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O.W. Griffith B.Sc. A.R.C.S.

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The special purpose of this note is to make clear that the fifth fundamental equation of electromagnetic theory may be derived from the four field equations and the principle of relativity without making any arbitrary convention as to the mass of a moving body. It is quite unnecessary to place the transverse mass of a moving body equal to $\frac{m_0}{\sqrt{1-\beta^2}}$ and the longitudinal mass equal $\frac{m_0}{(1-\beta^2)^{3/2}}$. The simple relation for the mass of a moving body $m = \frac{m_0}{\sqrt{1-\beta^2}}$, which was derived

directly from the principle of relativity by Lewis and Tolman (*loc. cit.*) and from ideas of light pressure by Lewis (*loc. cit.*) is sufficient.

The fact that the fifth equation can be derived by combining the principle of relativity with the four field equations is one of the chief pieces of evidence which support the theory of relativity.

University of Michigan,
Ann Arbor, Mich.,
November 11, 1910.

XXXVI. *A Note on the Measurement of the Refractive Index of Liquids.* By O. W. GRIFFITH, B.Sc., A.R.C.S.*

FOR some years past the author has been setting his students, as a laboratory exercise, to determine the refractive index of water by using an ordinary spherical flask filled with water as a convergent lens. The results have always been strikingly concordant, the error in the values obtained by different observers being small and fairly constant. It was therefore thought that an inquiry into the best conditions for the experiment might prove interesting, and this paper contains the results of such an investigation and indicates two very simple methods of determining the index of refraction of liquids. It will be seen that these methods are capable of giving accurate and reliable values.

As a rule the problem of refraction through a sphere either receives very meagre treatment in the ordinary text-book or is relegated to the collection of mathematical exercises at the end of the book. It is, however, usually demonstrated that the principal points of a sphere are coincident with its centre. So that if U and V are the reciprocals of the distances of conjugate foci measured from the centre of a transparent

* Communicated by the Author.

sphere, and R is the curvature of the sphere whose refractive index is μ , then (with the usual modern convention as to signs)

$$U + V = 2R \cdot \frac{\mu - 1}{\mu}.$$

We may call the right-hand side of this equation the converging power of the sphere. If then F is the power,

$$F = 2R \cdot \frac{\mu - 1}{\mu}.$$

Hence, since F and R can be easily measured, μ for the sphere may be as easily calculated.

In the case of a spherical flask filled with a liquid, there are two disturbing factors which vitiate the values of μ obtained from the above equation directly. They are (a) the thickness of the glass, and (b) the spherical aberration. It will be an advantage to consider these two sources of error separately.

(a) *Effect of the thickness of the glass.*

Consider the refraction of a narrow axial pencil through a sphere of index μ enclosed in a concentric spherical shell of index μ' . Let the pencil diverge from a point in air the reciprocal of whose distance from the centre of the sphere is U . Let V_1, V_2, V_3, V be the corresponding quantities for the successive conjugate points after refraction at the several surfaces; and let R_1 and R_2 be the external and internal curvature of the shell respectively. Then we have the following equations for the refraction at the different surfaces,

$$\begin{aligned}\mu'U + V_1 &= R_1(\mu' - 1) \\ \mu V_1 + \mu'V_2 &= R_2(\mu - \mu') \\ \mu'V_2 + \mu V_3 &= R_2(\mu - \mu') \\ V_3 + \mu'V &= R_1(\mu' - 1)\end{aligned}$$

$$\text{whence} \quad U + V = 2 \left(R_1 \frac{\mu' - 1}{\mu'} + R_2 \frac{\mu - \mu'}{\mu \mu'} \right) \quad \dots \quad (1)$$

If F is the power of the system and $K = R_2 - R_1$,

$$\text{then} \quad F = 2R_2 \frac{\mu - 1}{\mu} - 2K \frac{\mu' - 1}{\mu'} \dots \dots \quad (2)$$

In the case of a thin spherical flask containing a liquid, equation (2) shows that the effect of the glass is to decrease

the power of the system, acting as a thin divergent lens placed at the centre of the sphere and of power $-2K \frac{\mu' - 1}{\mu'}$.

