the colourless skin and the vivid scarlet of the exposed gills makes the appearance of this subterranean visitor striking in the extreme. It has four long, slender legs, that are gruesomely human in appearance, and are supplied with feet that are startlingly hand-like. The fore feet bear four fingers or toes and the rear ones have five, and though the legs are extremely slender, they possess a considerable amount of strength. Behind, the body terminates in a flattened tail that bears a fin like that of an eel.

In April 1899, two living specimens of this strange being were shipped by mail from San Marcos to the head office of the Fish Commission in Washington. They bore the journey of nearly 1800 miles, and reached their destination in good condition. They excited great interest, and for some time after their arrival a wondering group of spectators crowded about the aquarium into which they were put. These living specimens corrected several errors that had been made from observations of the dead bodies only. The legs are used for locomotion, and the animals creep along the bottom with a peculiar movement, swinging the legs in irregular circles at each step. They climb easily over the rocks piled in the aquarium, and hide in the crevices between them. All efforts to induce them to eat have been futile, as has also been the case with blind cave fish in captivity and they are either capable of long fasts or live on infusoria in the water.

From whence do these strange creatures come? The well is sunk in limestone, and that renders it likely that there may be some great cavern or subterranean lake communicating with it, but the rock through which the hole is bored is solid, except for a single channel two feet in diameter. The fact that the water rises nearly two hundred feet shows it to be under great pressure, and altogether this well affords material for study to geologists as well as zoologists.

Washington, D.C. CHARLES MINOR BLACKFORD.

Palæolithic Implement of Hertfordshire Conglomerate.

THE rudely-made Palæolithic implement, illustrated to half the actual size in the accompanying engraving, is probably unique in the highly intractable material from which it is made. It was found by me in May last with Palæolithic implements of flint in the Valley of the Ver, Markyate Street, near Dunstable: its weight is 1 lb. 6¼ oz.—1677 in my collection. Although rude, there is no doubt whatever as to its true nature; there is a large bulb of percussion on the plain side, as seen in the edge

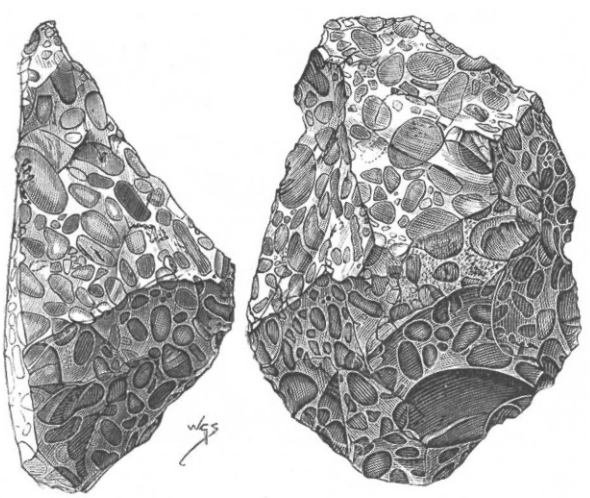

FIG. 1.—Palæolithic implement of Hertfordshire Conglomerate. One-half actual size.

view, and the hump-backed front is chipped to a rough cutting edge all round, each facet going right through the embedded pebbles. Its condition is totally different from a newly-broken block of Conglomerate, and indeed of Conglomerate broken in Roman times by quern-makers. It is faintly ochreous from being long embedded in clay, and sub-lustrous. Newly-broken Conglomerate is in colour a lustreless cold grey. The peculiar nature of the material would not admit of finer work: I have tried hard to flake Conglomerate without the slightest success; it breaks only after the heaviest blows, and then in the most erratic manner, the embedded pebbles often flying from the matrix. Sir John Evans has seen this example, and agrees with my conclusions as above expressed; he also informs me that several years ago he found what appears to be the point of a lanceolate implement of the same material and of Palæolithic character on the surface of a field near Leverstock Green.

Dunstable. WORTHINGTON G. SMITH.

On the Calculation of Differential Coefficients from Tables Involving Differences; with an Interpolation-Formula.

