



Director Circle of a Conic Inscribed in a Triangle

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then known, and constructed to indicate these correctly for 17,100 years. It lasted, in fact, but a very few years. During the troubles of the civil war it was cast away as old rubbish, and in 1646 was found by Sir Jonas Moore, and deposited in its dilapidated state at his house in the Tower.

From the time of the discovery of Algoristic arithmetic the pathway of practical science was northward across the Continent to England, and thus our early mathematicians were constantly on the watch to see what was going on in Italy and the Netherlands. Wright gave us a translation of a work of Stevin's called *The Haven-finding Art*, and he was the first to explain the principle upon which depended Mercator's projection, the inventor, Kauffman, having applied merely rule-of-thumb processes. In a historical dissertation prefixed to Robertson's *Navigation*, it is stated that nearly all that was done to advance navigation in the seventeenth and eighteenth centuries was the work of three men—Wright, Norwood, and Halley. With Napier, the inventor of logarithms, are associated the names of Briggs and Wright, as assisting in the promotion and improvement of this means of calculation. Wright translated into English Napier's great work on the *Description of Logarithms*.¹

There are some other matters of interest with which Wright had to do. He was the first to suggest the establishment of a standard length by the division of a meridian circle. The determination of the longitude was an unsolved problem in Wright's time. The idea of doing this by means of a clock had been suggested, but no clocks were accurate enough for the purpose. Wright elaborated a method which depended on the variation of the compass, sound and good in itself, but which improvements in clock-making rendered useless. The project of supplying London with water brought from springs near Ware, in Hertfordshire, was conceived and all but carried out by him. He was not, however, as able to finance schemes as to put them into shape for others to make use of. Wright was easily pushed aside, and Sir Hugh Middleton had all the glory of the famous work called the New River. No more fitting close to a notice of this too little-known mathematician can be given than the last few lines of the Latin paper

¹ Napier's other work on logarithms (*Mirifici Logarithmorum canonis constructio*) was first translated into English in 1889 by W. R. Macdonald.

above referred to: "Of him it may truly be said that he studied more to serve the public than himself, and though he was rich in fame and in the promises of the great, yet he died poor, to the scandal of an ungrateful age."

G. HEPPEL.

NOTES ON "A. I. G. T. SYLLABUS OF ELEMENTARY DYNAMICS." PART II.

(Macmillan & Co.)

The following notes may be found useful to those who are using the Syllabus.

Addendum to § 24, to follow line 6.—If two of the forces are parallel, this condition only necessitates that the third shall be parallel to them. Its line of action must be determined some other way, such as that of § 26, or of § 31.

Addendum to § 31.—If the two given forces are parallel, the above construction fails. But the theorem of moments still holds.

For, let p, q (Fig. i.) be the given parallel forces. Replace p by two components r, s , and let t be the resultant of q, s , so that r, t are equivalent to p, q . Then, generally, r, t will not be parallel. And therefore, by this section, the sum of the moments of p, q about any point = the sum of the moments of r, s, q = the sum of the moments of r, t = the moment of the resultant.

If the forces p, q had been equal and opposite parallel forces (Fig. ii.), the forces r, t would also have been equal and opposite parallel forces, so that in this case the new method also fails. For this case see §§ 33, 34.

DIRECTOR CIRCLE OF A CONIC INSCRIBED IN A TRIANGLE.

1. Let TP, TQ (Fig. i.) be tangents to a conic, C its centre, S, H its foci, $2a$ and $2b$ its axes. From S draw a perpendicular to TP and produce it to its image S' ; then we know that $S'T = ST$, angle $S'TP = PTS$, and $S'H = \text{major-axis} = 2a$. Similarly with a perpendicular drawn from H to TQ and produced to its image H' . Thus the two triangles $S'TH$ and STH' are equal, and the angles $S'TS, HTH'$, and therefore the halves of these angles, are equal, that is, "the tangents from T are equally inclined to the focal distances of T ."