Putting $\delta\mu$ for the error in the value of μ introduced by an error δF in measuring F , we find

$$\delta F = 2R_2 \frac{\delta\mu}{\mu^2}$$

or $\delta\mu = \frac{\mu^2 \delta F}{2R_2} \dots \dots \dots (3)$

But the numerical value of δF due to the thickness of the glass is given by

$$\delta F = 2K \cdot \frac{\mu' - 1}{\mu'}$$

Hence the error introduced by the glass envelope is

$$\delta\mu = \frac{\mu^2 K}{R_2} \cdot \frac{\mu' - 1}{\mu'}$$

Let t = the thickness of the glass shell, and r_1 the external radius, then

$$K r_1 = t R_2,$$

whence

$$\delta\mu = \frac{\mu^2 t}{r_1} \cdot \frac{\mu' - 1}{\mu'} \dots \dots \dots (4)$$

To calculate the magnitude of the error, assume the following values:

$$\mu = \frac{4}{3}, \mu' = \frac{3}{2}, t = .05 \text{ cm.}, \text{ then } \delta\mu = \frac{.03}{r_1};$$

and if t be taken = .04 cm.,

$$\delta\mu = \frac{.02}{r_1}.$$

So that in the case of a flask 20 cms. in diameter, and of thickness .05 cm., the error would be .003, and this is of the order observed when using a flask of the size mentioned. Even were this factor of no account, it appears from equation (2) that an additional small error is introduced owing to the difficulty of measuring accurately the internal radius of the flask. It would be a great advantage if it were possible to express the value of F in terms of the external radius. By slight re-arrangement of the terms in the right-hand side of equation (1) we get

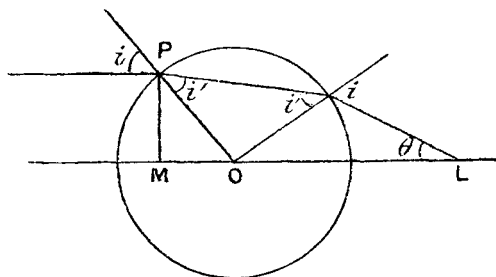
$$F = 2R_1 \frac{\mu - 1}{\mu} - 2K \left(\frac{1}{\mu} - \frac{1}{\mu'} \right), \dots \dots \dots (5)$$

which states that the system is equivalent to a water sphere of radius r_1 and a divergent lens of power equal to the second term. The value of this form of result will be evident when we have computed the effect of spherical aberration.

(b) *Effect of Spherical Aberration.*

Fig. 1 illustrates the refraction of a parallel beam through a sphere.

Fig. 1.



The focal length, measured from the centre, is

$$OL = \frac{OP \sin i}{\sin \theta}.$$

Also $\theta = 2(i - i')$ and $\sin i = \mu \sin i'$. Substituting for i' and expanding in terms of $\sin i$ by the Binomial Theorem, neglecting all terms above the second power, we get

$$\sin \theta = 2 \frac{\mu - 1}{\mu} \left[1 + \frac{3\mu - \mu^2 - 1}{2\mu^2} \cdot \sin^2 i \right] \sin i.$$

Putting $PM = a$, $OP = r$, then if f is the focal length,

$$f = \frac{\mu r}{2(\mu - 1)} - \frac{a^2}{r} \cdot \frac{3\mu - \mu^2 - 1}{4\mu(\mu - 1)}. \quad \dots (6)$$

The second term on the right-hand side of equation (6) represents the error due to spherical aberration, and its effect is to increase the converging power of the sphere. Applying this to the case of a thin glass flask filled with water, we may

assume that the greater part of the aberration is produced by the water sphere. Assigning the value $4/3$ to μ in the aberration term, we see that the error introduced in the value of f is numerically $\frac{11}{64} \frac{a^2}{r}$.

Now, reverting to equation (5) we see that the value of f , when the thickness of the glass is taken into account and the aberration is negligible, is given by

$$f = \frac{\mu r_1}{2(\mu - 1)} + \frac{K r_1^2 (\mu' - \mu) \mu}{2(\mu - 1)^2 \mu'}$$

or, since $K = \frac{t}{r_1 r_2}$,

$$f = \frac{\mu r_1}{2(\mu - 1)} + \frac{t r_1 (\mu' - \mu) \mu}{2 r_2 (\mu - 1)^2 \mu'} \quad \dots \quad (7)$$

