Zero-Window Varieties in the Collatz Carry Equation: The $(17, 27)$ Corridor Program Beyond the $(7, 11)$ Shell

Elias De Jesús

June 2026

Abstract

The seventeenth and eighteenth notes resolved the $(7, 11)$ shell: no proper cyclic zero-window can occur there, so every contact — actual or hypothetical — is locally isolated [1]. This note stress-tests the mechanism on the next major fully enumerated near-critical shell, $(17, 27)$, with wall $N = 2^{27} - 3^{17} = 5{,}077{,}565 = 5 \cdot 71 \cdot 14303$ (positive). All 5,311,735 residents were scanned over 17 positions and the essential lengths $1, \dots, 8$ — 722,395,960 window sums — and the essential scan finds *zero full-wall zero-windows at lengths* $1, \dots, 8$, against a uniform heuristic of ≈142.3. An extended all-length scan (1,444,791,920 sums, lengths $1, \dots, 16$) finds exactly 34 length-16 full-wall zero-windows — all on noncontact residents and irrelevant to contact isolation, since a contact's length-16 windows equal $-\tau_k$, nonzero units. The mechanism, however, is no longer one curve-versus-box disjointness but a layered sieve. The odd orders $\mathrm{ord}_{71}(2) = 35$ and $\mathrm{ord}_{14303}(2) = 7151$ forbid length-2 vanishing in those layers; the 14303-layer corridor excludes lengths 2 through 6 outright (zero abstract gap solutions); exactly three fatal gap tuples exist at lengths 7–8, realized by exactly 221 windows (13 full rotation orbits, $221 = 13 \cdot 17$); and none of the 221 also vanishes modulo 71. Consequently any hypothetical contact in $(17, 27)$ would be window-indecomposable and locally isolated — a conditional statement, since the shell contains no contact. The shell-wide isolation pattern of $(7, 11)$ thus generalizes at the theorem level while diversifying at the mechanism level. Two corrected indexing errors from the verification process are preserved in print. No proof of the Collatz conjecture is claimed.

1 Introduction

Does $(17, 27)$ behave like $(7, 11)$, where shell geometry excludes every proper zero-window, or does it break the pattern? The answer: *it behaves like* $(7, 11)$ *at the full wall, but by a layered sieve rather than one curve–box exclusion.* The central message:

$(17, 27)$ breaks the simple mechanism but preserves the theorem-level outcome.

2 Wall and prime-layer structure

$N = 2^{27} - 3^{17} = 5{,}077{,}565 = 5 \cdot 71 \cdot 14303$, a positive shell; 71, 14303, and $7151 = (14303 - 1)/2$ are prime. Layer data, with $3 \equiv 2^{\gamma_p} \pmod{p}$ in every layer:

$$\mathrm{ord}_5(2) = 4,\ \gamma_5 = 3; \qquad \mathrm{ord}_{71}(2) = 35,\ \gamma_{71} = 16; \qquad \mathrm{ord}_{14303}(2) = 7151,\ \gamma_{14303} = 5470.$$

3 Windows, essential lengths, exponent coordinates

For a resident $D = (d_0, \ldots, d_{16})$ with partial sums $s_j = \sum_{k<j} d_k$, the carry terms are $\tau_j = 3^{16-j} 2^{s_j}$ and the cyclic windows $W_{a,r} = \sum_{h=0}^{r-1} \tau_{a+h}$ (indices mod 17, using the wall periodicity of the seventeenth note). Essential lengths are $1 \le r \le \lfloor 17/2 \rfloor = 8$; for contacts, longer windows follow by complementarity. In each layer, with $e_j = s_j - \gamma_p j \bmod \mathrm{ord}_p(2)$, a window vanishing modulo p is a power-of-two zero-sum $\sum_h 2^{e_{a+h}} \equiv 0$.

4 Length two

By the Letter-Congruence Lemma: odd $\mathrm{ord}_p(2)$ makes length-2 vanishing impossible, while even order requires $d \equiv \gamma_p + \frac{1}{2}\,\mathrm{ord}_p(2)$. Applied here: mod 71 (order 35) and mod 14303 (order 7151): *impossible*; mod 5 (order 4): the condition is $d \equiv 1 \pmod 4$, satisfied by the realizable letters $1, 5, 9$, and indeed the mod-5 layer teems with length-2 zeros. Full-wall length-2 zero-windows are nonetheless impossible: the two odd-order layers forbid them.