(1) IN NATURE for July 20 (p. 271) Prof. Everett has given formulæ for calculating first and second differential coefficients in terms of differences. The formulæ can be more simply expressed in terms of "central differences." Let the values of a function u_x be given for $x = \ldots, -2, -1, 0, 1, 2, \ldots$; then, with the usual notation,

$$\Delta u_0 = u_1 - u_0$$
$$\Delta^2 u_0 = \Delta u_1 - \Delta u_0 = u_2 - 2u_1 + u_0,$$
&c.

Now write

$$\tfrac{1}{2}(\Delta u_0 + \Delta u_{-1}) = a_0$$
$$\Delta^2 u_{-1} = b_0$$
$$\tfrac{1}{2}(\Delta^3 u_{-1} + \Delta^3 u_{-2}) = c_0$$
$$\Delta^4 u_{-2} = d_0$$
&c.

Then $a_0, b_0, c_0, d_0, \ldots$ are the "central differences" of u_0. Take, for instance, the following table:—

y	e^y	Δ			
4·7	109·947	11563			
4·8	121·510	12780	1217	126	
4·9	134·290	14123	1343	143	17
5·0	148·413	15609	1486	155	12
5·1	164·022	17250	1641	174	19
5·2	181·272	19065	1815	189	15
5·3	200·337	21069	2004	213	24
5·4	221·406	23286	2217	231	18
5·5	244·692	25734	2448	259	28
5·6	270·426	28441	2707		
5·7	298·867				

Writing $y = 5{\cdot}2 + {\cdot}1x$, and $u_x = 10^3 e^y$, so as to get rid of decimals, we have the following values corresponding to $y = 5{\cdot}2$ $(x = 0)$:—

u_0	a_0	b_0	c_0	d_0	e_0
181272	18157½	1815	181½	15	2½

With this notation, the value of u_x for values of x between $-\frac{1}{2}$ and $+\frac{1}{2}$ is given by

$$u_x = u_0 + xa_0 + \frac{x^2}{2!} b_0 + \frac{x(x^2-1)}{3!} c_0 + \frac{x^2(x^2-1)}{4!} d_0$$
$$+ \frac{x(x^2-1)(x^2-4)}{5!} e_0 + \ldots\ldots \quad \text{(i.)}$$

This is a well-known formula. Differentiating with regard to x, and putting $x = 0$, we have (writing u for u_x)

$$\left(\frac{du}{dx}\right)_0 = a_0 - \tfrac{1}{6}c_0 + \tfrac{1}{30}e_0 - \tfrac{1}{140}g_0 + \ldots\ldots \quad \text{(ii.)}$$

Similarly, differentiating twice, and putting $x = 0$,

$$\left(\frac{d^2u}{dx^2}\right)_0 = b_0 - \tfrac{1}{12}d_0 + \tfrac{1}{90}f_0 - \tfrac{1}{560}h_0 + \ldots\ldots \quad \text{(iii.)}$$

Prof. Everett's formula for the "increase-rate" when fifth differences are negligible is obtained by taking the first two terms of (ii.).

(2) The advantage of these formulæ, as Prof. Everett points out, is their greater accuracy. The ordinary formula

$$\Delta - \tfrac{1}{2}\Delta^2 + \tfrac{1}{3}\Delta^3 - \tfrac{1}{4}\Delta^4 + \tfrac{1}{5}\Delta^5,$$

in the above example, would give for $y = 5{\cdot}2$

$$\frac{du}{dx} = 18131\tfrac{1}{2},$$

while, if the differences were taken backwards, we should get

$$\frac{du}{dx} = 18124\tfrac{1}{3}.$$

The formula (ii.), taken to the fifth central difference, gives

$$\frac{du}{dx}=18127\tfrac{1}{3},$$

the true value being

$$\frac{du}{dx}=18127{\cdot}224.$$

The inaccuracy in the ordinary formula is, of course, due to the fact that a table such as the above never gives the exact value of the function tabulated, but only the nearest integral multiple of a certain unit (in this case ·001). If we denote this unit by ρ, each tabulated value differs from the true value by some quantity lying between $-\frac{1}{2}\rho$ and $+\frac{1}{2}\rho$. It may be shown that this makes it possible for $\Delta-\frac{1}{2}\Delta^2+\frac{1}{3}\Delta^3-\frac{1}{4}\Delta^4$ to differ from its true value by as much as $\frac{16}{3}\rho$, while $a_0-\frac{1}{6}c_0$ cannot differ from its true value by more than $\frac{3}{4}\rho$. Hence this latter formula is more accurate than the ordinary one in the ratio of 64 : 9, or about 7 : 1, when fifth differences are negligible. When only seventh differences are negligible, the formula $a_0-\frac{1}{6}c_0+\frac{1}{30}e_0$ is more accurate than the ordinary formula, in the ratio of 832 : 55, or about 15 : 1.

