

AN INDUCTANCE AND CAPACITY BRIDGE.

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THE apparatus described herein, was developed, primarily, for the use of students in the Physical Laboratory of Brown University, the idea being to provide easily workable methods by which inductances and capacities can be compared, with an accuracy of about one tenth of one per cent., without the use of elaborate or expensive instruments. While the precision sought is considerably below that desirable in the most exact work, it is amply sufficient for the purpose and, indeed, for most practical work. With the exception of the resistance boxes, condenser and galvanometer, all of the necessary apparatus was built in the laboratory and, though somewhat roughly constructed, it gave results well within the desired limit. A brief description of its essential features is, accordingly, given in the hope that it may be found useful in other laboratories.

Maxwell's¹ method for the comparison of two inductances (L_1 and L_2) is illustrated in Fig. 1. P and Q are noninductive resistances; R represents the resistance of the coil L_1 plus the noninductive resistance in series with it between B and D , and S bears the same relation to $B E$. When the bridge is in balance for alternating or intermittent currents,

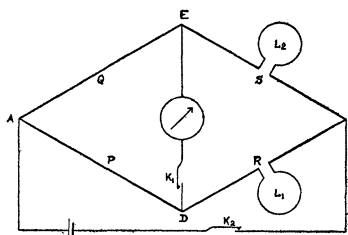


Fig. 1.

$$\frac{L_1}{L_2} = \frac{P}{Q} \quad (1)$$

provided the steady current relation

$$PS = QR \quad (2)$$

¹ Maxwell, "A Treatise on Electricity and Magnetism," third edition, Vol. II., p. 398.

is also fulfilled. Fig. 2 shows Maxwell's¹ arrangement for the comparison of an inductance with a capacity. The letters have the same significance as in Fig. 1 with the addition of C to represent the capacity of the condenser. When the resistances are so chosen that equation (2) is satisfied and the bridge is also in balance for alternating currents

$$L = CQR. \quad (3)$$

Under suitable conditions, these methods are very sensitive but, owing to the necessity of a double adjustment for direct and alternating current, their practical application is somewhat tedious. A

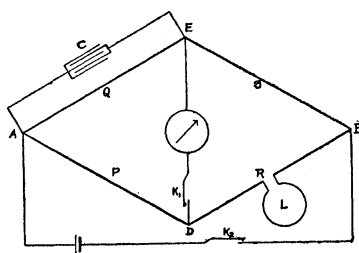


Fig. 2.

further difficulty arises from the fact that few resistance boxes are subdivided sufficiently to permit an exact balance. If the fine adjustment is accomplished with an ordinary slide wire, the battery and galvanometer leads cannot be kept in a fixed position and near together. Furthermore, the con-

nection wires of the bridge will include a considerable area. The error due to the inductances thus introduced would rarely exceed one tenth per cent., but it should be avoided if possible. The tedium of the methods may be considerably reduced by taking P equal to R and Q equal to S so that the bridge is always in balance for steady currents whatever the actual value of these resistances. Balance for alternating current may then be easily attained by altering P and R or Q and S by equal amounts.

The apparatus, designed to carry out these operations and to eliminate, as far as possible, all inductances save those compared, is illustrated, diagrammatically, in Fig. 3. It is, essentially, a connection board provided with a series of brass contact blocks and three double slide wires. The blocks, taken from an old rheostat, are about 1.5 by 2 inches on the top face and .75 of an inch thick. They are bolted securely to a slab of slate and may be connected by large well fitting plugs. Each block is provided with two bind-

¹ Maxwell, l. c., p. 425.

ing screws and a socket for the movable binding posts that terminate the battery and galvanometer leads. The slide wires D , C and D' are somewhat over one meter long and are stretched, parallel to one another, along the top of a wooden meter scale. D and D' are manganin, No. 22 B. S., and are soldered to small brass plates that are securely clamped to adjacent contact blocks by large binding screws. The copper wires C are insulated from D and D' , except at the sliding contacts, and may be connected to the galvanometer as indicated at G' . A longitudinal section of the sliders C_1 , C_2 and C_3 is exhibited, on a somewhat enlarged scale, at S . These sliders are, essentially, small brass tripods with feet grooved to fit the three wires and they are held firmly in place by inverted U 's, of vulcanized fiber, terminating in brass springs that bear against the under side of the wooden scale. The contact pressure is transmitted and regulated by an adjusting screw that passes through the bend of the U and engages the hole H . The resistance of these contacts was found to be very small and surprisingly constant.

