

Nieuwe Verhandelingen der Eerste Klasse von het Koninklijk-Nederlansche Instituut van Wetenschappen, Letterkunde en Schoone Kunsten te Amsterdam. Deel XII., Stuk 1.—*By the Institut.*

*Monday, 6th April 1846.*

SIR THOMAS M. BRISBANE, Bart., President, in the Chair.

The following Communications were read :—

1. On the Description of Oval Curves, and those having a plurality of Foci. By Mr Clerk Maxwell junior; with remarks by Professor Forbes. Communicated by Professor Forbes.

Mr Clerk Maxwell ingeniously suggests the extension of the common theory of the foci of the conic sections to curves of a higher degree of complication in the following manner :—

(1.) As in the ellipse and hyperbola, any point in the curve has the *sum* or *difference* of two lines drawn from two points or *foci* = a constant quantity, so the author infers, that curves to a certain degree analogous, may be described and determined by the condition that the simple distance from one focus *plus* a multiple distance from the other, may be = a constant quantity ; or more generally, *m* times the one distance + *n* times the other = constant.

(2.) The author devised a simple mechanical means, by the wrapping of a thread round pins, for producing these curves. See Figs. 1 & 2 (Plate II.) He then thought of extending the principle to other curves, whose property should be, that the sum of the simple or multiple distances of any point of the curve from three or more points or foci, should be = a constant quantity ; and this, too, he has effected mechanically, by a very simple arrangement of a string of given length passing round three or more fixed pins, and constraining a tracing point, P. See Fig. 3. Farther, the author regards curves of the first kind as constituting a particular class of curves of the second kind, two or more foci coinciding in one, a focus in which two strings meet being considered a double focus ; when three strings meet a treble focus, &c.

Professor Forbes observed that the equation to curves of the first class are easily found, having the form

$$\sqrt{x^2 + y^2} = a + b\sqrt{(x-c)^2 + y^2},$$

which is that of the curve known under the name of the First Oval of Descartes.\* Mr Maxwell had already observed that when one of the foci was at an infinite distance, (or the thread moved parallel to itself, and was confined in respect of length by the edge of a board,) a curve resembling an ellipse was traced; from which property Professor Forbes was led first to infer the identity of the oval with the Cartesian oval, which is well known to have this property. But the simplest analogy of all is that derived from the method of description,  $r$  and  $r'$  being the radiants to any point of the curve from the two foci;

$$m r + n r' = \text{constant},$$

which in fact at once expresses on the undulatory theory of light the optical character of the surface in question, namely, that light diverging from one focus  $F$  without the medium, shall be correctly convergent at another point  $f$  within it; and in this case the ratio  $\frac{n}{m}$  expresses the index of refraction of the medium.†

If we denote by *the power of either focus* the number of strings leading to it by Mr Maxwell's construction, and if one of the foci be removed to an infinite distance, if the powers of the two foci be *equal* the curve is a parabola; if the power of the nearer focus be *greater* than the other, the curve is an ellipse; if the power of the infinitely distant focus be the greater, the curve is a hyperbola. The first case evidently corresponds to the case of the reflection of parallel rays to a focus, the velocity being unchanged after reflection; the second, to the refraction of parallel rays to a focus in a dense medium (in which light moves slower); the third case to refraction into a rarer medium.

The ovals of Descartes were described in his *Geometry*, where he has also given a mechanical method of describing one of them,‡ but only in a particular case, and the method is less simple than Mr Maxwell's. The *demonstration* of the optical properties was given by Newton in the *Principia*, Book I., prop. 97, by the law of the sines; and by Huyghens in 1690, on the *Theory of Undulations* in his *Traité de la Lumière*. It probably has not been suspected that so easy and elegant a method exists of describing these curves by the use of a thread and pins whenever the powers of the foci are com-

\* Herschel on Light, Art. 232; Lloyd on Light and Vision, Chap. vii.

† This was perfectly well shewn by Huyghens in his *Traité de la Lumière*, p. 111. (1690.)

