

On a New Method of Determining Thermal Conductivity

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want of agreement of the theory with fact. On the theory the current flows from hot to cold in that metal of a pair whose abscissa is the greater. According to the observations the current flows from hot to cold in that metal of a pair whose ordinate is the greater. The two are only in agreement when the line on the diagram joining the two points corresponding to any two metals slopes upward to the right. It is obvious that the number of exceptions to this rule, even for the metals dealt with, must be almost as large as the number of agreements, and that the laws enunciated by M. Thomas are not supported by the observed facts.

XIX. *On a New Method of Determining Thermal Conductivity.*
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[Plate V.]

- (1) Introduction.
- (2) Theory.
- (3) Application to Liquids.
- (4) Apparatus.
- (5) Method of Experiment.
- (6) Results.
- (7) Calculation of Thermal Conductivity.
- (8) Conclusion.

(1) *Introduction.*

AN expression for the concentration of a solution in a vertical tube along which there is both movement of the solution and diffusion of the solute has been obtained by Dr. A. Griffiths (Phil. Mag. Nov. 1898), who pointed out to the author that a similar expression could be obtained in the case of thermal diffusion and suggested the present investigation, the object of which is:—

1st. To find the effect of impressed velocity on temperature gradient.

2nd. To test the practicability of using such artificially

* Read February 25, 1910.

produced temperature gradients as a means of determining thermal conductivity in absolute measure.

The author has extended Dr. Griffiths's results so as to take into account radiation, which plays an important part in most cases of conduction, and has carried out the researches suggested.

(2) Theory.

Consider a bar moving longitudinally with a constant velocity v as shown and passing through two constant temperature sources, and let

K = thermal conductivity,

s = specific heat,

ρ = density,

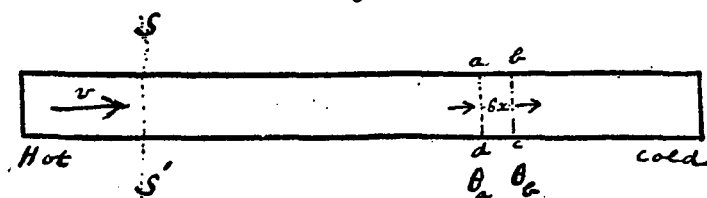
A = cross-section,

I = intrinsic heat in volume vA of the bar at the temperature of the enclosure,

θ = temperature above the enclosure at distance x from any plane in space normal to the bar.

Then the quantity of heat crossing a section SS' in space is due to conductivity $-KA \frac{d\theta}{dx}$, and due to the impressed velocity $vA\rho s\theta + I$.

Fig. 1.



Applying this to the small prism $abcd$ in space of width δx we have :—

$$\text{Heat entering } ad \text{ per sec.} = -KA \frac{d\theta_a}{dx} + vA\rho s\theta_a + I.$$

$$\text{Heat leaving } bc \text{ per sec.} = -KA \frac{d\theta_b}{dx} + vA\rho s\theta_b + I.$$

In the steady state the difference will be radiated, and since

$$\theta_b = \theta_a + \delta x \frac{d\theta}{dx},$$

this difference is given by

$$KA\delta x \frac{d^2\theta}{dx^2} - vA\rho s\delta x \frac{d\theta}{dx}.$$

But assuming Newton's Law of Emission and reckoning all temperatures from the room taken as zero, we may also express the heat radiated by the section $abcd$ by the expression $E p \theta \delta x$, where p is the perimeter of the bar and E the coefficient of emissivity. Equating we obtain:—

$$KA \frac{d^2\theta}{dx^2} - vA\rho s \frac{d\theta}{dx} = E p \theta.$$

The general solution of this differential equation is:—

$$\theta = e^{\frac{v\rho s x}{2K}} \left(A e^{\frac{x}{2} \sqrt{\frac{v^2\rho^2 s^2}{K^2} + \frac{4Ep}{KA}}} + B e^{-\frac{x}{2} \sqrt{\frac{v^2\rho^2 s^2}{K^2} + \frac{4Ep}{KA}}} \right),$$

where A and B are constants of integration.