5 Corridor tables and the three fatal tuples

A window $\tau_a, \ldots, \tau_{a+r-1}$ has its exponent gaps determined by the *interior letters* $d_a, \ldots, d_{a+r-2}$: $r-1$ consecutive positive letters whose sum, inside a length-17 word of total 27, is at most $r+9$. Enumerating all such gap tuples and counting solutions of $1 + 2^{x_1} + \cdots + 2^{x_{r-1}} \equiv 0 \pmod p$:

r	gap tuples	mod 5	mod 71	mod 14303
2	11	3	0	0
3	66	9	0	0
4	286	64	3	0
5	1001	171	14	0
6	3003	634	44	0
7	8008	1470	103	2
8	19448	4013	299	1

The 14303-layer is the bottleneck: *zero* solutions through length 6 — a corridor exclusion five lengths deep, covering all realizable interior blocks — and exactly three fatal tuples beyond:

$$(1,1,1,4,3,6), \qquad (6,1,1,2,3,2), \qquad (1,3,1,4,2,4,2).$$

Remark 5.1 (Corrected errors: two off-by-ones)**.** Two indexing slips from the verification process are preserved per program practice. First, a cross-check initially extracted the letter block $d_{a+1}, \ldots, d_{a+r-1}$ instead of $d_a, \ldots, d_{a+r-2}$; after the shift, every realized mod-14303 zero-window matches one of the three fatal tuples exactly (the census itself was never wrong). Second, the move-legality boundary letters for the window $\tau_a, \ldots, \tau_{a+r-1}$ are (d_{a-1}, d_{a+r-1}), not (d_a, d_{a+r-1}); correcting this flips the legality count of Section 8 from 187 to 34.

6 The census theorem

Theorem 6.1 (Essential zero-window census)**.** *Among all* $\binom{26}{16} = 5{,}311{,}735$ *residents of* $(17, 27)$, *scanned over all* 17 *positions and the essential lengths* $1, \ldots, 8$ *(*722,395,960 *window sums):*

zero full-wall zero-windows at lengths $1, \ldots, 8$, *against a uniform expectation of* $\approx$142.3*; and zero contacts, reconfirming the earlier census. Consequently any hypothetical contact would have no proper cyclic zero-window at* any *length — lengths* $9, \dots, 16$ *closing by complementarity — and would be locally isolated.*

Theorem 6.2 (Extended all-length census)**.** *Across all lengths* $1, \dots, 16$ *(*5,311,735 $\cdot$ 17 $\cdot$ 16 $=$ 1,444,791,920 *window sums), the only full-wall zero-windows occur at length* 16*: exactly* $34 = 2 \cdot 17$ *of them (two rotation orbits), all on noncontact residents, at positions where* $C(D) \equiv \tau_k \pmod{N}$*. They are irrelevant to contact isolation: for a contact, every length-*16 *window is the complement of a single unit term, hence equals* $-\tau_k \neq 0$*.*

The theorem has a structured proof independent of the bulk scan: length 1 by units; lengths 2–6 at the full wall by the 14303-layer corridor exclusion (the gap-tuple enumeration is complete over all realizable interior blocks); and lengths 7–8 by the finite endgame of Section 8. The 722-million-sum census then stands as independent confirmation.

7 Prime-layer census

Layer totals across the scan: mod 5: 166,804,068 (expected $\approx$144.5 million per uniform; a 16% excess); mod 71: 7,229,250 (a 29% deficit, with lengths $2, 3$ exactly empty, matching the corridor table); mod 14303: 221 against an expected $\approx$50,506 — a 228$\times$ deficit, empty through length 6, with 204 windows at length 7 and 17 at length 8.

8 The 221-window anatomy and the endgame

All 221 mod-14303 zero-windows realize exactly the three fatal tuples, with multiplicities

$$(1,1,1,4,3,6) : 17, \qquad (6,1,1,2,3,2) : 187, \qquad (1,3,1,4,2,4,2) : 17.$$

The multiplicities are combinatorially forced, not merely counted: fixing the interior block leaves the remaining $18 - r$ letters positive with sum $27 - \Sigma T$. For $(1,1,1,4,3,6)$ and $(1,3,1,4,2,4,2)$ the remaining letters are all forced to 1, giving one word per rotation: 17 each. For $(6,1,1,2,3,2)$, eleven letters sum to twelve — $\binom{11}{10} = 11$ placements of the single 2 — giving $11 \cdot 17 = 187$. Total $17 + 187 + 17 = 221$; and since window-vanishing is rotation-covariant and $\gcd(17, 27) = 1$ forces orbits of size 17 (Rotation-Quantization, note fourteen), $221 = 13 \cdot 17$ full orbits.

