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LII. Wave-lengths of electricity on iron wires

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LII. *Wave-Lengths of Electricity on Iron Wires.*
By CHARLES E. ST. JOHN, A.M.*

[Plate XII.]

THE question whether the magnetic properties of iron are called into play under extremely rapid alternations of the magnetizing forces is an interesting one, and has received various answers.

Hertz found negative results when he replaced one side of a rectangular copper resonator † by an equal iron wire; and in a later paper, on the “Finite Velocity of Electromagnetic Action” ‡, when he compares the rate of propagation along copper and iron wires, he concludes “that the rate of propagation in all wires is the same, and we are justified in speaking of it as a definite velocity. Even iron wires are no exception to this general rule; hence the magnetic properties of the iron are not called into play by such rapid disturbances” §.

Dr. Oliver J. Lodge attacked the question by means of his experiment on the alternate path. In his ‘Lightning Conductors and Lightning Guards’ (1892), he remarks:—“But everyone will say—and I should have said before trying—surely iron has more self-induction than copper. A current

* Communicated by Prof. J. Trowbridge.

† Wied. Ann. xxxi. p. 429 (1887). ‡ Ibid. xxxiv. p. 351 (1888).

§ ‘Electric Waves,’ p. 113.

going through iron has to magnetize it in concentric cylinders, and this takes time. But experiment declares against this view for the case of leyden-jar discharges."

Prof. John Trowbridge has shown* that the magnetic character of iron wires exercises an important influence upon the decay of electrical oscillations of high frequency, and that currents of such frequency as occur in leyden-jar discharges magnetize the iron. The spark in geometrically similar oscillating circuits of copper and iron was photographed by means of a revolving mirror, and the number of oscillations on the negatives compared.

Prof. J. J. Thomson has shown† that the presence of iron can affect the rapidly oscillating electric discharges through a rarefied gas by absorbing the energy of the discharges.

In a paper upon the "Absorption Power of Metals for the Energy of Electric Waves,"‡ V. Bjerknes has also given results that prove the great damping power of magnetic metals upon electric oscillations of very high frequency (100,000,000 double oscillations per second).

If the damping power of iron is due to the fact that its magnetic properties are brought into play under such rapidly alternating forces, it still remains an interesting question whether the self-induction of an iron circuit is measurably greater than that of a similar copper circuit, and whether the wave-length remains constant for oscillations of the same period.

In the determinations of the wave-length due to the Hertzian vibrator, the arrangement originated by Hertz§, and modified by Lecher|| and by Sarasin and De La Rive¶, has been very generally employed. In this arrangement secondary disks were placed face to face with the plates of the vibrator and near to them, to each secondary disk a long wire was attached, and these wires carried through the air parallel to each other, with, sometimes, an additional disk on the free ends.

With such an arrangement no exact adjustment of the length of the secondary circuit was required in order to excite powerful oscillations in it, for the direct electrostatic induction

* 'Proceedings of American Academy of Arts and Sciences,' xxv., May 27, 1891.

† Phil. Mag. [5] xxxii. p. 456, July 1891.

‡ Wied. Ann. xlv. p. 74 (1891).

§ Ibid. xxxiv. p. 551 (1888).

|| Ibid. xli. p. 850.

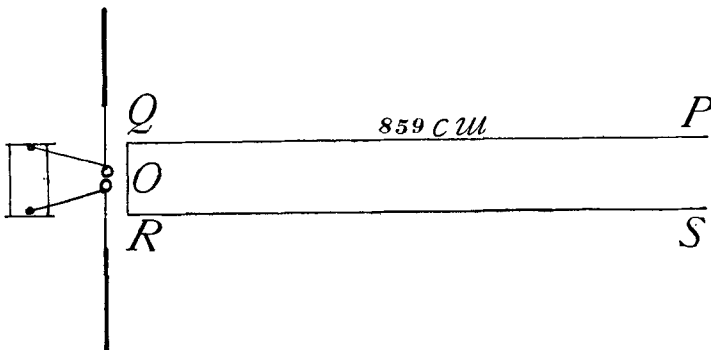
¶ Archives des Sciences Physiques, t. xxiii. p. 113 (1890).

between the plates of the secondary and the disks on the ends of the primary wires was so great that vigorous oscillations were produced along the secondary wires whatever their length might be, and several systems of waves could be detected which seemed to give experimental grounds for believing that the wave-system sent out from the Hertzian vibrator was very complex.

The capacity of the vibrator is increased by the presence of these secondary disks so near to the vibrator-plates, so that the wave-length found under these conditions is not that due to the simple Hertzian vibrator but that due to a very complex oscillating system with somewhat obscure internal reactions. Especially is this true when the wires are bridged as in the Lecher arrangement. The latter calls attention to the change in the sound of the primary spark when the secondary wires are bridged by a conductor. There is a very marked difference in the spark when the secondary circuit is removed entirely: the spark then loses much in body and explosive character. The secondary circuit under these conditions exerts apparently a strong reaction upon the primary.