Taking now θ_1 , θ_m , θ_2 as the temperatures above the room at the positions $x = 0$; $x = \frac{L}{2}$; $x = L$ respectively, we obtain

$$\frac{\theta_2 - e^{\frac{v\rho s L}{2K}} e^{-\sqrt{R}} \theta_1}{\theta_m - e^{\frac{v\rho s L}{4K}} e^{-\frac{1}{2}\sqrt{R}} \theta_1} = e^{\frac{v\rho s L}{4K}} e^{-\frac{1}{2}\sqrt{R}} (e^{\sqrt{R}} + 1) \quad \dots (1)$$

$$\text{where} \quad R = \frac{v^2\rho^2 s^2 L^2}{4K^2} + \frac{EpL^2}{KA}.$$

If, however, E is small this reduces to

$$\frac{\theta_2 - e^{\frac{-EpL}{2v\rho s A}} \theta_m}{\theta_m - e^{\frac{-EpL}{2v\rho s A}} \theta_1} = e^{\frac{v\rho s L}{2K} + \frac{EpL}{2v\rho s A}}, \quad \dots (2)$$

and if E could be neglected we should have

$$\frac{\theta_2 - \theta_m}{\theta_m - \theta_1} = e^{\frac{v\rho s L}{2K}} \dots (3)$$

It is hoped that with the aid of high-vacuum jacketed vessels equations (2) and (3) will be found useful when applied to liquids, but from the present point of view a particular case of equation (1) is of more importance.

Suppose θ_m = temperature of room = zero.

Then, whatever be the value of E , we obtain :—

$$\frac{-\theta_2}{\theta_1} = e^{\frac{v\rho sL}{2K}}$$

or $K = \frac{\frac{1}{2}v\rho sL}{\log_e \frac{-\theta_2}{\theta_1}} \quad . \quad . \quad . \quad . \quad . \quad (4)$

Hence the ratio of the temperatures above and below—at equal distances from the point in the bar which is at the same temperature as the enclosure—is independent of the value of the emissivity.

Equation (4) is used below for calculating the value of the thermal conductivity of mercury.

(3) *Application to Liquids.*

There are experimental difficulties, probably not insuperable, in determining the temperature at a given point fixed in space and situate within a moving solid, and it was thought advisable to experiment in the first place with a moving column of mercury.

When a column of mercury at a constant temperature moves slowly along a tube, the velocity is a maximum at the centre and a minimum at the circumference, viscosity being the main factor in determining the form of the movement. When, however, the top of such a column is heated and the lower end kept cold, the velocity of the mercury at the central part of the tube being greater than near the walls, the average temperature of the central parts is less than the average temperature of the outside parts, and therefore the average density of the central parts is greater than the average density of the outside parts. Thus the gravity head in the central part is greater than the gravity head elsewhere, and this tends to diminish the differences in velocity at various parts of the cross-section. A mathematical investigation leads to differential equations of an exceedingly complicated type, but the author is of opinion that, neglecting radial conductivity, the isothermal surfaces would be approximately horizontal to quite a considerable distance from the centre of the tube.

It is not unlikely, however, that radial conductivity will play an important part, and if the diameter of the tube is small in comparison with the length, and if the flow is small, the variations in temperature radially will undoubtedly be small in comparison with the longitudinal variations. If the radial conductivity were infinitely great, or if the diameter were exceedingly small, the isothermals would be horizontal and the gravity effect alluded to above would vanish and, in the opinion of the author, the flow would again approximate to the well known parabolic character.

So long, however, as the isothermals are approximately horizontal the nature of the flow does not affect the result:—

For let vp = the vertical component of the product of the velocity and density at a distance r from the axis ;

and let Q = quantity of heat crossing a horizontal plane per sec.

$$\text{Then } Q = -KA \frac{d\theta}{dx} + \int_0^R v\rho s\theta \times 2\pi r \cdot dr + I.$$

$$\begin{aligned} \text{But } \int_0^R v\rho s\theta \times 2\pi r \cdot dr &= s\theta \int_0^R 2\pi r \times v\rho \cdot dr \\ &= s\theta \times \text{mass crossing section} \\ &= s\theta \times v'\rho'A \quad [\text{per sec.}] \end{aligned}$$

where $v'\rho'$ may be called the average product of velocity and density. This, by the law of fluid continuity, is constant longitudinally, so the method will not be affected by variations of density in the parts of the liquid at different temperatures. We have then:—

$$Q = -KA \frac{d\theta}{dx} + v'\rho'A s\theta + I$$

and hence, just as in section (2), we obtain under conditions similar to those holding in equation (4)

$$K = \frac{\frac{1}{2}v'\rho'sL}{\log_e \frac{\theta_2}{\theta_1}} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