The endgame is symbolic. Every realization of a fatal tuple T has window value $W = \tau_a B_T$ with τ_a a unit and the bracket $B_T = 1 + \sum_h 2^{x_h}$ depending only on T, so per-layer vanishing is decided by three congruences:

T	$(1,1,1,4,3,6)$	$(6,1,1,2,3,2)$	$(1,3,1,4,2,4,2)$
B_T mod 71	57	23	58
B_T mod 5	0	4	1

All three brackets are nonzero modulo 71, so *every* mod-14303 fatal window is killed by the 71-layer — three congruences replacing a 221-item check. The single zero bracket modulo 5, at $(1,1,1,4,3,6)$, explains exactly which 17 windows (that tuple's orbit) also vanish modulo 5; this is moot at the full wall. Legality, with the corrected boundary convention of Remark 5.1, is also forced: the two all-ones complements put 1 at both boundaries (0 legal each), while the single 2 of $(6,1,1,2,3,2)$ sits at a boundary in exactly 2 of its 11 placements: $2 \cdot 17 = 34$ of the 221 would be legal moves. Legality alone would thus have eliminated 187; the mod-71 obstruction eliminates all 221 unconditionally.

9 Conditional Shell Isolation Theorem

Theorem 9.1 (Conditional isolation for $(17, 27)$)**.** *Any contact resident of the* $(17, 27)$ *shell, if one existed, would be window-indecomposable — lengths* 9–16 *closing by complementarity — and hence, by the Window Criterion of the seventeenth note, locally isolated in the one-unit move graph:*

any hypothetical contact in $(17, 27)$ *would be locally isolated.*

The statement is conditional: the complete census finds no contact in this shell.

10 Comparison with $(7, 11)$

$(7, 11)$ was resolved by a deterministic curve-versus-box disjointness in a single prime field. $(17, 27)$ is resolved by a layered sieve: odd-order layers kill length 2; the bottleneck 14303-layer excludes lengths 2 through 6; and the surviving 221 high-length windows are killed by the 71-layer. The shell-wide isolation pattern generalizes; the mechanism becomes multi-layered.

11 Baselines

Tier 3 diagnostics, not proofs: full-wall expectation 142.3 versus observed 0; the 228× mod-14303 deficit, explained by the corridor's smallness relative to the field (19,448 tuples at $r = 8$ against $p = 14303$, abstract solution counts tracking #tuples/p); the mod-5 excess and mod-71 deficit as letter-structure effects. The theorems are the exact corridor analysis and census.

12 Tier ledger

Tier 1 prime-layer structure; length-2 odd-order theorem (71, 14303) and mod-5 letter congruence; corridor exclusion mod 14303, lengths 2–6; Census Theorem (Thm. 6.1); the 221-window anatomy and orbit count and bracket endgame; the extended all-length census (Thm. 6.2); the conditional Shell Isolation Theorem.

Tier 3 uniform baselines; the 228× deficit interpretation; mod-5 excess and mod-71 deficit readings.

Tier 4 general corridor-exclusion theory for arbitrary shells; whether bottleneck layers are generic; unconditional primitive-contact isolation.

13 Limitations

No proof of the Collatz conjecture. No contact exists in $(17, 27)$, so the isolation statement is conditional. The enumeration is finite and shell-specific; prime-layer zero-windows are not full-wall zero-windows; the corridor method is not yet generalized to arbitrary shells; baseline expectations are heuristic diagnostics.

14 Conclusion

The $(17, 27)$ shell does contain far-end noncontact length-16 zero-windows — thirty-four of them — so the correct theorem is not global absence of all proper zero-windows. The correct theorem is stronger in the relevant sense: across more than 722 million essential window opportunities at lengths $1, \ldots, 8$, none vanishes at the full wall, and any contact would therefore be locally isolated by complementarity. The $(7, 11)$ result was not a one-off — in the next large fully mapped shell, essential partial vanishing is again forbidden, now by a layered prime sieve with a bottleneck layer and a three-congruence endgame rather than a single curve–box exclusion. If a contact existed in $(17, 27)$, it would be locally isolated; the shell contains none. The corridor program advances from a small-shell theorem to a multi-layer shell theorem.

References

[1] E. De Jesús, *Structured Sectors of the Collatz Carry Equation: Christoffel Towers, 2-Adic Windows, and Shell Residents — Series Notes*, Zenodo, 2026. `https://doi.org/10.5281/zenodo.20557441`

[2] J. C. Lagarias, *The $3x + 1$ problem and its generalizations*, Amer. Math. Monthly **92** (1985), 3–23.