Substituting $\mu = 4/3$ and $\mu' = 3/2$ in the second term on the right-hand side of equation (7), we find that the numerical value of that term is

$$\frac{2}{3} \cdot t \cdot \frac{r_1}{r_2}.$$

Hence the thickness of the glass tends to increase the focal length by an amount

$$\frac{2}{3} t \cdot \frac{r_1}{r_2},$$

and the spherical aberration tends to decrease it by

$$\frac{11}{64} \frac{a^2}{r_2}.$$

Hence it is possible by adjustment of the diameter of the aperture of the entrant beam to arrange that these errors should just compensate. This will be the case when

$$\frac{2t}{3} \cdot \frac{r_1}{r_2} = \frac{11}{64} \frac{a^2}{r_2},$$

$$\text{or} \quad \frac{a^2}{r_1} = \frac{128}{33} t = 4t, \text{ approximately.} \quad \dots \quad (8)$$

The following table gives the diameter of apertures that might be used with spheres of different radii.

Radius of sphere in cm.	Diameter of aperture in cm.	
	$t = \cdot 04$ cm.	$t = \cdot 05$ cm.
1	$\cdot 80$	$\cdot 90$
2	1.14	1.28
3	1.39	1.56
4	1.60	1.80
5	1.80	2.00

Small glass flasks of 6 cm. diameter and thickness $\cdot 04$ – $\cdot 05$ cm., painted over with dull black varnish except an aperture of about 1.5 cm. in diameter, seem to answer the purpose very well, and good values of the index of refraction of a liquid can be obtained by their use. It is essential that the exposed parts of the flask should be spherical, and this may be tested by measuring the radius of curvature at the aperture with a spherometer and comparing it with the distance between the two diametrically opposite apertures, as measured with a vernier calipers. The parallel beam is obtained by means of a collimator, and the position of the focus determined by the optical bench eyepiece, or by means of a travelling microscope, the source of light being the slit of the collimator illuminated with radium light. By using a short length of platinum wire heated with an electric current as a source and a thermopile or bolometer as detector, the method can be applied to obtain a rough estimate of the refractive index for heat radiation of quartz in the form of a sphere, or of a liquid (such as a solution of iodine in carbon bisulphide), enclosed in a fused silica flask.

It is beyond the scope of this note to discuss the general question of spherical aberration, but the accompanying diagrams are interesting in that they compare the aberrations produced in a sphere and in the usual types of convergent lenses. The effect of the aberration may be represented in two ways. If F is the power of the system, f its focal length, R the curvature of a surface and a the radius of the aperture, and μ the index of refraction, then the error produced by

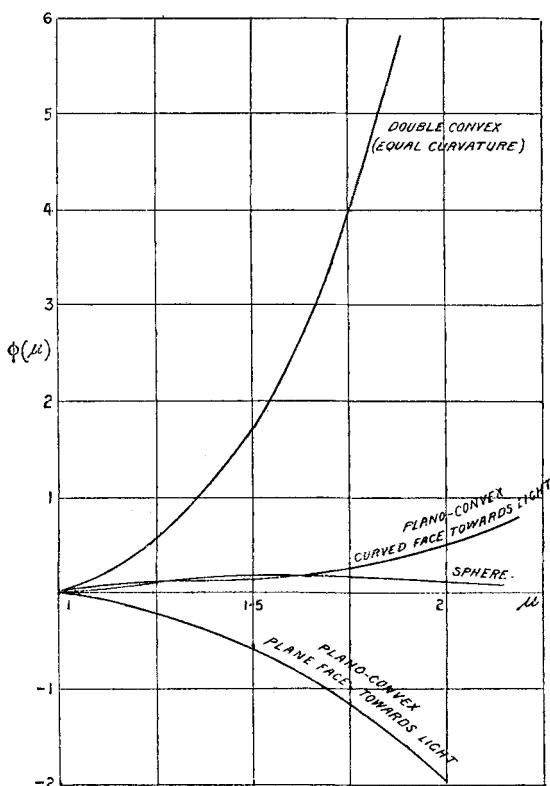
the spherical aberration of a parallel beam may be written