‡ Edit. 1683. *Geometria*, Lib. II., p. 54.

mensurable. For instance, the curve, Fig. 2, drawn with powers 3 and 2 respectively, give the proper form for a refracting surface of glass, whose index of refraction is 1.50, in order that rays diverging from  $f$  may be refracted to  $F$ .

As to the higher classes of curves with three or more focal points, we cannot at present invest them with equally clear and curious physical properties, but the method of drawing a curve by so simple a contrivance, which shall satisfy the condition

$$m r + n r' + p r'' + \&c. = \text{constant},$$

is in itself not a little interesting; and if we regard, with Mr Maxwell, the ovals above described, as the limiting case of the others by the coalescence of two or more foci, we have a farther generalization of the same kind as that so highly commended by Montucla,\* by which Descartes elucidated the conic sections as particular cases of his oval curves.

## 2. On the Influence of Contractions of Muscles on the Circulation of the Blood. By Dr Wardrop.

In this paper, Dr Wardrope states that he has endeavoured to shew, by a series of observations and experiments, that the muscles, besides being the active organs of motion, perform, by their contractions, an important office in the circulation of the arterial as well as venous blood; an office which has not hitherto been described by physiologists, but which appears to be capable of explaining several interesting phenomena in the living body, of which no satisfactory account has yet been given.

## 3. On the Solubility of Fluoride of Calcium in Water, and the relation of this property to the occurrence of that Substance in Minerals, and in recent and Fossil Plants and Animals. By Dr G. Wilson.

After a preliminary reference to the existence of fluorine in recent and fossil bones, Dr Wilson stated that he had made a series of experiments with a view to discover what solvent carried fluoride of calcium into the tissues of plants and animals. His first trials were made with carbonic acid, which was passed in a current through water containing pure fluor-spar in fine powder suspended in it. The fluor was by this treatment dissolved, yielding a solution which precipitated oxalate of ammonia, and when evaporated left a residue

\* *Histoire des Mathematiques. First Edit. II., 102.*

Fig. 1. Two Foci. Ratios 1:2.

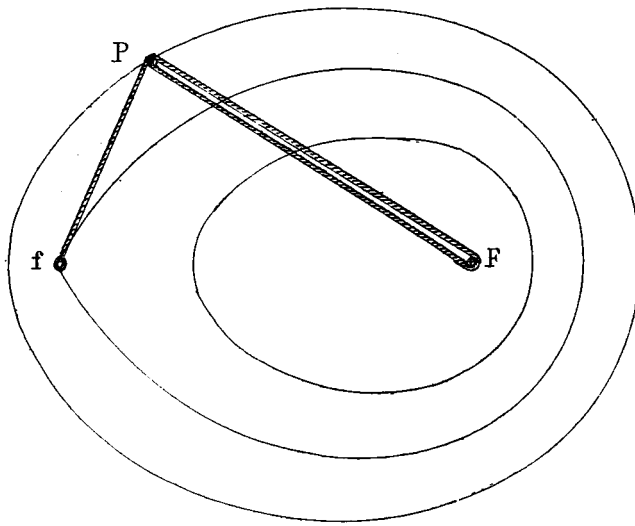


Fig. 2. Two Foci. Ratios 2:3.

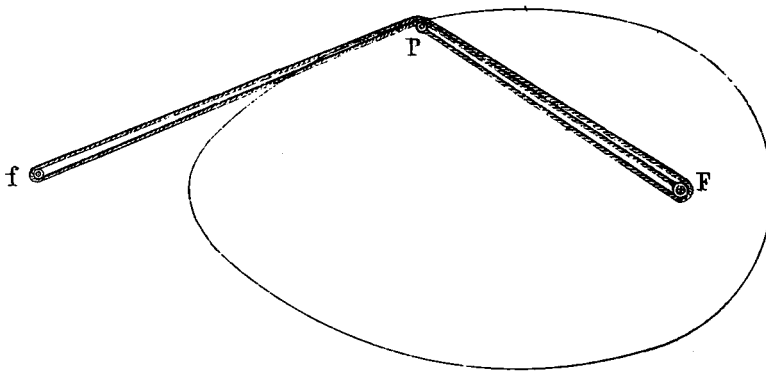


Fig. 3. Three Foci, Ratios of Equality.