It seemed desirable to devise some form of secondary depending more directly upon the principle of electrical resonance, the use of which would not increase the capacity of the vibrator, and whose reaction upon it would be a minimum. This was done by omitting the secondary disks and using simply a long wire as shown in fig. 1.

Fig. 1.

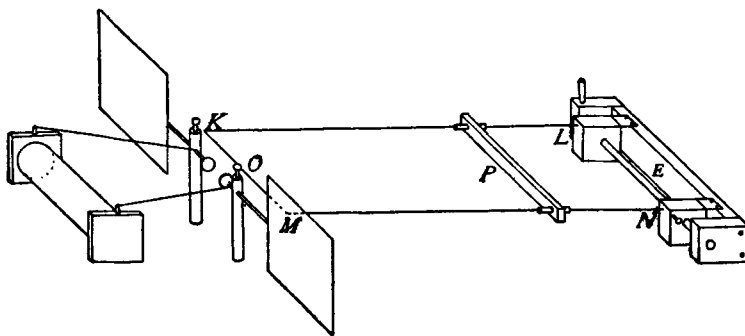


The secondary circuit consists of the long rectangle PQRS,

which is carefully adjusted to resonance before any other measurements are made.

For determining the occurrence of resonance and for exploring the wires to obtain the wave form, the bolometer as described by Paalzow and Rubens* was used in connexion with Rubens's† adaptation of it. The exploring terminals of the bolometer are shown at P, fig. 2. They consist of two

Fig. 2.



capillary glass tubes set in a frame of wood; the tubes slide over the wires to be explored, and around each tube is wrapped, by a single turn, one of the lead wires to the bolometer. Electric oscillations in the secondary circuit cause inductively alternating currents along the lead wires through one arm of a balanced Wheatstone bridge, which forms the bolometer. This arm of the bridge is made of fine iron wire, and so arranged that the bridge current and these oscillating currents traverse it without affecting each other. The bridge is thrown out of balance by the increase of resistance caused by the heat generated from the alternating currents, and a corresponding throw of the galvanometer is produced.

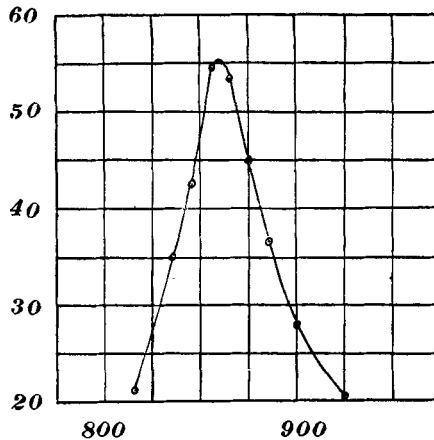
To adjust the circuit to resonance the exploring terminals were placed at P S (fig. 1). The induction-coil was put in action, and the reading of the bolometer taken for this length of wire. A few centimetres of wire were cut off and the reading again taken. This operation was repeated until a maximum

* "Anwendung des bolometrischen Principals auf electrische Messungen," Wied. *Ann.* xxxvii. p. 529.

† "Ueber stehende electrische Wellen in Drähten und deren Messung," *Ibid.* xlii. p. 154.

point was passed. The wires were renewed, and the operation repeated again and again. A sharp and unmistakable maximum was found when P Q was 859 centim. long. (fig. 3). The effect fell off rapidly either when the wires were lengthened or shortened from this point. The result is shown graphically in fig. 2 (Pl. XII.), where distances from Q are used as abscissas and deflexions of the galvanometer as ordinates. When the

Fig. 3.



circuit is thus arranged, there is little effect produced upon it by the vibrator unless it is near the point of resonance. The period of the vibrator is the controlling factor, and the influence of the secondary circuit is greatly reduced.

To determine the character of the vibration along the wire, the lengths Q P and R S (fig. 1) were fixed at 859 centim., the exploring terminals were moved along the rectangle, and the bolometer-readings taken for each position of the exploring terminals. The graphic representation of the results is given in fig. 1 of Pl. XII., where, as in all the curves, the abscissas are the length of the sides of the rectangle and the ordinates the bolometer-readings. The character of the curve indicates a simple form of vibration. The total length of the wire is equivalent to 7 half wave-lengths. The minimum points occur at nearly equal intervals, and the distance from the minimum at 748 centim. to the centre O (fig. 1) of the side Q R may be taken as 3 half wave-lengths. This furnishes a ready means of calculating the half wave-length.

