



Notes on Elementary Dynamics. III

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to the three diagonals of the octohedron, and eight regular hexagonal faces in the planes of its faces. It will easily be seen that this solid has thirty-six equal edges, and twenty-four three-faced corners, each solid angle being contained by two hexagonal, and one square, face. Of the edges twenty-four are the intersections of a hexagonal and a square face, and twelve of two hexagonal faces. The dihedral angle between two hexagonal faces is the dihedral angle of a regular octohedron, and that between an hexagonal and a square face is the supplement of half the same, so that two of these latter and one of the former make up four right angles, and thus three "Kelvin fourteen-faces" may meet in a common edge, so as exactly to fill up the space about that edge.

It will have been observed that all the points, lines, and planes which occur in connection with the solids discussed above, have reference to (A) three rectangular directions in space, or the directions of the edges of a cube, (B) six directions bisecting the angles between each pair of the first three directions, or the six directions of the face diagonals of the cube, and (C) four directions equally inclined to the first three, or the four directions of the diagonals of the cube.

Thus the rhombic twelve-face cells in the partition of space have their four-face corners on lines parallel to the (A) directions, and their three-face corners on lines parallel to the (C) directions, while their faces are normal to the (B) directions, and the points of contact of the inscribed spheres are on lines in the same directions, and their edges are parallel to the (C) directions.

The general problem, of which it has only been possible here to touch the fringe, has been discussed by Bravais in the *Journal de l'Ecole Polytechnique*, tome 19, 1850; and by Lord Kelvin in his *Mathematical and Physical Papers*, vol. 3, as well as in his paper referred to above. Also by Sohncke, *Punksysteme als Grundlage einer Theorie der Krystallstruktur*; Karlsruhe, 1876.

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NOTES ON ELEMENTARY DYNAMICS. III.

COLLISION.

WE have seen that for solving problems on the simultaneous impact of more than two bodies, we require in addition to the laws of Abstract Dynamics, not only a *generalization* of Newton's Empirical Law as to change of relative velocity, but also the assumption that the impacts are simultaneous in the special sense of having their greatest compression at the same instant. In other words, it is only in a particular set of special cases that elementary methods can lead to a solution.

This seems to have been overlooked by many examiners in setting questions of this sort without the necessary restrictions.

I now proceed to touch on a few points regarding the theory of impacts which may be of interest to some who have learned the subject only from the elementary text books.

The use of such terms as "coefficient of elasticity," and "perfectly elastic body" in the theory of impact is perhaps to blame for the confusion that exists in the minds of many learners as to the true nature of the theory of collision. There is a vague impression that these terms imply a close connection between the Theory of Collision and that of Elastic Bodies. To prevent this confusion Thomson and Tait substituted the term "coefficient of restitution" for "coefficient of elasticity." Lord Kelvin has done much to give precision to our ideas regarding elasticity, and has summarized his views in the article on that subject in the *Encyclopædia Britannica*. Following him we may note the distinction between *statical* Perfect Elasticity, which implies that the forces required to keep a body at rest in a given state of *strain* or deformation are independent of the states of strain through which the body may have passed in reaching the state in question; and *kinetic* Perfect Elasticity, which implies that the forces are also independent of the rate at which the strain of the body may be changing, when not at rest. The former kind of Perfect Elasticity is possessed by many bodies, within certain limits, the latter probably by none, though for comparatively slow motions some bodies possess it approximately, within certain limits of strain. Lord Kelvin found by experiment that in wires subjected to torsional vibrations well within the limits of perfect statical elasticity, there is still a loss of energy due to a kind of internal resistance which he calls "viscosity" (as depending on the strain-velocity). This showed that the kinetic elasticity of such bodies is far from being perfect; and there is reason to believe that the same is true of all terrestrial substances.

The so-called "Perfect Elasticity" of bodies in the elementary theory of Impact is not to be identified with either of the above-defined qualities of bodies. It is attributed to bodies in which the "coefficient of restitution" is equal to 1, and so far as it can be related to the Theory of Elastic Bodies, is really a much more complex quantity than either. This can be best explained by considering more closely the physical nature of a collision.

When two or more bodies strike one another, there is in general a state of vibration set up in each body,* while at the

* This is perceived by the sense of touch when a batsman strikes a cricket ball, and by the ear when a bell is tolled.

same time the translational and rotational velocities of each body as a whole are changed. Broadly speaking, we may say that (1) part of the energy of the bodies concerned is turned into molecular vibrations, *i.e.* into heat, at the instant of collision, (2) part appears as energy of vibration of the bodies, passing continually from the kinetic to potential forms, and *vice versa* (as in the case of bells and other musical percussion-instruments), and (3) the remainder appears as kinetic energy in the motions of the bodies as wholes.