(3) The formulæ (ii.) and (iii.) give the first and second differential coefficients for the values of the "argument" shown in the table. It is often more useful to have them for the *intermediate* values. This requires a modification of the method of central differences. Let us write

$$\begin{aligned}\tfrac{1}{2}(u_1+u_0)&=\mathrm{V}\\ \Delta u_0&=\Delta_1\\ \tfrac{1}{2}(\Delta^2u_0+\Delta^2u_{-1})&=\Delta_2\\ \Delta^3u_{-1}&=\Delta_3\\ \&\text{c.}\end{aligned}$$

Thus for the interval from 5·2 to 5·3, in the above example, we have

V	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
$190804\frac{1}{2}$	19065	$1909\frac{1}{2}$	189	$19\frac{1}{2}$	9

With this notation, it may be shown that, for any value of x from 0 to 1,

$$\begin{aligned}u_x=&\left\{\mathrm{V}-\frac{1-2x}{2}\Delta_1\right\}\\ &-\frac{x(1-x)}{2\,!}\left\{\Delta_2-\frac{1-2x}{6}\Delta_3\right\}\\ &+\frac{x(1-x^2)\,(2-x)}{4\,!}\left\{\Delta_4-\frac{1-2x}{10}\Delta_5\right\}\\ &-\frac{x(1-x^2)\,(4-x^2)\,(3-x)}{5\,!}\left\{\Delta_6-\frac{1-2x}{14}\Delta_7\right\}\\ &+\ \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(iv.)}\end{aligned}$$

Or, if we write $x=\frac{1}{2}+\theta$, then for values of θ from $-\frac{1}{2}$ to $+\frac{1}{2}$,

$$\begin{aligned}u_{\frac{1}{2}+\theta}=&\{\mathrm{V}+\theta\Delta_1\}\\ &-\frac{1-4\theta^2}{2^2.2\,!}\left\{\Delta_2+\tfrac{1}{3}\theta\Delta_3\right\}\\ &+\frac{(1-4\theta^2)\,(9-4\theta^2)}{2^4\,.\,4\,!}\left\{\Delta_4+\tfrac{1}{5}\theta\Delta_5\right\}\\ &-\ \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(v.)}\end{aligned}$$

Differentiating this last expression twice with regard to θ, and putting $\theta=0$ we find

$$\left(\frac{du}{dx}\right)_{\frac{1}{2}}=\Delta_1-\frac{1}{24}\Delta_3+\frac{3}{640}\Delta_5-\frac{5}{7168}\Delta_7+\ \ldots\ldots \quad \text{(vi.)}$$

$$\left(\frac{d^2u}{dx^2}\right)_{\frac{1}{2}}=\Delta_2-\frac{5}{24}\Delta_4+\frac{259}{5760}\Delta_6-\frac{3229}{322560}\Delta_8+\ \ldots\ldots \quad \text{(vii.)}$$

Thus for $y=5{\cdot}25$, in the above example, we find

$$\frac{du}{dx}=19057{\cdot}17,$$

the true value being

$$\frac{du}{dx}=19056{\cdot}63.$$

(4) The ormula (iv.) is useiul for constructing tables by means of interpolation. For halving the intervals in a table, it gives

$$u_{\frac{1}{2}}=\mathrm{V}-\tfrac{1}{8}\Delta_2+\frac{3}{128}\Delta_4-\frac{5}{1024}\Delta_6+\frac{35}{32768}\Delta_8-\ \ldots \quad \text{(viii.)}$$