Fig. 3 also shows connections arranged for the comparison of two inductances L_1 and L_2 . R_1 , R_2 , R_3 and R_4 are variable non-inductive resistances, of the common plug cutout pattern, guaranteed, by the makers, to be accurate within one tenth per cent. The galvanometer is connected at G and G' and the battery at B and B' through a secohmmeter. In practice, equal resistances are included in R_3 and R_4 , plugs are placed at 2 and 6, and balance for steady current is obtained by adjusting R_1 , R_2 and X . After removing the plugs from 2 and 6, balance is again produced by altering R_3 and Y . Finally, with the secohmmeter in motion, balance for alternating current is attained by moving the contact C_3 and, if necessary, making equal changes in R_3 and R_4 . Unless the resistances in the boxes at R_3 and R_4 are very well adjusted, it will generally be necessary to repeat the two latter operations but in this case, the final adjustment can always be made, with the contacts C_2 and C_3 , so that exact balance is observed with either direct or alternating current.

For the comparison of an inductance with a capacity the coil L_2 is removed, plugs are placed at 2 and 6, the condenser is connected

where X , Y and Z are bridge readings; a_1 is the sum of the resistance per unit length of the manganin wires at X ; a_2 bears the same relation to Y ; and a_3 is the resistance per unit length of the wire D at Z . Also, in virtue of the adjustments described above, P equals R , Q equals S , and equations (1) and (2), or (2) and (3), are satisfied.

For the purpose of testing the accuracy attainable with the apparatus, three coils (A , B and C) were inter-compared by the first of the above methods and then separately compared with several different capacities by the second method. During these comparisons the sliding contacts were set to an exact balance, but readings were taken to the nearest millimeter, since the error thus introduced was always much less than one tenth of 1 per cent. of either P or Q . Reduction of the observations was facilitated by the use of graphs, constructed with bridge readings as abscissas and corresponding resistance as ordinates. These graphs were drawn to such a scale that the products a_1X , etc., could read off directly with the required degree of precision. This was accomplished, on a sheet of convenient size, by drawing several parallel lines, each representing about 20 centimeters of the wire.

The results of the first series of observations are given in Tables I., II and III. L_A , L_B and L_C represent the inductance of the coils A , B and C , respectively, and the other letters refer to Fig. 1. The values under P and Q were calculated by equations (4) and (5), and those under L_B/L_C , etc., by equation (1). The average deviation of a single observation from the mean is .064 per cent. in Table I., .084 per cent. in Table II., .061 per cent. in Table III.

TABLE I.

L_1	L_2	P	Q	L_B/L_C
C	B	2.102	3.619	1.7217
"	"	3.110	5.361	1.7238
"	"	12.110	20.882	1.7244
"	"	102.083	175.671	1.7210
B	C	4.060	2.357	1.7226
"	"	7.056	4.094	1.7235
"	"	5.102	2.960	1.7236
"	"	6.040	3.500	1.7257
			Mean	1.7233

TABLE II.

L_1	L_2	P	Q	L_A/L_B
B	A	7.074	10.025	1.4171
"	"	13.060	18.518	1.4179
"	"	6.105	8.665	1.4193
"	"	8.021	11.370	1.4176
A	B	5.081	3.574	1.4217
"	"	6.744	4.753	1.4189
"	"	9.865	6.952	1.4194
"	"	13.873	9.764	1.4209
			Mean	1.4191

TABLE III.

L_1	L_2	P	Q	L_A/L_C
A	C	13.978	5.708	2.4489
"	"	7.860	3.212	2.4470
"	"	8.731	3.570	2.4456
"	"	24.105	9.842	2.4492
C	A	4.116	10.057	2.4434
"	"	8.137	19.897	2.4452
"	"	12.113	29.621	2.4454
"	"	6.120	14.974	2.4466
			Mean	2.4464

A comparison of the individual coils with the sections of a mica condenser, by Elliott Bros., is exhibited in Table IV. Column C gives the nominal value of the capacities used. The sub-letters on the P 's and Q 's indicate the coil connected at L , Fig. 2, or L_1 , Fig. 3, during the corresponding observation. The ratios given in the last three columns were each calculated from pairs of observations, tabulated in the same horizontal line, by the equation

$$\frac{L_A}{L_B} = \frac{P_A Q_A}{P_B Q_B}, \quad (6)$$

which follows directly from (3) when P is taken equal to R . The average of the individual deviations from the mean is .032 per cent. of L_A/L_C , .049 per cent. of L_A/L_B , and .070 per cent. of L_B/L_C . Since the actual capacity of the condenser was unknown, it was impossible to determine the exact value of the inductances but an idea of their magnitude was obtained from the observations given in the

first two rows of Table IV. The reduction was made by equation (3), using the nominal value of the capacity. The results were .0161 Henry for coil *A*, .0113 Henry for coil *B*, and .00658 Henry for *C*.

