The Anatomy of the -17 Cycle: Time-Axis and Finite-Field Structure of the First Nondegenerate Contact

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Abstract

We analyze the -17 cycle of the accelerated odd-only Collatz map as the first known *nondegenerate* wall contact: the earlier known cycles -1, $+1$, and -5 live on shells with degenerate walls $|N| = 1$, where every resident is trivially a contact, while the -17 cycle's shell $(7, 11)$ has the prime wall $|N| = 139$. In $\mathbb{F}_{139}$, both 2 and 3 are primitive roots with $3 = 2^{41}$ and $3^7 = 2^{11}$, so dividing the carry by 3^6 converts contact into a vanishing sum of seven powers of 2: the -17 word realizes the exact zero-sum

$$2^0 + 2^{18} + 2^{37} + 2^{58} + 2^{77} + 2^{98} + 2^{117} \equiv 0 \pmod{139},$$

a near-arithmetic exponent progression perturbed just enough to land on zero, while the mechanical Christoffel word's exponents $\{0, 19, 39, 59, 78, 98, 118\}$ sum to 74. Since $7 \nmid 138$, the identity is not a root-of-unity phenomenon. The singular necklace is isolated in the local-move graph (every neighbor nonsingular), but its 84 neighbor-moves include a residue at distance 2 from zero — no protective moat — and exactly two shortest arithmetic channels connect the mechanical word to the cycle word. No ordering statistic detects contact; only the congruence does. No claim about the Collatz conjecture is made, and one nondegenerate shell does not give a general theory.

1 Introduction

The ninth note [1] gave the complete census of the $(7, 11)$ shell: 210 residents, exactly seven contacts — the rotation class of the -17 cycle word — a vacuous 2-adic window, and a nonsingular mechanical word two moves away. That note located the boundary of the Christoffel framework; this note crosses it and asks the frontier's first concrete question: *how, exactly, does the -17 cycle touch the wall?* The answer has a precise finite-field form. The series' first eight notes explain *missed* windows; the -17 shell is the first nontrivial odd-prime *hit*, and its thesis is:

the -17 cycle is the first known nondegenerate contact, and its contact is a finite-field zero-sum event.

2 Degenerate-wall classification of the known cycles

Theorem 2.1. *The known cycles of the accelerated odd-only map have the following shells and walls:*

cycle	*word*	*shell* (L, S)	$\lvert N\rvert = \lvert 2^S - 3^L\rvert$
$+1$	(2)	$(1, 2)$	1
-1	(1)	$(1, 1)$	1
-5 *(elements* $-5, -7$*)*	$(1, 2)$	$(2, 3)$	1
-17	$(1, 1, 1, 2, 1, 1, 4)$	$(7, 11)$	139

The cycles $+1$*,* -1*,* -5 *lie on* degenerate *walls* $|N| = 1$*, where every shell resident is trivially a contact. Hence:*

$$\boxed{-17 \textit{ is the first known cycle on a nondegenerate wall.}}$$

Proof. Direct computation of each orbit's valuation word and wall gives $|2^2 - 3| = 1$, $|2 - 3| = 1$, $|2^3 - 3^2| = 1$, and $|2^{11} - 3^7| = 139$. On a $|N| = 1$ shell the contact congruence is vacuous. □

This is a statement about *known* cycles only; it classifies the inventory, not the possibilities. Its force is organizational: the odd-prime layer of this series has nontrivial content for the first time at $(7, 11)$, which is why the frontier opens there.

3 The $(7, 11)$ shell, reviewed

From the ninth note [1]: $|\mathcal{D}_{7,11}| = 210$, $|N| = 139$ prime; exactly 7 contacts, the rotation class of $D_{-17} = (1, 1, 1, 2, 1, 1, 4)$; the mechanical word

$$X_{(7,11)} = (1, 2, 1, 2, 1, 2, 2) = X_3 X_4$$

has $C = 3767 \equiv 14 \pmod{139}$ and is nonsingular with all its rotations. Thus mechanical nonsingularity does not imply shell nonsingularity.

4 Time-axis tables for the seven rotations

For each rotation, the term residues $3^{6-j} 2^{s_j} \bmod 139$ and the cumulative walk are listed below. All walks end at 0.

