# The $(20, 31)$ Shell in the Collatz Carry Equation: Contact-Free Census, Mixed-Bracket Tracers, and Isolation Before Enumeration

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**Abstract**

The $(20, 31)$ shell is resolved by the two-track method in its intended order. First (twenty-first note [1]), the mixed-bracket tracer proved essential zero-window exclusion at all lengths $1, \dots, 10$, so any hypothetical contact would be locally isolated — 587,858 bracket evaluations, no enumeration. Second (this note), the complete census of all $\binom{30}{19} = 54{,}627{,}300$ residents proves the shell is *contact-free*: $C_{20}(D) \not\equiv 0 \pmod{|N|}$ for every resident, $|N| = 3^{20} - 2^{31} = 1{,}339{,}300{,}753 = 7 \cdot 191{,}328{,}679$. Layer counts: 7,803,720 words vanish modulo 7 (expectation 7,803,900; exactly $20 \cdot 390{,}186$, rotation quantization's seal), and *zero* vanish modulo the large prime (expectation 0.286). The minimum nonzero lattice distance is $r^* = 14$, typical against the uniform-minimum expectation $\approx$12.3, with no resident within distance 10. The mechanical word and all 21 standard-product residents from the closed Christoffel sector are profoundly nonsingular. Existence and isolation are separate facts, separately proved: nonexistence by census, pre-isolation by tracer; neither was inferred from the other. The contact-free ledger extends to 129,525,250 censused residents across 33 shells, with the $-17$ necklace still the unique primitive nondegenerate contact in the enumerated universe. No proof of the Collatz conjecture is claimed.

## 1 Shell data and the tracer inheritance

$(L, S) = (20, 31)$, negative wall $|N| = 3^{20} - 2^{31} = 1{,}339{,}300{,}753 = 7 \cdot 191{,}328{,}679$ (both prime), $|\mathcal{D}_{20,31}| = \binom{30}{19} = 54{,}627{,}300$. From the twenty-first note: neither layer supports exponent-coordinate reduction ($\mathrm{ord}_7(2) = 3$ with $3 \notin \{1, 2, 4\}$; the large prime has odd order $(p-1)/2$ with 3 a certified non-residue); the dlog-free mixed bracket $B_T = \sum_h 3^{r-1-h} 2^{\sigma_h}$ replaced discrete logarithms; the large-prime layer is fatal-tuple-empty at every essential length $1 \le r \le 10$; hence the shell has no essential full-wall zero-window, and any hypothetical contact would be window-indecomposable and locally isolated.

## 2 The census theorem

**Theorem 2.1** (Contact-free census)**.** *All* 54,627,300 *residents of* $(20, 31)$ *were enumerated, matching the binomial exactly, and*

$$C_{20}(D) \not\equiv 0 \pmod{|N|} \qquad \textit{for every } D \in \mathcal{D}_{20,31} :$$

*the* $(20,31)$ *shell is contact-free.*

## 3 Layer counts and quantization

$$\#\{C \equiv 0 \bmod 7\} = 7{,}803{,}720 \quad (\text{exp. } 54{,}627{,}300/7 = 7{,}803{,}900),$$
$$\#\{C \equiv 0 \bmod 191{,}328{,}679\} = 0 \quad (\text{exp. } 0.286), \qquad \#\{C \equiv 0 \bmod |N|\} = 0 \quad (\text{exp. } 0.041).$$

Since $\gcd(20,31) = 1$, zero classes are rotation-closed with full orbits of size 20, and indeed $7{,}803{,}720 = 20 \cdot 390{,}186$ — the quantization seal on the mod-7 layer, exercised even though the full-wall count is zero. The contact-free outcome was statistically likely (joint expectation 0.041); the census converts likely into theorem.

## 4 Near misses and the mechanical word

The minimum nonzero lattice distance is

$$r^* = 14, \qquad \text{at } (2,1,3,1,2,1,1,1,1,2,2,3,1,1,2,2,1,2,1,1),$$

typical against the uniform-minimum expectation $|N|/(2M) \approx 12.3$ with $M = 54{,}627{,}300$; no resident lies within distance 1, 2, 5, or 10 (uniform expectations 0.08–0.82). The lower-mechanical word $(1,2,1,2,1,2,1,2,1,2,2,1,2,1,2,1,2,1,2,2)$ has $C \equiv 394{,}556{,}971 \pmod{|N|}$ — profoundly nonsingular.

## 5 Closed-sector product residents

The block equations $2a+5b+12c = 20$, $3a+8b+19c = 31$ over $X_3, X_4, X_5$ have exactly two solutions: $(a,b,c) = (4,0,1)$ and $(5,2,0)$, i.e. the multisets $X_5X_3^4$ and $X_4^2X_3^5$. Their $5+21 = 26$ arrangements yield 21 distinct resident words (5 coincidences via Christoffel concatenation identities). All 21 are nonsingular, with minimum lattice distance 35,424,725 — the finite-Christoffel sector's residents sit far from this wall, as the closed sector's certificates predict.

## 6 Three claims, kept separate

Contact nonexistence: proved by census.

Essential zero-window exclusion: proved by tracer.

Conditional isolation: tracer plus contact complementarity.

Contact absence is not inferred from zero-window exclusion — the mod-7 layer carries 42,019 fatal tuples and the joint expectation was a nontrivial 0.041 — and zero-window exclusion is not inferred from contact absence: the tracer proved it before a single resident was enumerated.

## 7 Three shells compared

$(7,11)$: one primitive nondegenerate contact, isolated and explained by curve–box geometry. $(17,27)$: contact-free, essential zero-window-free by a layered sieve with a bottleneck layer and a three-congruence endgame. $(20,31)$: contact-free, essential zero-window-free by a pure mixed-bracket bottleneck — the large prime empty at every essential length. The pattern holds across three mechanisms; the mechanisms share nothing but the wall.

## 8 Updated ledger

Adding $(20, 31)$, the contact-free census ledger reaches exactly

$$74{,}897{,}950 + 54{,}627{,}300 = 129{,}525{,}250$$

censused residents across 33 shells. The $-17$ necklace remains the unique primitive nondegenerate contact in the enumerated universe.

## 9 Tier ledger and limitations

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| **Tier 1** | the contact-free census (Thm. 2.1); the mod-7 count and its quantization; the large-prime zero count; $r^* = 14$; the mechanical-word residue; the 21 product-resident verifications. |
| **Tier 3** | uniform-expectation comparisons; near-miss typicality. |
| **Tier 4** | extrapolation to all shells; any global claim. |

Finite shell only; no proof of the Collatz conjecture; no statement about arbitrary future shells; no claim that all proper noncontact windows (lengths $11, \ldots, 19$) are absent; no contact anatomy, because no contact exists.

## 10 Conclusion

The $(20, 31)$ shell is empty at the wall. The tracer program had already shown that any contact would have been isolated; the census shows there is no contact to isolate. This is the first shell resolved in the program's intended order: identify the local geometry first, then ask whether the wall is touched. It is not — the nearest resident stands fourteen residues away, and behind it, fifty-four million more, none closer.

## References


[1] E. De Jesús, *Structured Sectors of the Collatz Carry Equation: Christoffel Towers, 2-Adic Windows, and Shell Residents — Series Notes*, Zenodo, 2026. `https://doi.org/10.5281/zenodo.20557441`

[2] J. C. Lagarias, *The $3x + 1$ problem and its generalizations*, Amer. Math. Monthly **92** (1985), 3–23.