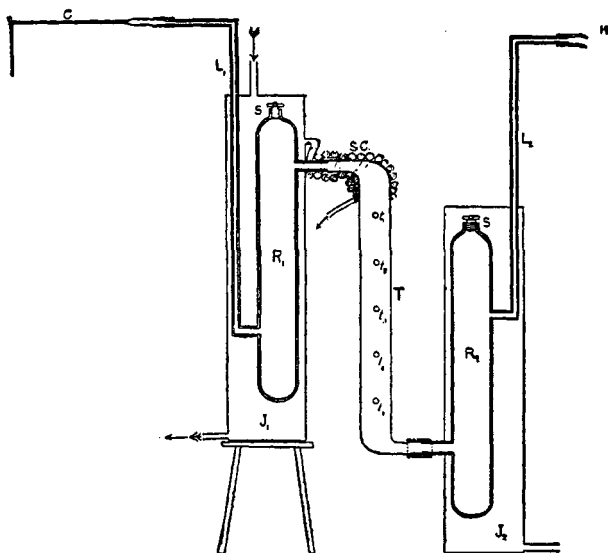
the product $v'\rho'$ being equal to the mass of liquid outflowing per sec. divided by the cross-section of the tube.

(4) *Apparatus.*

Preliminary experiments showed the importance and the difficulty of maintaining the ends of the experimental tube at constant temperatures in spite of the inflow of mercury.

The form eventually adopted consists of two specially cast iron reservoirs R_1 R_2 (fig. 2) about 40 cms. long and of 5 cms.

Fig. 2.



internal diameter, to which by strong rubber tubing bound by wire the glass tube T was attached. This latter a little less than 3 cms. in diameter and 40 cms. long—its ends being bent as shown—had five side limbs through which thermometers t_1-t_5 were inserted and fastened in with cement. The thermometers had cylindrical bulbs of small radius and were about 6 cms. apart.

Two limbs L_1 L_2 were attached to the iron reservoirs and these latter were surrounded with tin-jackets J_1 J_2 . Screws S_1 and S_2 were provided so as to allow the air to escape in filling the apparatus, these screws being inserted with the aid of cement as soon as the mercury reached the top of the reservoirs.

A capillary tube C was attached to the top of the limb L_1 , and to L_2 pressure tubing was attached leading to a vessel in which the head of mercury was kept constant by an overflow pipe. By raising or lowering this vessel the head could be varied and a constant outflow of mercury from the capillary C could be obtained.

A spiral SC of flexible tin tubing leaving the steam jacket J_1 was found useful for keeping up the temperature, thus enabling a larger temperature-slope to be investigated; for by turning a tap the steam would leave the jacket J_1 by the spiral as well as by the main tap at the bottom of the jacket.

Both tin jackets were surrounded with cotton-wool which was also bound carefully round the experimental glass tube, the latter being also shielded from the jackets by a thick-walled asbestos enclosure. A thermometer hung in this enclosure gave the temperature of the air around the experimental tube. Fig. 1 (Plate V.) gives a general view of the apparatus.

(5) *Method of Experiment.*

The apparatus having been filled with pure mercury, ice was placed in the jacket J_2 and steam was injected into J_1 . When the top of the glass tube T was warm enough and there was no danger of breaking it, steam was allowed to circulate through the spiral coil.

In about 3 hours the steady state was obtained and the temperatures were noted. The head H was then raised till a suitable flow up the tube T and out of the capillary resulted. The weight flowing out in a definite time—usually 15 minutes—was taken, and when in two or three more hours' time the new steady state was attained this flow was remarkably constant, the weighings not differing by more than .5 gm. or .6 gm. in 300 gms. In this connexion it is interesting to note that there is an automatic tendency to maintain constancy in the flow; for any slight increase of head—due for example to vibration—temporarily causing a faster flow would introduce a higher counterbalancing pressure in the experimental tube the mercury in which on

the whole would become slightly colder and heavier. Thus the equilibrium is stable.

Under the new steady state the temperatures are again noted as well as the temperature of the enclosure. All the readings were corrected so as to give the actual excesses above the temperature of the surroundings. To do this readings were taken when all the thermometers were *in situ* and at the same temperature. They then showed slight differences due to various causes, one of which was the hydrostatic pressure of the mercury.

It was found that if the ordinary stationary state was not required the steady state under flow could be more quickly obtained by starting the flow, when, in the variable state, the top thermometer reached a temperature near that desired.

If a flow from hot to cold was desired the head H and capillary C were interchanged.

(6) Results.

(a) Mean distances between thermometers by cathetometer.

