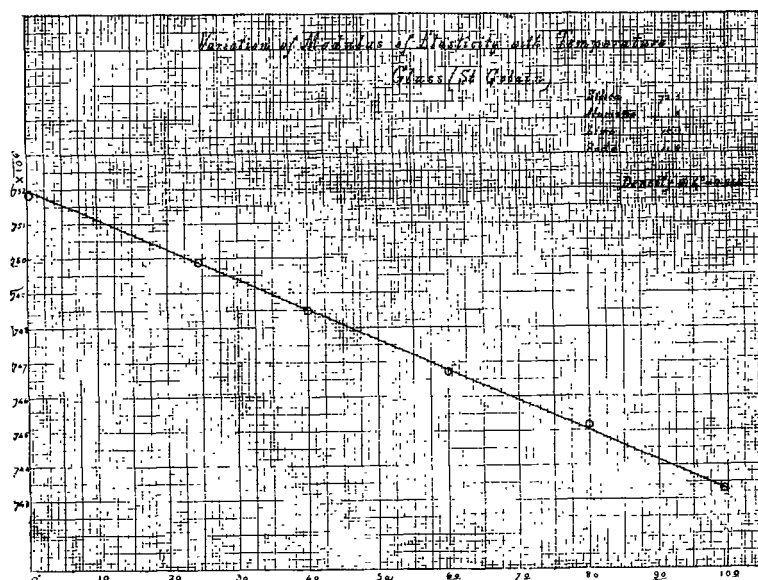


Fig. 17.



Results obtained by other Experimenters on the change of the Modulus of Elasticity with change of Temperature.

I have found five researches on this subject.

Wertheim, 1844. *Ann. de Chim. et de Phys.*

IRON.

Modulus 5·2 per cent. greater at 100° than at 18°.

Modulus 19·1 per cent. less at 200° than at 100°.

IRON WIRE.

Modulus 4·9 per cent. greater at +10° than at -11°·6.

Modulus 7·42 per cent. greater at 100° than at 18°.

WIRE OF ENGLISH CAST-STEEL.

Modulus 23·23 per cent. greater at 100° than at 18°.

Modulus 9·46 per cent. less at 200° than at 100°.

Modulus at 200° is 11·57 per cent. higher than modulus at 18°.

STEEL WIRE TEMPERED TO BLUE.

Modulus 1·97 per cent. higher at +10° than at -10°.

Modulus 5·1 per cent. higher at 100° than at 18°.

CAST-STEEL.

Modulus 2·8 per cent. less at 100° than at 18°.

Modulus 5·73 per cent. less at 200° than at 100°.

SILVER.

Modulus 5 per cent. less at +10° than at -13°·8.

Modulus 1·87 per cent. greater at 100° than at 18°.

Modulus 12·87 per cent. less at 200° than at 100°.

COPPER.

Modulus 6·53 per cent. less at +10° than at -15°.

Modulus 6·58 per cent. less at 100° than at 18°.

Modulus 20 per cent. less at 200° than at 100°.

WIRE OF BERLIN BRASS (Cu=67·55, Zn=32·35).

Modulus 7·95 per cent. less at +11° than at -10°.

Kupffer, 1856. *Mem. de l'Acad. de St. Pétersb.*

Modulus of iron wire 5·5 per cent. less at 100° than at 0°.

Modulus of copper wire 8·2 per cent. less at 100° than at 0°.

Modulus of brass wire 3·9 per cent. less at 100° than at 0°.

Kohlrausch and Loomis, 1870. *Pogg. Ann.*

Modulus of iron wire 5 per cent. less at 100° than at 0° .

Modulus of copper 6 per cent. less at 100° than at 0° .

Brass 6.2 per cent. less at 100° than at 0° .

H. Tomlinson, 1887. *Phil. Mag.* xxiii.

Says, "my own experiments show that both the torsional and longitudinal elasticities of iron and steel are decreased by about $2\frac{1}{2}$ per cent. when the temperature is raised from 0° to 100° ."

M. C. Noyes, 1895. *The Physical Review.*

Modulus of a piano wire of $\frac{4}{10}$ mm. diam. 5 per cent. less at 100° than at 0° .

The results of Wertheim's experiments giving an increase to the modulus, as the temperature rises, of iron, iron wire, wire of English cast-steel, steel wire drawn to blue, and silver, have not been confirmed in any instance by subsequent experiments; only for cast-steel *rod* and copper did he obtain a diminution of modulus for a rise of temperature from 18° to 100° . Yet he found that a *wire* of English cast-steel had a modulus 23 per cent. higher at 100° than at 18° .

On the Acoustical Properties of Aluminium.