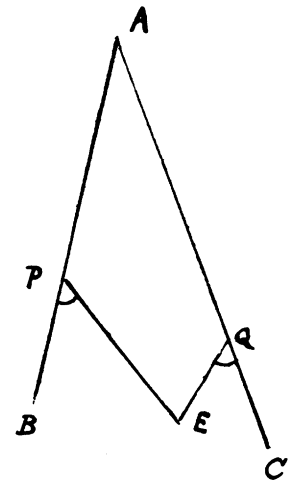
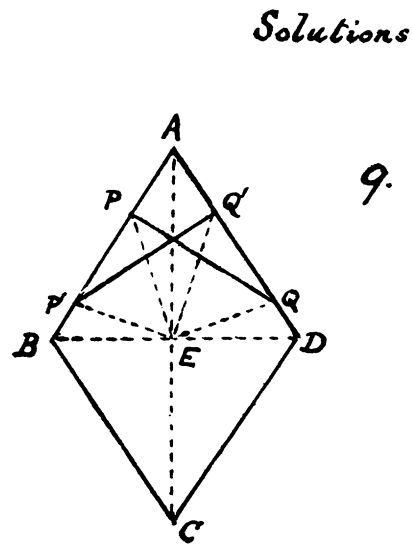
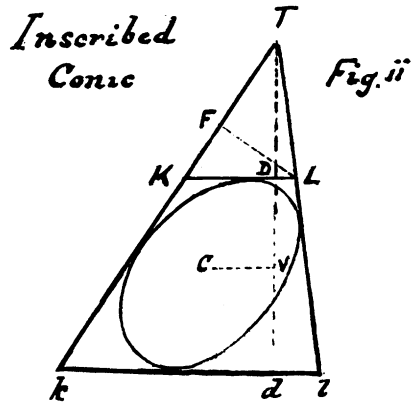
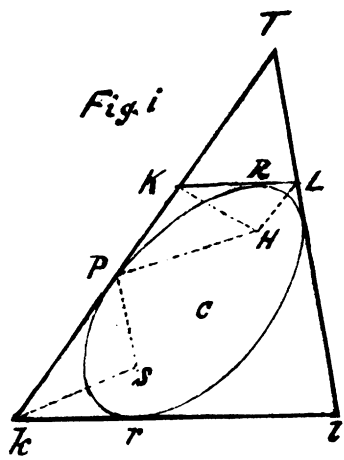
Taking either of the above triangles, we have

$$4a^2 = ST^2 + TH^2 - 2ST \cdot TH \cos \angle PTQ$$

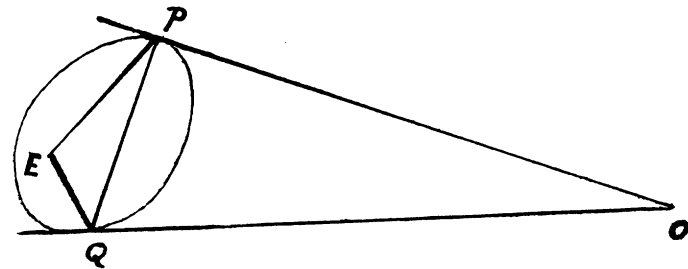
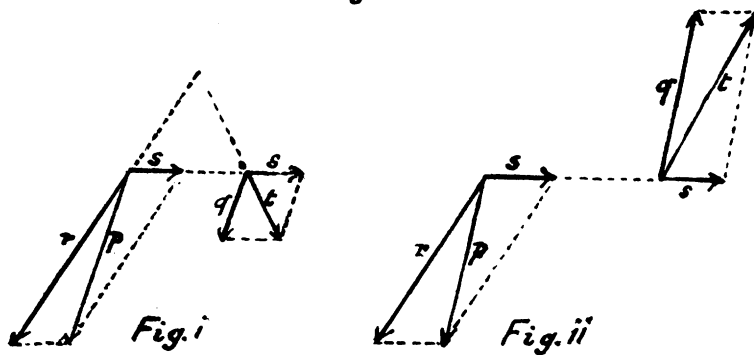
$$= 2CS^2 + 2CT^2 - 2ST \cdot TH \cos T,$$