Part 2 soon gets dissipated, partly in setting up sound vibrations in the air, and partly into heat, in virtue of the "viscosity" or want of perfect elasticity of the substances, but not as a rule before the collision is over. Part 3 only is dealt with in the elementary theory of Impact.

It will be seen that even on the supposition of perfect static and kinetic elasticity in the bodies considered, it does not follow that the coefficient of restitution would be unity. The problem of calculating the exact nature of the motion after impact, even with such an ideal assumption, is a difficult one, and even for the simple case of the direct impact of two equal homogeneous spheres, the solution cannot be given in finite terms. One case only has been solved in finite terms, *viz.*, that of two uniform cylinders colliding directly in the line of their common axis. There it is found that if the cylinders be equal, they will *exchange* velocities, and separate without vibration, while if unequal, the shorter will have no vibration after impact, but the longer will be set in vibration longitudinally. The result when the cylinders are equal corresponds so far with the elementary theory of the impact of bodies whose coefficient of restitution is unity, but it has been found by experiment that the time of duration of impact is much greater than that given by the above calculation; and it is probable that the assumption of "perfect kinetic elasticity" for the rapid strain-changes during impact is quite erroneous.

Various other attempts have been made, with more or less success, to explain Newton's Empirical Law by the abstract Theory of Elastic Bodies with the aid of certain assumptions. These have been well explained by A. E. H. Love in his *Treatise on Elasticity*, Vol. II. Enough has been said, however, to show that the Elementary Theory of Impact stands on an empirical basis of its own, and that the terms "coefficient of elasticity" and "perfect elasticity" would be better omitted entirely from its nomenclature.

It may be added that the term "coefficient of elasticity," or, briefly, "elasticity," is applied by writers on the theory to certain quantities connecting the components of stress with those of strain in a given substance. The term "Modulus of

Elasticity" is defined by Thomson and Tait as "the number obtained by dividing the number expressing a stress by the number expressing the strain it produces." It is sometimes inaccurately called a "coefficient of elasticity." The term "resilience" is sometimes used as a substitute for "coefficient of restitution" in elementary Impact-theory: but though not so misleading as the term "coefficient of elasticity," is perhaps better reserved for other uses. See "Elasticity," §§ 53-66, in the *Encyc. Brit.*

R. F. MUIRHEAD.

MATHEMATICAL NOTES.

32. On the proof of the formula $S=ut+\frac{1}{2}ft^2$.

I should like to elicit opinions from mathematical teachers as to how far ordinary students can be expected to grasp what is implied in the proofs of this formula which dispense with the notion of infinitesimals, by introducing instead the conception of 'mean velocity.' It seems to me that, properly understood, these proofs imply all that is explicitly stated in such a proof as that given in Todhunter's elementary book; and that if the new proof appears simpler to the student, it is only because he does not understand it—because to him it is merely "a fudge." In the *Elements of Dynamics*, by the Rev. J. L. Robinson, for example, the proof commences with the statement (p. 47):

"Since the velocity increases *uniformly* throughout the given time, the *mean* velocity during the interval will be *half the sum of the extreme velocities*."

I am sure most students would accept this statement as a mere truism. Very possibly they would even fail to reproduce it if asked to write out the proof, and merely say:

$$\begin{array}{ll}\text{Velocity at beginning} & = u, \\ \text{,, end} & = u + at, \\ \therefore \text{Mean velocity} & = u + \frac{1}{2}at, \\ \therefore \text{Space described} & = ut + \frac{1}{2}at^2.\end{array}$$

And if they were asked to find the space described under uniformly increasing acceleration they would cheerfully proceed to do so, thus:

$$\begin{array}{ll}\text{Acceleration at beginning} & = a, \\ \text{,, end} & = a + \beta t, \\ \therefore \text{Mean acceleration} & = a + \frac{1}{2}\beta t, \\ \therefore \text{Velocity at time } t & = u + at + \frac{1}{2}\beta t^2.\end{array}$$

But velocity at beginning = u ,

$$\begin{array}{ll}\therefore \text{Mean velocity} & = u + \frac{1}{2}at + \frac{1}{4}\beta t^2, \\ \therefore \text{Space described} & = ut + \frac{1}{2}at^2 + \frac{1}{4}\beta t^3.\end{array}$$

It is true that in another part of the book in question a careful definition of 'mean velocity' is given; but the apparent simplicity of the proof is due to the fact that this definition is forgotten, or its force ignored. To really convince oneself that the 'mean velocity' is the arithmetic mean of the extreme velocities *because* the acceleration is uniform, one has to go through a process of reasoning not less complex than that given by Todhunter.