$QR = 30$ centim. $748 + 15 = 763$ centim. $763 \div 3 = 254.3$ centim. = a half wave-length. The distance from this minimum to the end of the wire P should be a fourth wave-length of 127.15 centim. The actual distance is $859 - 748 = 111$ centim., so that the correction for the free end of the wire is about 16 centim.

To adjust the length of the wire under this arrangement was a work of considerable difficulty, but the possibility of using a single-wire circuit free from disturbing capacities overbalanced much inconvenience. To remove some of this, the ends P and S were wound upon wooden bobbins, so that shortening and lengthening could be produced without cutting the wire. This was a marked gain in convenience; but the changing size and form of the coils, as the wire was shortened or lengthened, altered the capacity at the end slightly and somewhat irregularly. This led to the adoption of the arrangement shown in fig. 2.

The secondary circuit consists of the rectangle KLMN with the side LN open; the lengths of the sides KL and MN can be varied between 15 centim. and 1000 centim. The ends are really formed by the small copper boxes L and N. These were 10 centim. square and 4 centim. thick, and mounted upon the wooden bar E by insulating supports. Within the boxes were wooden bobbins fixed on a hard rubber axle, and each capable of holding 10 metres of the largest wire experimented upon. In the front of each box was a small opening for the passage of the wire, but, to insure a firm contact between the wire and the boxes, a brass block was soldered on the inner side of the front and a binding screw passed in from the side of the box. The bar E was fastened to a wooden support resting upon a car which ran on a wooden track extending the entire length of the room. The car carried a brake so that the wires could be drawn taut, and the wooden screw held the axle from turning. With this arrangement the length of the wire could be varied at pleasure while the end capacities remained constant.

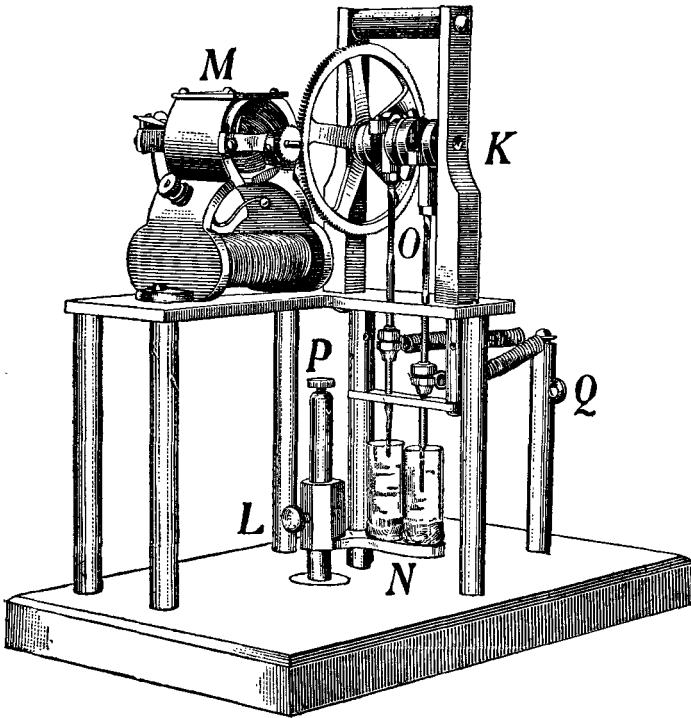
The end capacities are not a desirable feature, for their own sake, since they destroy the perfect simplicity of the plain rectangular circuit, and seem to detract somewhat from the sharpness of the maxima; but the gain in convenience and the possibility of obtaining a large number of observations whose average values can be used may overbalance these considerations.

For making and breaking the current through the induction-coil, an interruptor which would work with certainty and

regularity was much needed. With the assistance of the mechanician of the laboratory I devised an interruptor which gave very satisfactory results.

A small electric motor *M* (fig. 4) was used to produce the necessary motion. This was actuated by the current from two storage-cells, and ran at a fairly constant speed. The armature of the motor was in three sections, and was free from dead points, giving it the great advantage for the present purpose that it could be set in action simply by closing the circuit, making it possible to control it from the observer's station.

Fig. 4. $\frac{1}{4}$ Scale.