$$\therefore a^2 + b^2 = CT^2 - ST \cdot TH \cos T.$$

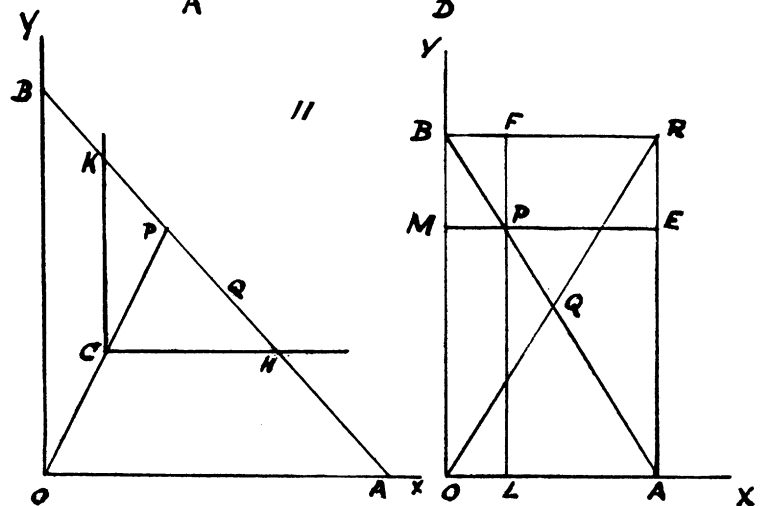
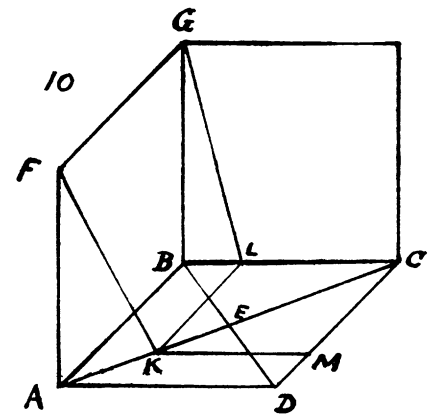
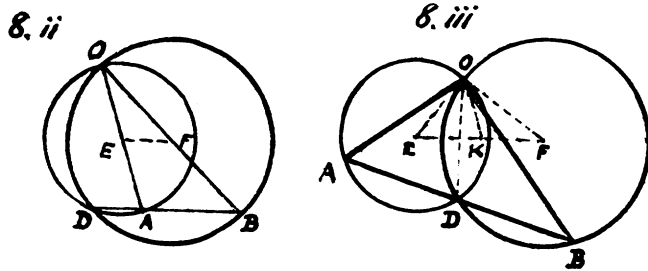
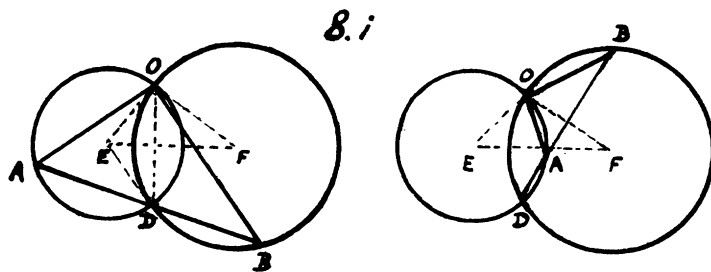
2. Let the tangents TP, TQ be cut in K, L , by the tangent at a third point R , and in k, l by the tangent



Note on Syllabus



Solutions



parallel to it. Then the rectangle $kP \cdot PK$
 $= sq. \text{ on radius parallel to } PK$
 $= SP \cdot PH$. Also angles SPk , HPK are equal.
 \therefore the triangles SPk , HPK are similar
 \therefore angle $PkS = PHK = \frac{1}{2}PHR = \frac{1}{2}PHQ - \frac{1}{2}QHR$
 $= THQ - LHQ = THL$;

and the angles kTS , HTL have been shown equal (1),
 therefore these triangles kTS , HTL are similar and the
 rectangle $kT \cdot TL = ST \cdot TH = \text{constant}$.

3. The result of Article 1 may now be written

$$a^2 + b^2 = CT^2 - kT \cdot TL \cos T.$$

Now draw LF (Fig. ii.) at right angles to TK , and TD
 at right angles to KL , passing through the orthocentre O ,
 and cutting kl in d . Also draw CV parallel to the tan-
 gents, and therefore bisecting Dd at right angles: and let us
 suppose the triangle TKL acute-angled, so that O falls
 within it.

The above result now becomes

$$\begin{aligned} a^2 + b^2 &= CT^2 - kT \cdot TF \\ &= CT^2 - TO \cdot Td \\ &= TO \cdot TD + CT^2 - TO(TD + Td) \\ &= TO \cdot OD + TO^2 + CT^2 - 2TO \cdot TV \\ &= TO \cdot OD + CO^2. \end{aligned}$$

[If, instead of TKL , we had taken the circumscribed
 triangle Tkl and o its orthocentre, we should have found
 in the same way $a^2 + b^2 = To \cdot od + Co^2$.]