$$\Delta F = R^3 a^2 \phi(\mu), \quad (9)$$

$$\Delta f = -R a^2 \psi(\mu), \quad (10)$$

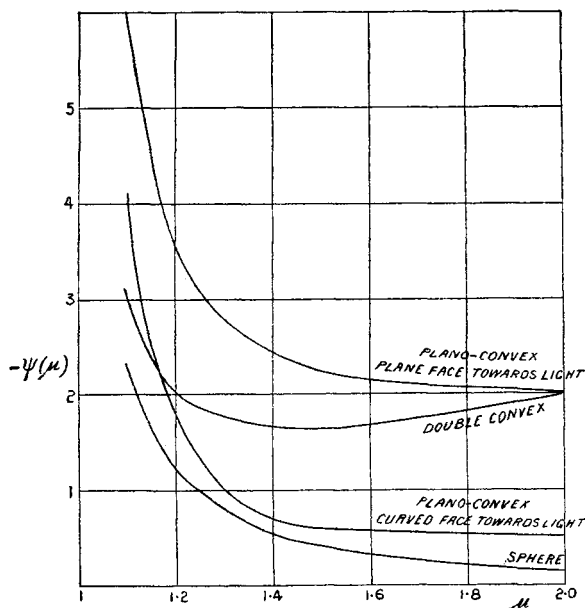
where $\phi(\mu)$ and $\psi(\mu)$ are certain functions depending on the type of lens employed. The curves are drawn for (1) a sphere, (2) a plano-convex lens with light incident on curved face, (3) a plano-convex lens with light incident on plane face, (4) double convex lens with faces of equal curvature. Fig. 2 represents the graph of equation (9) for values of μ between

Fig. 2.



1 and 2, and $R^3 a^2$ being unity in all cases. Fig. 3 gives the graph of equation (10) for values of μ between 1 and 2, the value of $R a^2$ being unity in all cases. It is noticeable that the errors in the case of the sphere are considerably less than they are for any of the other lenses.

Fig. 3.

*Second method of determining μ for a liquid.*

Arrange two plano-convex lenses so as to form a telescopic system of minimum aberration, that is so that a parallel incident beam may emerge as a parallel beam. Let the outer faces of the lenses be of equal curvature R , and let t be the thickness of each lens, and μ' the refractive index of its material. Then the equations representing the successive refraction of a parallel beam through the two surfaces of the first lens are

$$\begin{aligned}\mu'V_1 &= -R(\mu' - 1), \\ \mu V_2 &= \mu'V_1/(1 + V_1t),\end{aligned}$$

V_1 and V_2 having the usual meanings, and μ being the refractive index of the medium adjoining the plane surface of the lens. This gives

$$\mu V_2 = -\frac{\mu'(\mu' - 1)R}{\mu' - Rt(\mu' - 1)}.$$

Hence, if d = distance between the plane-faces of the lenses,

$$\frac{2\mu}{Rd} = \frac{\mu'(\mu' - 1)}{\mu' - Rt(\mu' - 1)}.$$

Let $d=d'$ when $\mu=1$.

Then evidently $d=\mu d'$.

For glass lenses of 10 cm. focus, $d=20$ cm., $d'\doteq 27$, when $\mu=4/3$. Therefore it is quite possible to measure both of these lengths to a high degree of accuracy. In the apparatus used, the two lenses are fitted on the ends of two tubes sliding one within the other through a stuffing-box. The apparatus is mounted on the table of a spectrometer whose telescope and collimator have been previously focussed for parallel light. The distance between the lenses is adjusted so that a clear image of the collimator slit is observed in the telescope with the tube (a) full of air, (b) full of water or other liquid. Index marks are placed on the tubes and the distance between these may be measured with vernier calipers, or by means of a travelling microscope if great accuracy is desired.

XXXVII. *The Destruction of the Fluorescence of Iodine and Bromine Vapour by other Gases.* By R. W. WOOD*.

[Plate III.]

AN extended study of the fluorescence of sodium, potassium, mercury and iodine vapour has shown that the intensity of the emitted light is greatly reduced if air, or some other chemically inert gas, is present. A quantitative study of the phenomenon, showing the relation between the intensity of the fluorescence and the pressure and molecular weight of the foreign gas, is much to be desired as a means of testing any hypothesis which may be made regarding the action of the gas upon the radiating molecules. The vapour of iodine is especially suited to the work, since its fluorescence can be observed at room temperature in glass bulbs, and the conditions of pressure, density, &c. can be accurately determined, which is nearly or quite impossible with sodium vapour.

A satisfactory theory of the phenomenon should not only explain the destruction of the fluorescence by the inert gas, but also the failure of bromine to show any trace of fluorescence when under the same conditions as iodine vapour. Its absorption spectrum is very similar, and yet it usually remains quite dark even under the most powerful excitation.

Some years ago I suggested the hypothesis that the molecule might be capable of storing up a certain amount of

* Communicated by the Author.