Similarly, for subdivision of the intervals into fifths,

$$\left.\begin{aligned}u_{\frac{1}{5}}&=\mathrm{V}-{\cdot}3\Delta_1-{\cdot}08\Delta_2+{\cdot}008\Delta_3+{\cdot}0144\Delta_4-{\cdot}000864\Delta_5\\ &\quad-{\cdot}0029568\Delta_6+{\cdot}00012672\Delta_7+{\cdot}000642048\Delta_8-\ \ldots\\ u_{\frac{2}{5}}&=\mathrm{V}-{\cdot}1\Delta_1-{\cdot}12\Delta_2+{\cdot}004\Delta_3+{\cdot}0224\Delta_4-{\cdot}000448\Delta_5\\ &\quad-{\cdot}0046592\Delta_6+{\cdot}00006656\Delta_7+{\cdot}001018368\Delta_8-\ \ldots\\ u_{\frac{3}{5}}&=\mathrm{V}+{\cdot}1\Delta_1-{\cdot}12\Delta_2-{\cdot}004\Delta_3+\ \&\text{c.}\\ u_{\frac{4}{5}}&=\mathrm{V}+{\cdot}3\Delta_1-{\cdot}08\Delta_2-{\cdot}008\Delta_3+\ \&\text{c.}\ ;\end{aligned}\right\}\text{(ix.)}$$

the terms in $u_{\frac{3}{5}}$ and $u_{\frac{4}{5}}$ being the same as in $u_{\frac{2}{5}}$ and $u_{\frac{1}{5}}$, but with signs alternately alike and different; and the sequence of signs in each case being . . . + + − − + + . . . The corresponding formulæ for subdivision into tenths might be found: but it is simpler to subdivide into halves and then again into fifths.

When several differences have to be taken into account, the above method of direct calculation is less troublesome than the ordinary process of building up the table by calculation of the sub-differences.

In the formulæ (ix.) the terms due to V and Δ_1 have been given in the form $\mathrm{V}-{\cdot}3\Delta_1$, $\mathrm{V}-{\cdot}1\Delta_1$, . . . ; but in practice these terms would be obtained by successive additions of $\cdot2\Delta$ to u_0, so that it is not necessary to calculate V.

August 16. W. F. SHEPPARD.

Apparent Dark Lightning Flashes.

ON the evening of the 5th of the present month we were visited by a severe thunderstorm, which passed practically over this place. The lightning was very vivid and at times occurred at intervals of only a few seconds. In order to photograph some of the flashes I placed a camera on my window sill and exposed four films for consecutive periods of 15 minutes each.

During the exposures I was observing the sky, and repeatedly found that after nearly each bright flash I could see distinctly a *reversed image* of each flash *in any part of the sky* to which I turned my head. These apparent dark flashes, or rather the images on my retina, lasted for sometimes 5 to 10 seconds. At the time I wondered whether dark flashes had ever been noticed before, and thought that this phenomenon was not uncommonly observed, but seeing Lord Kelvin's letter in your issue of August 10, I send this note in case it may prove of interest.

Westgate-on-Sea, August 13. WILLIAM J. S. LOCKYER.

Subjective Impressions due to Retinal Fatigue.

IN reading the interesting optical experience as described by Lord Kelvin in NATURE of August 10, it occurred to me that a somewhat similar effect on the eye, as noticed by myself, might be of interest.

Frequently late in the evening, and with a dull cloudy sky, I have seen my own figure, at least in part, apparently projected in gigantic form high up on the cloudy background.

This happened in the following manner. Going to the door of the house, and standing there with the strong light from the lobby or hall lamp shining out upon the gravel-walk in front, I saw my figure in shadow strongly defined upon the illuminated pathway. On raising my eyes quickly to the sky, I there saw the same form marked out on the dark clouds, but in a lighter shade.

The effect on the eye, as in Lord Kelvin's experience, is doubtless that of fatigue: in my experience, however, the form observed being very dark as compared with the illuminated background, I received the complementary impression of a light-coloured figure on a dark background.

The time during which this impression remained when looking at the clouds might be a couple of seconds.

August 14. W. J. MILLAR.

Mathematics of the Spinning-Top.

IT should have been stated on p. 321 that, while θ_3 is the angle between HQ and HQ′ in Fig. 1, p. 347, the angle between HS and HS′ is θ_2. At the same time this opportunity is available for some corrections, for which the printers are not responsible. On p. 321 the values of sin θ_3 and sh θ_1 should be interchanged; on p. 348, after equation (35), read ... "MX is the harmonic mean of MT, MT′ and of M*m*, M*m*′, ..."

August 12. A. G. GREENHILL.