Now make O the centre of a circle with radius
 $= \sqrt{TO \cdot OD}$, and the conclusion runs thus: "The director
 circle of any conic touching the sides of an acute-angled
 triangle cuts this circle at the extremities of a diameter."

If the triangle be obtuse-angled, so that O falls outside it
 (in which case the above circle is called the polar circle of
 the triangle), a similar investigation leads to the result
 $a^2 + b^2 = -TO \cdot OD + CO^2$, and the interpretation is, "The
 director circle of any conic touching the sides of an obtuse-
 angled triangle cuts the polar circle of the triangle
 orthogonally."

4. Hence follow easy proofs of some well-known pro-
 positions.

α . If the conic be a parabola, the director circle becomes
 the directrix, and we have, "The directrix passes through
 the orthocentre."

β . If the conic be an in-circle or ex-circle and r its
 radius, $a^2 + b^2$ becomes $2r^2$, and we have the property
 which has been used to prove Feuerbach's theorem that
 the nine-point circle touches each of the in- and ex-circles
 (see for example Richardson and Ramsey's *Modern Plane*
Geometry, pp. 32, 34).

From this property we can deduce a solution of a
 problem set in a scholarship paper of King's College, Cam-
 bridge, for December 1892. "Given C the centre of the
 inscribed circle of a triangle, and P its point of contact
 with a side, and O the orthocentre, construct the triangle."
 Through P draw a perpendicular to PC , and from O draw
 OD at right angles to it. With centre C and radius

$= CP \sqrt{2}$ describe a circle cutting OC produced both ways
 in $G \cdot G'$. The point where a circle through GDG' cuts
 OD again is a vertex of the triangle, and the rest follows
 at once.

γ . Let a conic touch the sides of a quadrilateral. Com-
 plete the quadrilateral. The above investigation applies
 to the orthocentre of each of the four triangles formed
 by the tangents, and to each of the conics touched by
 them; and as one of these conics can be a parabola we
 have this result: "The orthocentres of the four triangles of
 a complete quadrilateral lie on one straight line," viz. the
 directrix of the parabola.

δ . Except in the case of right-angled triangles, which
 we may neglect as being easy, we can see from a figure
 that two at least of these four triangles are obtuse-angled.
 Let us confine our attention to two obtuse-angled triangles
 and the corresponding polar circles. As the director circles
 of all inscribed conics cut these polar circles orthogonally, it
 follows that "the centres of the director circles, and, there-
 fore, of the conics, lie on a straight line." This line is the
 radical axis of the polar circles, and is therefore perpen-
 dicular to the line of orthocentres. Also, it is easily seen
 that "each of the director circles cuts the line of ortho-
 centres at two fixed points."

ϵ . Each diagonal of a complete quadrilateral is the
 ultimate form of an inscribed ellipse. Hence "The middle
 points of the three diagonals lie on a straight line," and
 "The centre of an inscribed conic lies on the straight line
 joining the middle points of the diagonals."

ζ . Lastly, the director circle corresponding to each of
 these diagonals is the circle which has that diagonal for its
 diameter, for the expression $a^2 + b^2$ becomes $a^2 + O$. Hence
 "The circles on the three diagonals of a complete quadri-
 lateral pass through the same two points," viz. the two
 fixed points on the line of orthocentres.

I reserve for another paper the propositions which can
 be deduced from a special case of Article 2.

E. P. ROUSE.

SOLUTIONS OF EXAMINATION QUESTIONS.

*The Editor will be glad to avail himself of the help of all
 classes of readers towards making this section of the Gazette as
 useful as possible. MATHEMATICAL TUTORs are invited to
 send neat solutions; STUDENTs to call attention to classes of
 problems presenting exceptional difficulties, and EXAMINERs who
 sympathise with us to forward copies of their papers. The help
 of foreign readers is especially requested in obtaining copies of
 papers set in the public examinations of other countries.*

7. A string hangs vertically from one side of a horizontal
 circular cylinder of given radius, and carries a heavy particle at
 its lower end. Find the least velocity with which the particle
 must be projected horizontally in order that the string may wrap
 itself round the cylinder, never becoming slack.

[Inter. Arts Hons. 92.]