rotation	Q	terms mod 139	cumulative walk
$(1, 1, 1, 2, 1, 1, 4)$	-17	$34, 69, 46, 77, 10, 53, 128$	$34, 103, 10, 87, 97, 11, 0$
$(1, 1, 2, 1, 1, 4, 1)$	-25	$34, 69, 46, 15, 10, 53, 51$	$34, 103, 10, 25, 35, 88, 0$
$(1, 2, 1, 1, 4, 1, 1)$	-37	$34, 69, 92, 15, 10, 7, 51$	$34, 103, 56, 71, 81, 88, 0$
$(2, 1, 1, 4, 1, 1, 1)$	-55	$34, 138, 92, 15, 80, 7, 51$	$34, 33, 125, 1, 81, 88, 0$
$(1, 1, 4, 1, 1, 1, 2)$	-41	$34, 69, 46, 60, 40, 73, 95$	$34, 103, 10, 70, 110, 44, 0$
$(1, 4, 1, 1, 1, 2, 1)$	-61	$34, 69, 90, 60, 40, 73, 51$	$34, 103, 54, 114, 15, 88, 0$
$(4, 1, 1, 1, 2, 1, 1)$	-91	$34, 135, 90, 60, 40, 7, 51$	$34, 30, 120, 41, 81, 88, 0$

The Q-column is the accelerated orbit

$$-17 \to -25 \to -37 \to -55 \to -41 \to -61 \to -91 \to -17.$$

The rotation order in the table follows the left-rotation sequence of the word; the orbit visits the values in the order shown by repeated application of the rotation-carry identity of the ninth note. The walks are not gentle: the -17 representative's cumulative trace

$$34, 103, 10, 87, 97, 11, 0$$

ranges over most of $\mathbb{F}_{139}$ before the final large-letter term (128) closes it. *Contact arises from whole-axis cancellation, completed by the last term — not from local residue calm.*

5 Canonical time-axis representative

Among rotation normalizations (lexicographically minimal; max-letter-last; minimum-Q; maximum-Q), the word

$$(1, 1, 1, 2, 1, 1, 4)$$

is simultaneously the *lexicographically minimal* and the *max-letter-last* representative of the singular necklace. The normal form of the cycle accumulates small divisions and places the 4-valuation burst at the end — the time-axis signature of the orbit step

$$-91 \to -17, \qquad 3(-91) + 1 = -272 = -17 \cdot 2^4,$$

which discharges four divisions at once.

6 Finite-field reduction

Lemma 6.1. *In $\mathbb{F}_{139}$:*

$$3^7 = 2^{11};$$

both 2 and 3 are primitive roots of order 138; and

$$3 = 2^{41}.$$

Proof. The wall identity gives $3^7 - 2^{11} = 139 \equiv 0$. Since $138 = 2 \cdot 3 \cdot 23$, primitivity of 2 follows from the three maximal-subgroup checks

$$2^{69} \equiv 138, \qquad 2^{46} \equiv 96, \qquad 2^6 \equiv 64 \pmod{139},$$

none of which equals 1. Similarly, for 3 one checks

$$3^{69} \equiv 138, \qquad 3^{46} \equiv 42, \qquad 3^6 \equiv 34 \pmod{139}.$$

The discrete logarithm $3 = 2^{41}$ is verified directly. □

Proposition 6.2 (Exponent form of contact)**.** *For $D \in \mathcal{D}_{7,11}$ with partial sums s_j, contact $C(D) \equiv 0 \pmod{139}$ is equivalent to*

$$\sum_{j=0}^{6} 2^{e_j} \equiv 0 \pmod{139}, \qquad e_j \equiv s_j + 97j \pmod{138},$$

i.e. a vanishing sum of seven powers of 2.

Proof. Since 3^6 is a unit modulo 139, $C(D) \equiv 0$ is equivalent to

$$\sum_{j=0}^{6} 3^{-j} 2^{s_j} \equiv 0 \pmod{139}.$$

By Lemma 6.1,

$$3^{-j} = 2^{-41j} = 2^{97j}$$

in the exponent group $\mathbb{Z}/138\mathbb{Z}$. □

7 The zero-sum theorem

Theorem 7.1. *The* -17 *word*

$$D_{-17} = (1, 1, 1, 2, 1, 1, 4)$$

has exponent multiset

$$\{e_j\} = \{0,\ 18,\ 37,\ 58,\ 77,\ 98,\ 117\} \pmod{138},$$

and realizes the exact zero-sum

$$2^0 + 2^{18} + 2^{37} + 2^{58} + 2^{77} + 2^{98} + 2^{117} \equiv 0 \pmod{139}.$$

Proof. The partial sums of D_{-17} are

$$0, 1, 2, 3, 5, 6, 7.$$

Applying $e_j = s_j + 97j \bmod 138$ gives, in order of j,

$$0, 98, 58, 18, 117, 77, 37,$$

which is the stated multiset. The vanishing of the power sum is a finite verification, equivalent to

$$139 \mid C(D_{-17}) = 2363 = 17 \cdot 139.$$

$\square$

This is the odd-prime congruence of the ninth note rewritten in exponent form: *the* -17 *cycle is a vanishing configuration of seven powers of* 2 *in* $\mathbb{F}_{139}$.