The low density (2.7) of aluminium combined with a modulus of elasticity of only 712×10^6 render this metal easy to set in vibration; a transverse blow given to a bar of this metal causes it to vibrate with an amplitude of vibration greater than that which the same energy of blow gives to a similar bar of steel or of brass. This fact has given rise to the popular opinion that aluminium has sonorous properties greatly exceeding those of any other metal. This opinion is erroneous. If a bar of aluminium and a bar of brass having the same length and breadth and giving the same note, are struck transversely so that the bars have the same amplitude of vibration, the bars give equal initial intensity of sounds; but the bar of aluminium from its low density and because of its internal friction will vibrate less than one-third as long as the bar of brass. Thus, a bar of aluminium and a bar of brass of the same length and width and of such thickness that they gave the same note, SOL₄ of 768 v. d., were vibrated so that the sounds at the moment of the blows were, as near as could be judged, of the same intensity. The duration of the sound of the brass bar was 100 seconds; the sound of the aluminium bar lasted 30 seconds.

The readiness with which a bar of aluminium vibrates when acted on by aerial vibrations of the same frequency as those

given by the bar, gives one the means of making many charming experiments in which "sympathetic vibrations" come into play.

I here describe an experiment which I devised to show the interference of sound in a manner similar to analogous experiments in the case of light. The resonant box on which Kœnig mounts his UT_5 (1024 v. d.) fork is open at both ends and has a length of nearly a half wave of the sound of the fork. If this resonant box is held with its axis vertical, above an aluminium bar in tune with the vibrating fork, the bar does not enter into sympathetic vibration with the fork, because the sonorous pulses, on reaching the aluminium bar from the two openings of the resonant box, differ in phase by one half wave-length. But if the axis of the box is held parallel to the axis of the bar, then the sonorous waves reaching the bar have travelled over equal lengths from the openings at the ends of the box, and these waves conspire in their action and the aluminium bar enters into sympathetic vibration.

As this experiment is an interesting one I here give details as to the manner of making it. The bar of aluminium has a large surface, having a length of 17 cms. and a width of 5 cms. The two nodal lines, which are at a distance from the ends of the bar equal to $\frac{2}{3}$ ths of its length, are drawn on the bar. The bar is supported under these nodal lines on threads stretched on a frame. This frame is of such a height that the under surface of the aluminium bar is 8.4 cms., or one quarter wave-length, above the surface of the table, so that the vibrations of the bar and those of the waves reflected from the table will act together. The upper surface of the bar is covered with a piece of thick cardboard, in which is cut a rectangular aperture, having for length the distance between the nodal lines and a width equal to that of the bar. As this piece of cardboard rests on supports which lift it a slight distance above the surface of the bar, the latter, when it vibrates, does not send to the ear the vibrations of the surfaces of the bar included between its nodal lines and its ends, which vibrations are opposed in phase to those given by the central area of the bar. Thus the sound emitted by the bar is much increased and the experiment rendered more delicate and improved in every way. I have found that the experiment succeeds best when the centre of the resonant box is held about 58 cms., or $7\frac{\lambda}{4}$ above the surface of the aluminium bar.

This experiment works best in the open air, away from the action of sound-waves reflected from the walls and ceiling of a room.

The fact that aluminium gives, from a comparatively slight blow, a great initial vibration, and that its vibrations last for a short time, render this metal peculiarly well suited for the construction of those musical instruments formed of bars which are sounded by percussion and the duration of whose sounds is not desirable.

I had hoped that aluminium would prove to be a good substance out of which to make plates on which to form the acoustic figures of Chladni. Experiments have shown that aluminium is not suited to this purpose. I had plates of aluminium carefully cast, with 2 cms. of thickness. These plates were turned down on the face-plate of a lathe to thicknesses of 2 mm. and 3·8 mm. Three of these plates were quite homogeneous in elasticity, for the Chladni figures when obtained on them were symmetrical. Yet the Chladni figures were difficult to produce, because it is difficult to obtain a pure tone from an aluminium plate. The sound is generally more or less composite; therefore the plate in its vibration tends to form two or more figures at the same time, and the consequence is that either no figure is formed or one is given that is not sharply defined. One square plate of 30·8 cms. on the side and 3·8 cm. thick, gave quite clearly the three following tones:— UT_2 (1), SOL_2 (2), and SOL_4 (3). Corresponding respectively to the Chladni figures of (1) two lines drawn between opposite points of the centre of sides of plate; (2) figure formed of the two diagonals drawn between the corners of plate; (3) figure similar to (1) but with corners of plate cut off by curved lines. Figure 3 corresponded so nearly to the sound of SOL_4 that a vibrating SOL_4 fork when held near the plate set the latter into vigorous vibration.

Another difficulty met with in using plates of aluminium for Chladni's figures is that sand, even when entirely free from salt and from the globular grains of wind-blown sand, does not move freely over a vibrating surface of aluminium, whether this surface has been polished or has been slightly tarnished and roughened by the action of alkali.

There is one serious objection to the use of aluminium in the construction of musical and acoustical instruments, and that is the great effect that change of temperature has upon its elasticity. If a bar of aluminium and a bar of cast-steel be tuned at a certain temperature to exact unison, a change from that temperature will affect the frequency of vibration of the aluminium bar $2\frac{1}{2}$ times as much as the same change of temperature will affect the bar of cast-steel.