The motor was geared to the two-crank shaft *K* by means of a wheel and pinion. The speed of the shaft *K* was about 750 revolutions per minute, so that about 25 breaks were made per second. The plunging-rods were thinned at *O* so that

they were flexible and gave the required freedom of motion. They ran through the bed-plate and the brass bar below, which served as guides. The plunging-rods carried bind-screws by which the flexible coils leading the current from the brass post Q were attached. The lower ends of the plungers were of No. 18 platinum wire. The brass bottoms of the glass mercury-cups screwed into the brass arm N, which was adjustable by means of the collar and binding-screw L along the pillar P. At P was attached one pole of the battery actuating the coil and also one pole of the condenser in the base of the coil, and at Q was attached one pole of the coil and the other pole of the condenser. The cups were filled with mercury to a height of 8 millim. and then filled with alcohol to within a few millimetres of the top. They usually required cleaning only after several hours' use, when the surface of the mercury consisted of very fine globules, and sharp breaks were not made at each stroke of the plunger as was indicated by the occasional failure of the spark. The character of the spark depended much upon the exact height of the mercury-cups. The adjustment was best made while the coil was in action by raising or lowering the cups until the spark had a white body and a peculiar snap.

The plates of the Hertz vibrator were 40 centim. square and fixed at 61 centim. apart. The spark-gap was supplied with platinum-faced balls (3 centim. in diameter), which worked with less trouble than the usual brass ones. The side K M (fig. 2) of the secondary circuit was parallel to the conductor forming the vibrator plates, and fixed at 6 centim. distance with its centre O opposite the spark-gap. The long sides of the rectangular secondary lay in a horizontal plane, and ran through the centre of the room at a height of 1.6 metre above the floor. They were held by their end supports at 30 centim. apart. The induction-coil was 53 centim. long, 19 centim. in diameter, and was excited by the current from five storage-cells. A sparking distance of about 6 millim. was most effective in producing oscillations in the secondary circuit.

The following method was pursued in taking the observations. The interruptor was set in action, the circuit closed through the induction-coil, and an observation taken of the first swing of the needle. The circuits were broken as soon as the needle reached the end of its first swing, and the extent of this excursion was the reading recorded. In accordance with the experience of Paalzow and Rubens, it was found that a steady deflexion could not be obtained, but this first

swing was, under like conditions, satisfactorily constant; and a preliminary calibration of the instrument by passing currents of known strength through it, showed that the square root of the deflexion taken in this manner was in a constant ratio to the current.

The same copper wire (diameter 0.1201 centim.) that had been used in the secondary without end capacities was used for the rectangle K L M N in fig. 2, and, with the exploring terminals close to L and N, the maximum point was found by lengthening and shortening the wire. Bolometer-readings were taken for each length used. To insure the constant activity of the spark, a convenient length was taken as a point of reference, and observations taken at this point before and after a series of readings. If the spark had remained constant, the readings were retained. A maximum point was found when K L was 818 centim. The sides were fixed at this length, and the form of the wave was obtained by sliding the exploring terminals along the wire, and taking bolometer observations for each position. The result is shown in fig. 3 of Pl. XII. The critical points were determined several times, and the steadiness of the spark insured by choosing, as before, a point of reference. The curve shows three minima at 240, 496, and 752 centim., starting from O. These give half wave-lengths of 255, 256, and 256 centim., with an average of 255.6 centim. The third minimum at 752 centim., was determined with care, as it was to be used as a basis for calculating the half wave-length. An error in determining the position of this minimum would be divided by three, since the distance from O to this minimum was three half wave-lengths. The total length of the circuit was seven half wave-lengths, and it was the equivalent of one fourth of a wave-length from the third minimum to the end. The actual distance to the end was $818 - 752 = 66$ centim. $127.8 - 66 = 61.8$ centim. = the equivalent of the capacities in centimetres of wire.

A comparison of the curve (fig. 1, Pl. XII.) obtained from the plain wire circuit with the curve (fig. 3) obtained when capacities were fixed on the free ends, shows a quite satisfactory agreement, which tends to create confidence in both methods. The half wave-length by the first is 254.3 centim. and by the second it is 255.6, values which differ by about one-half of 1 per cent. There is a marked difference, as was to be expected, in the form of the curve next the free ends. When end capacities were used, the accumulation of charges seemed largely confined to them, out of reach of the

exploring terminals, while with the plain wire it seemed distributed over a greater distance. In each case the effect of the ends was to make the curve depart from the normal form along the free wire.

The theory of my investigations rests upon the principle of electrical resonance. The sides of the rectangle K L M N (fig. 2, p. 428) were shortened to a few centimetres in length, so that it could be safely assumed that the period of the secondary was considerably shorter than that of the vibrator. The exploring terminals were kept at L N, and bolometer observations taken for each small addition to the length of the sides K L and M N. When best resonance was found with the shortest length of the secondary circuit that gave a maximum, it was assumed that the secondary had the same period as the vibrator and that its equivalent length was a half wave-length, its actual length depending upon the effect due to the free ends. The occurrence of resonance is a very marked phenomenon even with a vibrator that damps as rapidly as the Hertzian. The accompanying table shows two series of readings for the first maximum when an iron wire was used:—