TABLE IV.

<i>C</i>	P_C	Q_C	P_B	Q_B	P_A	Q_A	L_A/L_C	L_A/L_B	L_B/L_C
1.0	3.132	2,103	5.092	2,229	7.062	2,280	2.4446	1.4186	1.7232
1.0	6.137	1,072	7.098	1,597	11.066	1,454	2.4457	1.4195	1.7230
.5	6.139	2,150	13.094	1,737	11.066	2,917	2.4456	1.4193	1.7232
.5	9.137	1,445	17.098	1,329	24.056	1,342	2.4452	1.4207	1.7211
.2'	9.137	3,573	43.093	1,306	24.056	3,320	2.4464	1.4192	1.7239
.2'	16.137	2,022	19.100	2,945	34.064	2,343	2.4460	1.4189	1.7239
.2	16.137	2,040	19.100	2,971	34.064	2,365	2.4472	1.4197	1.7237
.2	12.132	2,714	33.094	1,716	54.051	1,490	2.4460	1.4182	1.7249
.1	22.132	2,986	33.100	3,446	54.067	2,993	2.4487	1.4187	1.7260
.1	42.129	1,570	43.100	2,640	104.048	1,554	2.4447	1.4211	1.7203
						Mean	2.4460	1.4194	1.7233

The observations in Table IV. may also be used, in an obvious modification of equation (3), to calculate the ratio of the capacities given in column *C*. The ratio of each of the subdivisions to the total capacity of the condenser was thus determined and the results are given in Table V. The first column gives the coil used in making the comparisons and the numbers over the other columns indicate the nominal capacity of the sections. The average deviation of a single observation from the mean does not exceed .07 per cent. in any of these determinations.

TABLE V.

Coil.	.5	.2'	.2	.1
<i>C</i>	.49902	.20176	.20008	.09967
"	.49830	.20163	.19980	.09946
<i>B</i>	.49904	.20167	.20002	.09952
"	.49885	.20152	.19960	.09962
<i>A</i>	.49880	.20160	.19986	.09948
"	.49850	.20160	.19979	.09952
Mean	.49875	.20163	.19986	.09954

By a slight modification of the connections given in Fig. 3, the apparatus may be adapted for Anderson's¹ method of comparing

¹ Anderson, Phil. Mag., Vol. XXXI., p. 329, 1891.

inductance and capacity illustrated in Fig. 4. For this purpose, the plug at P , Fig. 3, is removed and a noninductive resistance R_5 is connected between the blocks it joined. The coil L_2 is removed, plugs are placed at 2 and 7, the galvanometer terminal G' is removed from E to M , and the condenser is connected between A and

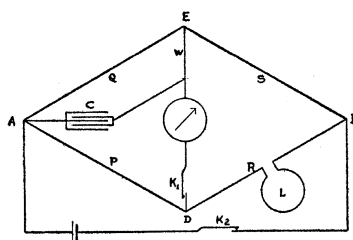


Fig. 4.

M . R_3 is taken equal to R_5 and balance for steady current is obtained by adjusting R_1 , R_2 and X . Balance for alternating current is then secured by altering R_4 and Z . After these adjustments, we have on comparing Figs. 3 and 4

$$S = R_5 = R_3 = Q, \quad (7)$$

$$R = P = R_2 + a_1 X, \quad (8)$$

$$W = R_4 + 2a_3 Z, \quad (9)$$

where the significance of the letters is apparent from the figures and the discussion of equations (4) and (5). Hence the inductance L may be determined in terms of the capacity C by the well-known formula

$$L = C\{W(P + R) + PS\}, \quad (10)$$

or, since R is equal to P , by the formula

$$L = CP(2W + S). \quad (11)$$

The foregoing examples are sufficient to show the application of the apparatus in methods depending on the principle of Wheatstone's bridge. The advantages gained by its use arise from the compact arrangement of the connections and the employment of noninductive slide wires. Provided the resistances, included in R_1 ,

R_2 , etc., are properly constructed, the bridge is practically free from inductance except in the coils under comparison. The necessary adjustments can be rapidly and accurately made without changing the relative position of the battery or galvanometer leads. The results, quoted in the above tables, show that inductances and capacities can be compared, by the methods described, with a precision equal to that of the available resistances.

BROWN UNIVERSITY,
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