$t_1 - t_2$	6.12 cms.
$t_1 - t_3$	11.91 cms.
$t_1 - t_4$	17.86 cms.
$t_1 - t_5$	23.55 cms.

(b) In the table below V denotes the weight of mercury outflowing in 15 minutes. The spiral steam coil was used in every case. The corrected temperatures $t_1 - t_5$ and the enclosure temperature are given.

Magnitude of flow.	t_1 .	t_2 .	t_3 .	t_4 .	t_5 .	Enclosure.
V=0 (ord. steady state)	70.9	52.0	38.2	27.1	18.5	19.5
V=114.6 gms.	66.2	45.5	31.6	22.1	15.0	18.8
V=141.9 gms.	64.8	43.5	30.2	20.9	14.2	18.5
V=323.0 gms. A	52.8	30.5	19.5	13.1	9.3	19.0
V=399.0 gms. B	47.7	26.1	16.4	11.1	8.0	19.5
V=481.25 gms. C	43.9	22.6	14.4	10.1	7.4	19.2
V=497.97 gms. D	43.1	21.5	13.5	9.6	7.1	18.0
V=543.88 gms. E	40.5	19.7	12.5	9.0	6.5	18.0
V=820.0 gms.	21.1	10.0	7.4	5.5	4.5	15.0
V=+268 gms. (flow hot to cold).	76.7	63.6	51.6	41.1	30.5	14.0

The above are shown plotted as curves (Pl. VII. fig. 2) temperatures being represented by ordinates, and distance from the first thermometer (multiplied by five) being taken as abscissæ.

(c) When the flow was from hot to cold the action of the spiral was harmful. Accordingly a result is given which was obtained earlier in the work with an experimental tube fitted with thermometers 7 cms. apart, the first and fifth being at the extreme top and bottom respectively of the vertical portion of the tube. This result, when plotted, shows that with a sufficient downward flow the temperature-distance curve is concave as regards the origin—a result in accordance with theory.

State of vertical column.	t_1 .	t_2 .	t_3 .	t_4 .	t_5 .	Enclosure.
Ordinary steady state	75.0	53.5	35.4	22.1	10.3	19.0
Flow of 39 c.cms. in 13 mins. (hot to cold) }	88.0	79.0	67.4	54.1	29.0	19.0

(7) Calculation of Thermal Conductivity.

The curves in section (6) *b* marked A—E fall within the range suitable for calculating the thermal conductivity of mercury. Remembering that the velocity is negative equation 5 of section (3) may be written for practical purposes:—

$$K = \frac{1}{2} \frac{s}{At} \times .4343 \times \left(\frac{VL}{\log_{10} \frac{\theta_1}{\theta_2}} \right)$$

where *V* is the weight of mercury escaping in the time *t*, which was always 15 minutes, and θ_1, θ_2 are temperatures on the curve in question at distances $\frac{L}{2}$ on the left and right respectively of the point on the curve at the same temperature as the room.

s the specific heat of mercury = .0333.

A the cross-section of the tube was found = 5.98 sq. cms.

t = 15 × 60 seconds.

The following are the values of $\frac{5LV}{\log_{10} \frac{\theta_1}{-\theta_2}}$ in each case,

for each curve three different values of L being taken.