Length of sides of rectangle...	15	25	35	40	42.3	45	50	60	75
Deflexions of Galvanometer...	107	145	156	194.3	199.2	181.5	140	81	42
" " " ...	94	119	161	185	191	178	136	76	34

There can be no free motion of electricity at the ends of the secondary circuit, but an accumulation alternately positive and negative, and a resulting alternation of potential, the phase at L being always opposite to the phase at N in case of resonance. Elsewhere along the circuit the electricity moves with more freedom and less accumulation. The point O may be called the electrical middle of the circuit, where the accumulation is least and the movement most unrestrained. The electromotive impulses from the vibrator act directly upon the side K M, so that O remains a point of free motion or the ventral segment of the wave, while L and N are always places of no electric movement, or the nodal points. The shortest circuit being a half wave-length, a second resonating circuit ought to be found by increasing each side of the rectangle by a half wave-length, making the circuit three half wave-lengths long, and a third when the circuit is five half wave-lengths, and so on*. It is known that the change of period

* J. J. Thomson, 'Recent Researches in Electricity and Magnetism,' p. 297.

produced by replacing copper by iron does not exceed 2 per cent. The difference in length between a copper and an iron circuit of the same period would be very small with circuits a half wave-length long; but this difference would be three times as great with circuits three half wave-lengths long, and there might be a cumulative difference that would become measurable by the use of circuits of still greater length. To examine this question, a copper wire (diameter 0.1201 centim.) was used as the secondary circuit in fig. 2. The sides were taken 15 centim. long, and gradually lengthened to 875 centim., and bolometer-readings taken for each addition, the exploring terminals being always at the ends L and N. The result is shown graphically by the upper curve in fig. 4, Pl. XII. The critical points in the curve are the results of many separate determinations. The unsteadiness of the spark on the vibrator made the determinations somewhat laborious, though a single series of observations would locate a maximum very closely. After this had been done, the space of about a metre, including the maximum point, was worked over forward and back; the constancy of the spark was insured by choosing a convenient point of reference, as already described.

An examination of the curve shows four maxima occurring when the sides of the rectangle were 45, 306, 562.5, and 818 centim. long. The additions of wire for the successive maxima after the first were 261, 256.5, and 255.5. These should be half wave-lengths. The last two agree well, but the first differs from the average of the last two (256 centim.) by 5 centim. The sides were fixed at 818 centim. and the wave-form, fig. 3 (Pl. XII.), was obtained. From this the half wave-length was found to be 255.6 centim., and the total length of the circuit seven half wave-lengths. By fixing the sides of the rectangle at 562.5 centim. and 306 centim., a similar investigation showed the circuits to be respectively 5 and 3 half wave-lengths long.

An explanation of the fact that the distance between the first and second maxima was anomalously large may possibly be this: for the first maximum the sides of the rectangle were but 45 centim. long, so that the effect of the closed end was relatively great and the maximum appeared earlier than it otherwise would, but when the rectangle was 300 centim. long, the influence of the closed end became relatively small, and the second and future maxima came in the normal positions. Besides, in the first case the capacity was largely local, while in the others it was mainly distributive. This same effect appeared in every case, and seemed a constant phenomenon.

The maximum I omitted from the above discussion was not constantly present, but appeared when the primary spark was especially active, and seems to belong to a circuit whose period is to the period of the vibrator in the ratio of 5:3. The sides of the rectangle were 127·5 centim. long and the end capacities equivalent to 62 centim. of wire. The half wave-length was $30 + 127\cdot5 \times 2 + 62 \times 2 = 409$ centim. $409 \div 255\cdot6 = 1\cdot6$ nearly.

This was the only indication of complexity in the vibration of the Hertz vibrator; and it may be explained by supposing that such a vibration is superposed upon the fundamental rate of the oscillator, or that the oscillator when particularly active can excite a circuit having such a ratio to itself. This last seems the simpler, but it is difficult to see why a circuit of the ratio 3:2 should not be excited as well. The maximum I would indicate the presence of an undertone rather than an overtone. This point requires more investigation than it has been possible to give it, and will be left undecided for the present.

An annealed iron wire (diameter 0·1186 centim.) was put in place of the copper, and the same series of observations repeated. The results are shown in the lower curve of the upper pair in fig. 4, Pl. XII. The maxima E F G H appear at 42·5, 301, 553, and 805 centim., in each case before the corresponding maximum with the copper; and the difference is seen to increase with the length of the circuits. The successive additions were 258·5, 252, and 252 centim., the last two agreeing, but the first, as with the copper, is much larger. With the sides of the rectangle fixed at 805 centim., the form of the wave was found as shown in fig. 2 of the plate. The third minimum occurs at 740 centim. Calculated as before, the half wave-length is $740 + 15 = 755$. $755 \div 3 = 251\cdot6$ centim. This agrees well with the value 252 given above by the last two additions, but differs by 4 centim. from the value found when the copper was used.

