

NOTE ON THE LAW OF DISTRIBUTION OF VELOCITIES AMONG GAS MOLECULES.

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LET us consider one molecule. Let this molecule pass from a point A , to a point B , the time of passing being such that the molecule shall have come in contact with as many molecules as there are in the unit of volume. By taking a unit of volume large enough, the molecule considered will, when it reaches B , have passed through all possible velocities, and through each velocity in proportion to the frequency of that velocity among the molecules, *i. e.*, according to the law of distribution, if such exists.

The system consisting of the single molecule and all the other molecules is a conservative system. The sum of the kinetic energy of the single molecule and that of all the others, remains constant. The principle of least action applies.

Any particular velocity being represented by c , the number of times the molecule will have a velocity between c and $c + dc$ in passing from A to B , will depend on N , the total number of molecules in unit of volume, on some function of c , $\varphi(c)$, the form of which is to be determined, on dc , and on a constant A dependent on the temperature of the gas and the molecular weight.

The kinetic energy at any instant will be expressed by $ANc^2\varphi(c)dc$ and the action by $AN \int_0^\infty c^2\varphi(c)dc$.

That this may be a minimum, $\varphi(c)$ must have the form $\frac{\sqrt{h}}{\sqrt{\pi}} e^{-hc^2}$.

Hence the expression for the action becomes

$$\frac{AN\sqrt{h}}{\sqrt{\pi}} \int_0^\infty c^2 e^{-hc^2} dc.$$

As the variation in the action due to changes of temperature is involve in A , this expression is solely dependent on the number of velocities the molecule passes through, *i. e.*, it equals N . $\therefore A = 4h$. And the law of distribution is

$$\frac{4N\sqrt{h}}{\sqrt{\pi}} e^{-hc^2} c^2 dc.$$

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