8 Mechanical comparison

The mechanical word $X_{(7,11)}$ has partial sums

$$0, 1, 3, 4, 6, 7, 9.$$

Its exponent multiset is

$$\{0, 19, 39, 59, 78, 98, 118\},$$

with

$$\sum 2^e \equiv 74 \pmod{139},$$

so it does not vanish. The cyclic gap sequences are

$$\text{mechanical: } (19, 20, 20, 19, 20, 20, 20),$$

$$\text{singular: } (18, 19, 21, 19, 21, 19, 21).$$

Both are near-arithmetic progressions of step $138/7 \approx 19.7$ — the Christoffel structure visible *in exponent space* — with the singular multiset a slight perturbation of the mechanical one that lands exactly on zero. Since $7 \nmid 138$, $\mathbb{F}_{139}$ contains no seventh roots of unity: the zero-sum is *not* a root-of-unity identity but a genuinely arithmetic coincidence, which is what makes it a cycle rather than a symmetry.

9 The local-destruction theorem

Theorem 9.1. *Every one-unit redistribution neighbor of every singular resident of* $(7, 11)$ *is nonsingular. The* 84 *neighbor-moves of the necklace realize* 57 *distinct residues modulo* 139*, and the minimum distance of any neighbor residue from* 0 *is exactly* 2*.*

Proof. Exhaustive verification over all neighbors of all seven rotations, using the ninth note's local-move formula for the residue jumps. □

The necklace is isolated — but *not* protected by a residue moat: a neighbor sits at distance 2 from contact. Isolation here is a fact of where the move windows happen to land, not of any large arithmetic gap.

10 Two channels from mechanical to singular

Theorem 10.1. *The move-graph distance from* $X_{(7,11)}$ *to* D_{-17} *is* 2*, and there are exactly two shortest paths, through the middle words*

$$(1, 1, 1, 2, 1, 2, 3) \qquad (C \equiv 128 \pmod{139}),$$

and

$$(1, 2, 1, 2, 1, 1, 3) \qquad (C \equiv 36 \pmod{139}).$$

Proof. Exhaustive enumeration of common neighbors. □

Two arithmetic channels, not one: the residue traces

$$14 \to 128 \to 0$$

and

$$14 \to 36 \to 0$$

both connect the Christoffel word to the cycle in two moves, refining the ninth note's single printed trace into the complete statement.

11 Full-shell statistics

Comparing the 7 contacts with the 203 noncontacts: descents (median 2 vs 2), maximum letter (4 vs median 3), carry concentration (0.30 vs median 0.25), late-mass concentration (0.5 vs 0.5). The contacts are statistically unremarkable except for elevated carry concentration — a feature shared by many nonsingular residents (49 words have maximum letter ≥ 4; seven are singular). *No ordering statistic detects contact; only the congruence does.*

12 What this says about the bit analyzer

$$\ell(D_{-17}) = v_2(Q + \Xi_\alpha) = 2.$$

The cycle word leaves c_α at its second letter, the window is vacuous against shell quotients of size up to ≈158, and the analyzer of notes five through eight supplies nothing here. The bit analyzer is not wrong; it is out of its intended regime. The -17 resident lives where the odd-prime layer is fully active and the 2-adic window layer is blind — the division of labor between the series' two layers, exhibited at the one place a known cycle touches a nontrivial wall.

13 Tier ledger

Tier 1 degenerate-wall classification (Thm. 2.1); time-axis tables; finite-field reduction (Lem. 6.1, Prop. 6.2); zero-sum theorem (Thm. 7.1); mechanical comparison; local-destruction (Thm. 9.1); two-channel paths (Thm. 10.1).

Tier 3 full-shell statistics; near-miss interpretation; carry-concentration patterns.

Tier 4 any general theory of unbalanced residents; positive cycles; larger nondegenerate shells.

14 Limitations

No proof of the Collatz conjecture is claimed. Only one nondegenerate contact shell exists in the known inventory and only it is studied here. The zero-sum identity is exact but not yet generalized: nothing here says what zero-sums on larger walls must look like. Statistical diagnostics do not classify contacts. Positive cycles and larger nondegenerate shells remain open.

15 Conclusion

The -17 cycle is the first known nondegenerate contact. Its wall contact is a finite-field zero-sum of seven powers of 2 in $\mathbb{F}_{139}$ — a near-arithmetic exponent progression perturbed onto zero — not a Christoffel-window phenomenon. The cycle's normal form accumulates small divisions and discharges them in one burst; its necklace is isolated but within distance 2 of a near miss; and two arithmetic channels connect it to the Christoffel word of its shell. This, in exact terms, is how a cycle touches a wall — and the proper starting point for the time-axis/unbalanced-resident program.

References

[1] E. De Jesús, *Structured Sectors of the Collatz Carry Equation: Christoffel Towers, 2-Adic Windows, and Shell Residents — Series Notes*, Zenodo, 2026. `https://doi.org/10.5281/zenodo.20557441`

[2] J. C. Lagarias, *The $3x+1$ problem and its generalizations*, Amer. Math. Monthly **92** (1985), 3–23.