The same series of observations was repeated with a second pair of finer wires (diameter of copper wire 0·07836 centim., diameter of iron wire 0·0785 centim.). The results are shown in the lower pair of curves in fig. 4 of the plate. A comparison of the curves shows the same general result, which appears more distinctly from the following table :—

	1st Maximum.			2nd Maximum.			3rd Maximum.			4th Maximum.		
	Cu.	Fe.	Diff.	Cu.	Fe.	Diff.	Cu.	Fe.	Diff.	Cu.	Fe.	Diff.
Upper pair...	45	42.5	2.5	306	301	5	562.5	553	9.5	818	805	13
Lower pair...	40	37.5	2.5	300	294	6	552	540	12	799	784	15

The successive differences should be in the ratio of 1, 3, 5, 7, if the theory of the present investigation is correct. The differences for the first two maxima are very small, so that the experimental error in their determination would be relatively large, and in the case of the fourth maximum the damping was so great that it was difficult to fix the point with certainty. The difference for the third maximum was relatively large and the determination of the point was sharp. Taking this difference as a point of reference, the calculated and observed values are shown in the accompanying table:—

	1st Maximum.		2nd Maximum.		3rd Maximum.		4th Maximum.	
	Calcu- lated.	Ob- served.	Calcu- lated.	Ob- served.	Calcu- lated.	Ob- served.	Calcu- lated.	Ob- served.
Upper pair...	1.9	2.5	5.7	5	9.5	9.5	13.3	13
Lower pair...	2.4	2.5	7.2	6	12	12	16.8	15

The observed half wave-lengths for the four wires are :—

{	Copper (diameter 0.1201 centim.)	255.6 centim.
{	Iron („ 0.1186 centim.)	251.6 centim.
{	Copper („ 0.07836 centim.)	251.6 centim.
{	Iron („ 0.07850 centim.)	246.8 centim.

The wires in each pair were as near the same diameter as could be found, the iron of the larger pair having slightly the smaller diameter, but the copper being the smaller one in the second case. In other respects the circuits compared were as nearly identical as possible. The capacity per unit length being the same for wires of the same diameter, the shortening

of the wave-length when iron displaced copper must be caused by an increased self-induction due to the magnetic properties of the iron. This implies that the magnetization of iron can be reversed 115 million times per second. This reduced the "time lag" of magnetization to very narrow limits, if forces of such duration can magnetize the iron.

In the case of extremely rapid oscillations Prof. J. J. Thomson has shown ('Recent Researches in Electricity and Magnetism,' sec. 295) that approximately $y^2 = \frac{2}{L'C}$, where $\frac{y^2}{4\pi^2}$ is the square of the frequency, and L' is the self-induction for very rapid oscillations and C the capacity of the system. It is easy from this to calculate an approximate value for the ratio between the self-induction per unit length of the iron and copper circuits.

Let L = the self-induction of the copper per unit length.

L' = " " " iron " "

C = the capacity of either per unit length. "

Using as a basis of calculation the data from the third maximum of the curves in fig. 4, Pl. XII., the total length of the copper circuit (diameter 0.1201 centim.) is:—

The sides, 562.5×2 = 1125 centim.

The closed end = 30 "

The equivalent of the end capacities $62 \times 2 = 124$ "

1279 centim.

For the wire (diameter 0.1186 centim.) the length is:—

The sides, 553×2 = 1106 centim.

The closed end = 30 "

The equivalent of the end capacities $61 \times 2 = 122$ "

1258 centim.

Since the two circuits have the same frequency, the products of the self-induction and capacity are equal.

$$1258^2 L'C = 1279^2 LC,$$

$$\frac{L'}{L} = 1.034.$$

In the same manner, for

$$\begin{cases} \text{Copper (diameter 0.08840 centim.),} \\ \text{Iron (" 0.08847 centim.),} \end{cases} \quad \frac{L'}{L} = 1.041.$$

$$\begin{cases} \text{Copper (diameter 0.07836 centim.),} \\ \text{Iron (" 0.07850 centim.),} \end{cases} \quad \frac{L'}{L} = 1.043.$$

By the use of Lord Rayleigh's formula for inductance under very rapid oscillations, it is easy now to calculate a value for the permeability of the iron.

Lord Rayleigh's formula is:—

$$L' = l \left(A + \sqrt{\frac{\mu R}{2pl}} \right),$$

where l is the total length of circuit, A a constant depending only on the form of the circuit, or lA is the inductance of a similar copper circuit; μ the permeability; R the ohmic resistance; $p = 2\pi n$, where n is the number of complete oscillations per second.