Curve.	V.	5L.	θ_1 .	$-\theta_2$.	Abscissa of room temp.	$\frac{5LV}{\log_{10} \frac{\theta_1}{-\theta_2}}$	Mean.
A	323	$\left\{ \begin{array}{l} 2 \times 26.3 \\ 2 \times 39.3 \\ 2 \times 51.3 \end{array} \right.$	$\left\{ \begin{array}{l} 28.3-19.0 \\ 35.2-19.0 \\ 43.4-19.0 \end{array} \right.$	$\left\{ \begin{array}{l} 19.0-13.4 \\ 19.0-11.4 \\ 19.0-9.9 \end{array} \right.$	61.3	$\left\{ \begin{array}{l} 77120 \\ 77240 \\ 77380 \end{array} \right. =$	77250
B	399	$\left\{ \begin{array}{l} 2 \times 35 \\ 2 \times 25 \\ 2 \times 48 \end{array} \right.$	$\left\{ \begin{array}{l} 36.5-19.5 \\ 30.1-19.5 \\ 47.7-19.5 \end{array} \right.$	$\left\{ \begin{array}{l} 19.5-12.0 \\ 19.5-13.6 \\ 19.5-10.3 \end{array} \right.$	48	$\left\{ \begin{array}{l} 78610 \\ 78430 \\ 78750 \end{array} \right. =$	78600
C	481.25	$\left\{ \begin{array}{l} 2 \times 25 \\ 2 \times 32 \\ 2 \times 40 \end{array} \right.$	$\left\{ \begin{array}{l} 31.1-19.2 \\ 36.5-19.2 \\ 43.9-19.2 \end{array} \right.$	$\left\{ \begin{array}{l} 19.2-13.4 \\ 19.2-12.3 \\ 19.2-11.3 \end{array} \right.$	40	$\left\{ \begin{array}{l} 77100 \\ 77160 \\ 77770 \end{array} \right. =$	77340
D	497.97	$\left\{ \begin{array}{l} 2 \times 25 \\ 2 \times 35 \\ 2 \times 41 \end{array} \right.$	$\left\{ \begin{array}{l} 29.5-18.0 \\ 36.9-18.0 \\ 43.1-18.0 \end{array} \right.$	$\left\{ \begin{array}{l} 18.0-12.5 \\ 18.0-11.2 \\ 18.0-10.4 \end{array} \right.$	41	$\left\{ \begin{array}{l} 77750 \\ 78500 \\ 78690 \end{array} \right. =$	78310
E	543.88	$\left\{ \begin{array}{l} 2 \times 24 \\ 2 \times 30.5 \\ 2 \times 35.5 \end{array} \right.$	$\left\{ \begin{array}{l} 29.8-18.0 \\ 34.9-18.0 \\ 40.5-18.0 \end{array} \right.$	$\left\{ \begin{array}{l} 18.0-12.5 \\ 18.0-11.6 \\ 18.0-11.0 \end{array} \right.$	35.5	$\left\{ \begin{array}{l} 78750 \\ 78670 \\ 76150 \end{array} \right. =$	77860

Mean=77870

Or mean value of $\frac{LV}{\log_{10} \frac{\theta_1}{-\theta_2}} = 15574.$

This gives a mean value for $K = .0209.$

Each curve was drawn three times by an assistant and the one lying intermediate between the other two accepted for calculation of K. In such graphic methods a repetition by different people, or even by the same person, leads to slightly different results; but in the present experiments, where the size of the thermometer-bulbs must have an appreciable effect on the uniformity of cross-section of the tube, it would be out of place to use elaborate mathematical methods of interpolation which will most probably be necessary in future work. A general consistency for the value of K is

obtained using different velocities, and this value is of the right order.

(8) *Conclusion.*

The curves obtained agree closely with those which can be deduced from theory, and promise to lead to a new method of determining thermal diffusivity directly. This method has the advantage of requiring only temperature measurements, quantity of heat not entering directly but being inferred from a knowledge of specific heat. In improving the method the first thing will be the substitution of thermojunctions—at much more frequent intervals—for thermometers. The cross-section of the tube need hardly then be affected, and many more points on the curve could be obtained, thus diminishing very much errors in graphic interpolation.

The author is, however, presenting his research at this stage, embracing the theory and the work which can be done with thermometers, as he is anxious to perform experiments under the best possible conditions, and hopes to receive advice and criticism before proceeding with more delicate work.

The author wishes to express his thanks to the Principal and Council of Birkbeck College for facilities for carrying out the work, and more especially to Dr. A. Griffiths who has watched the work with interest from the beginning and has given valuable help and encouragement throughout.

FIG. 1.

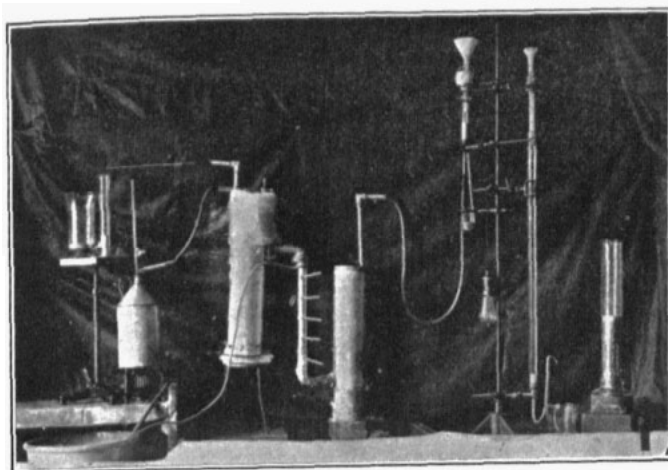


FIG. 2.

