

"The objects of the understanding, considered as representations, are symbols, that is, signs that are at least potentially general. But the rules of logic hold good of any symbols, of those that are written or spoken, as well as of those that are thought."

"Symbols which directly determine only their imputed qualities are but sums of marks, or *terms*;

"Symbols which further independently determine their objects by means of other term or terms, and thus, expressing their own objective validity, become capable of truth or falsehood, are *propositions*;

"Symbols which still further independently determine their interpretants, and thus the minds to which they appeal, by premising a proposition or propositions which such a mind is to admit, are *arguments*."

Mr. Peirce seems to have regarded it as *essential* to an argument that it should appeal to the interpreting mind to judge of it independently. Thus, he says, "an argument will here denote a body of premises considered as such," for it must distinctly show what the interpretation of the premises is expected to be, yet, in so far as the argument is a rational appeal, the conclusion which embodies this, interpretation is not put as an *assertion*, but is only formulated and submitted to the interpreting mind to judge.

Mr. Peirce has always been careful to exclude from logic, matter that he considers psychological, and therefore it is not surprising that he did not explain to what mind the appeal of the argument is addressed when one reasons with oneself. But it would seem to be plain from the above extracts, and is rendered perfectly clear in the papers referred to, that he not only considered all logical thought as an operation upon symbols consisting in substitution, but that he undertook to demonstrate this and to show *how* the same is true.

I may add that Peirce does not in the papers referred to say that substitution, which he makes the one hinge of all reasoning, is an *indecomposable* operation, and that in Baldwin's *Dictionary of Philosophy and Psychology*, Article "Symbolic Logic," he shows that no operation of substitution is valid unless the operations of insertion and subsequent omission into which it can be resolved are both valid.

FRANCIS C. RUSSELL.

THE PLACE OF MATHEMATICS IN EDUCATION.

The present rector of the University of Munich, Professor Ferdinand Lindemann, has devoted his official rectorate lecture to the important subject of the significance of mathematics in the higher schools. At present the curriculum of the German gymnasia is based upon the principle that education consists first of all in a knowledge of classical philology and history. Pro-

fessor Lindemann is fully convinced that a knowledge of Greek thought is indispensable for any educated man. But we must not forget that the leading philosopher of ancient Greece wrote over his school the significant words

μηδεις ἀγεωμέτρητος εἰσίτω,

and Melancthon quotes this famous maxim of Plato in his preface to the Latin edition of Euclid (Basel, 1537). How different is the classical conception from the modern treatment which mathematics receives! It is now considered dry, monotonous and tedious, and the mathematician is generally eschewed, being stigmatised by the saying, *mathematicus non est collega*, "the mathematician is unsocial."

In order to point out the value of carefully elaborated mathematical exercises, Professor Lindemann quotes Helmholtz as saying, "In my judgment, a true comprehension of mathematics is attained by working out mathematical propositions on paper and accurately revising each statement that is given. When one simply thinks out something in his mind, there is always a possibility of error, of disregarding some important term which he will never notice until he writes it down. I consider this most excellent practice in order to arrive at really clear logical thought, and to understand mathematics. For if students do not work out their mathematics and write it down they will never positively understand it."

How little consideration is given mathematics among leading experts on ancient and classical times, appears from Mommsen's famous dictum to which he gave utterance in his speech before the Royal Academy of Sciences, Berlin, in 1884. "We shall, furthermore, continue to call the ideal culture of mankind in good Latin, humanity; and the man who would in time replace Homer by the doctrines of conic sections, in good Greek, banausic." In answer, Professor Lindemann says, "Mommsen misunderstands the facts. We agree with him perfectly that Greek reflection and Roman thought continue to sway even to-day, consciously and unconsciously, our humanistic culture, and we too designate the ideal of human civilisation as humanism, but this ideal comprehends not only the development of art, politics, literature and history, but of the exact sciences as well. The innumerable theorems of conic sections certainly constitute mathematics as little as the recitation of Homeric songs can pass for classical scholarship. But if elements of the theory of conic sections have lately been introduced into the program of our higher schools, this step has an ulterior purpose. The treatment of conic sections in methods of analytical geometry familiarises the student with an instance of the general laws of interdependence; it is the general idea of functions as here introduced in geometrical form, which has directed and controlled the development of mathematics during these latter centuries, and upon which rest the great discoveries of Newton and Leibnitz.

Professor Lindemann further calls attention to the application of mathematics in technical occupations and sciences, especially in astronomy, physics, and of late even in chemistry. He points out that the only road to success in the sciences in modern times passes through the gate of higher mathematics, and mentions in connection therewith such names as Kepler, Newton, Comte, Mayer, Helmholtz, Clifford, Hertz, Mach, Pearson, Poincaré, and Herbart.

Wilamovitz has made progress in his method of teaching the classics by introducing bits of Euclid in his textbooks; but, argues Professor Lindemann, will a classical philologist be able to explain the subject-matter of the seemingly most simple statements of mathematics referring to definitions, axioms, etc.? Do the philologists have an idea of the vast literature which of late has grown out of the discussion of these simple propositions, since Bolyai, Lobatchevsky, and Gauss? There are quite a number of mathematical textbooks which still retain the false ground that it is possible to improve upon Euclid, and in spite of the discussions and lectures held at almost every University on the subject, they continue to offer definitions and even demonstrations which long since have been shown to be insufficient.

Professor Lindemann declares that mathematical instruction in gymnasias, corresponding in America to undergraduate courses in college and university, should not cover all the details of mathematical branches, but should be so arranged as to enable the student to gain a proper comprehension of the grand edifice of mathematics and its solid foundation. Teachers of mathematics should be equipped to satisfy these conditions and should be familiar with the methods by which the science of mathematics has been worked out. They should know its history, not only in general, but some of its main problems; for instance, how mankind happened to be interested in the trisection of the angle and the squaring of the circle. He should have a command of the basic ideas of analytic mechanics; should at least have become acquainted with the exact execution of certain experiments, such as the motion of the pendulum; and should also have clear ideas concerning the field of applied mathematics and its significance in practical life. It is these aims that the leading mathematicians have had in mind since the beginning of the last century.

THE SLAV INVASION.

MR. FRANK JULIAN WARNE'S VIEW OF THE SITUATION.

While other nations are waging wars, causing loss of life, property, and money, the United States is passing through industrial struggles which are not less expensive. The anthracite strike commission estimated the loss of the last strike at one hundred million dollars. Mr. Frank Julian Warne,