The value of $p = 2\pi n = 36 \times 10^7$.

R for iron wire (diam. 0.1186 centim.) = 1.328 ohm per metre.

" " " (" 0.08847 ") = .227 " "
 " " " (" 0.0785 ") = .301 " "

For iron (diameter 0.1186 centim.) :—

$$L' = 1.034 L = l \left(A + \sqrt{\frac{\mu R}{2pl}} \right),$$

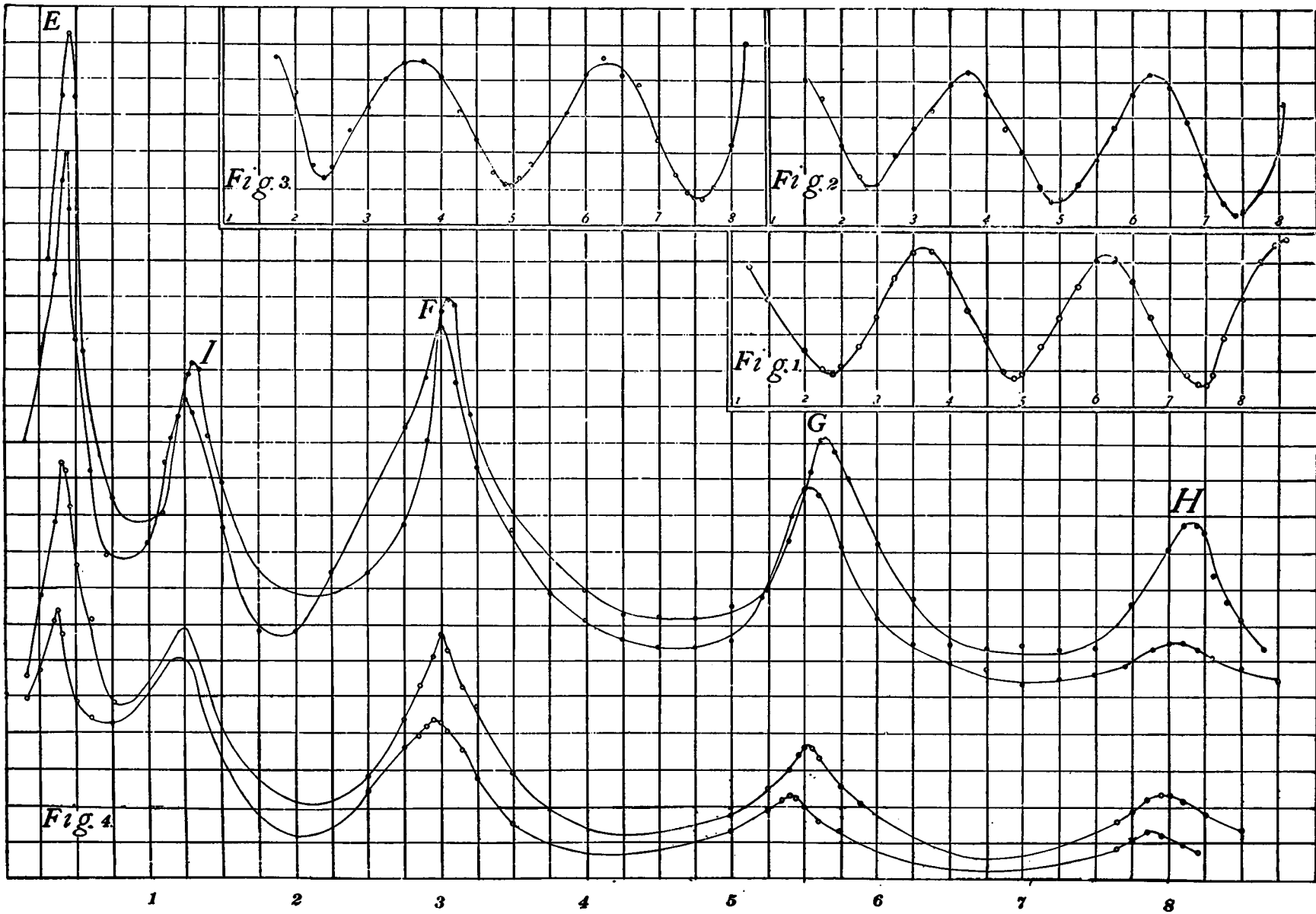
$$L + .034 L = L + l \sqrt{\frac{\mu R}{2pl}},$$

$$.034 L = l \sqrt{\frac{\mu R}{2pl}}.$$

Calculating the value of L for a similar copper circuit l units long, substituting the value in the above equation, and solving for the three cases, we get :—

For the iron wire, diameter 0.1186 centim., $\mu = 430$,
 " " " 0.08847 " $\mu = 389$,
 " " " 0.0785 " $\mu = 336$.

These values for the permeability all fall within a reasonable limit, and have for an average $\mu = 385$. Those are the values found for different specimens of wire made by the same company, but the specimens were wound and unwound and stretched many times during the series of observations. Besides the shortening of the wave-length there is shown a decided increase in the damping, as has already been observed by Trowbridge and Bjerknes. In fig. 4 (Pl. XII.) the curves for iron fall below the corresponding ones for copper, but owing to the change in the activity of the spark no exact measurement was made. It was only observed that the bolometer throws with the copper circuit were always



greater than with the iron circuit of the same dimensions, when the spark was constant as far the eye and ear could judge.

A value can readily be calculated for the damping factor $e^{-\frac{Rt}{L}}$ in the case of the iron and copper. Lord Rayleigh's formula for the resistance under very rapid oscillations is :—

$$R' = \sqrt{\frac{1}{2} p l \mu R}.$$

For the iron wire circuit (diameter 0·1186 centim.), $l = 1258$; $\mu = 430$; $R = 1·67 \times 10^9$; $p = 36 \times 10^7$, whence $R' = 403 \times 10^9$, $L = 34 \times 10^3$.

The damping factor becomes $e^{-6 \times 10^6 t}$ approximately.

The time required for the amplitude to fall to one half its maximum value is $t = 0·000000115$ sec. On the basis of 115×10^6 alternations per second, the number of complete oscillations during this time is 6·5. A like calculation for the corresponding copper circuit gives about 60 times as many.

The following table shows the results when copper circuits are compared in which wires of different diameters are used:—

					3rd Maximum.
Copper wire (diameter 0·1201 centim.)					562·5 centim.
"	"	("	0·0884	") 553·5 "
"	"	("	0·07836	") 552·0 "
"	"	("	0·03915	") 535·0 "

The half wave-lengths calculated from this maximum are :—

Copper (0·1201 centim.)	255·8 centim.
" (0·0884 ")	252·2 "
" (0·07836 ")	251·6 "
" (0·03915 ")	244·8 "

There are found by dividing the total length of the circuit by 5 :—

$$535 \times 2 = 1070 \text{ length of sides.}$$

$$30 \quad \quad \quad \text{,, closed end.}$$

$$62 \times 2 = \frac{124}{1224} \text{ equivalent of end capacities.}$$

$$1224 \div 5 = 244·8 \text{ centim.} = \text{half wave-length.}$$

The range of wires suitable for the study of the phenomena is rather limited. If the wires have a greater diameter than 1 millim. the difference between iron and copper is slight; while with wires less than 0·5 millim. in diameter the damping is so great that long wires cannot be used, and advantage cannot be taken of the cumulative effect.

I wish to express my great obligation to Prof. John Trowbridge for the encouragement and suggestions that I have received from him.

Conclusions.

1. The self-induction of iron circuits is greater than that of similar copper circuits under very rapid electric oscillations (115×10^6 reversals per second). The change in self-induction varies from 3.4 to 4.3 per cent. in the present investigation and increases with decreasing diameters.

2. The increase in self-induction produces greater damping, and a shortening of the wave-length of 1.5 to 2 per cent.

3. The permeability μ of annealed iron wire under this rate of alternation is about 385.

4. For oscillations of the same period, the wave-length along parallel copper wires varies directly with the diameter of the wires. (Range of wires used 0.03915 centim. to 0.1201 centim.) The maximum decrease observed is 5 per cent.

Jefferson Physical Laboratory,
July 24, 1894.

LIII. *Change of Period of Electrical Waves on Iron Wires.*
By JOHN TROWBRIDGE*.

IN an investigation upon the damping of electrical waves on iron wires †, I endeavoured to detect a change of periodicity as well as a damping of these waves. On account, however, of the strong damping effect exerted by the magnetic nature of the conductors, not a sufficient number of oscillations could be set up in them to enable one to make conclusive measurements. With more powerful means of experimenting I returned to the subject, and I have detected a marked change in the period of electrical waves which is produced by the magnetic nature of the wire. At the same time Mr. Charles E. St. John, working in the Jefferson Physical Laboratory, by an entirely different method, has shown a change in wave-length on iron wire even for the very rapid period of the Hertz vibrator. It will be remembered that Hertz believed that iron wires behaved like copper wires when transmitting very rapid electrical oscillations. Stefan ‡ in a recent paper gives an analysis of electrical oscillations, in which he proves that rapid electrical waves on iron-wire circuits have the same wave-length as those on copper circuits of the same geome-

* Communicated by the Author.

† Phil. Mag. Dec. 1891.

‡ Wied. Ann. xli. 1890, p. 422.