The Distinguished Reproducing Kernel

Exact Closure and Kernel-Generated Geometry

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Abstract

This paper introduces the Distinguished Reproducing Kernel as a primitive exact-closure principle for kernel-generated geometry and as the physical realization of logicality. The central object is not a reproducing kernel on a previously completed background. It is a maintained kernel K, a candidate state Σ, and a closure transform T_K co-determined by the same closure datum:

$$T_K(\Sigma) = (K, \Sigma'), \qquad K \subsetneq \Sigma, \quad K \subsetneq \Sigma'.$$

The equation is pair-valued, nondegenerate, and nonprojective: it reproduces the maintained kernel exactly while producing a closed state not exhausted by that kernel. It distinguishes ordinary reproducing-kernel idempotency from exact closed-state reproduction. From the datum itself, admissibility follows: a state on which T_K is asserted has already survived the tests needed for the closure transform of the maintained kernel to be formed. Reproduction is the further requirement that closure produce an admissible state over the same kernel. In analytic realizations this means that the regularized closure family has a stable, presentation-independent value; integral well-posedness is generated by exact closure rather than imported as a prior background domain. The same framework gives nonzero kernel states, local logarithmic potentials, normalized kernel overlaps, and the canonical metric identity

$$g_{\widehat{\Phi}} = \frac{1}{2} g_K$$

between the raw kernel geometry and the normalized comparison geometry. The result is a compact foundation for treating geometry as the residue of exact kernel closure. Classification and physical applications are natural further directions.

1 Introduction

Reproducing kernels are usually studied after the ambient analytic setting has already been chosen. One specifies a domain, a measure, a Hilbert space of functions, and then studies the kernel that reproduces evaluation in that space. The Distinguished Reproducing Kernel reverses this order. The kernel is not merely an auxiliary object placed on top of a background geometry. It is the maintained object whose closure generates the geometry on which its own reproduction is tested.

The essential change is that exact closure is not the scalar equation $T_K(\Sigma) = K$. That equation records only the maintained-kernel projection. The full closure statement is pair-valued:

$$T_K(\Sigma) = (K, \Sigma'), \qquad K \subsetneq \Sigma, \quad K \subsetneq \Sigma'.$$

The maintained kernel is preserved, but it does not exhaust either the input state or the closed output state. The second component Σ' contains the realized branch content: response, residue, transition, and state information beyond the representative kernel itself.

This distinction is the main foundation needed for the physics. Ordinary Hilbert-space reproduction says that a kernel reproduces functions in a space. Exact DRK closure says that a candidate state carrying the maintained kernel closes, under the transform of that kernel, to a closed state that still contains the same maintained kernel. Ordinary reproduction is therefore below DRK closure. It tests the kernel component, not the whole closed state.

The second distinction is admissibility. A candidate state is not first admitted by an independent background and then tested by closure. In the DRK formulation, the assertion of

$$T_K(\Sigma) = (K, \Sigma')$$

already says that the state has survived the requirements needed for the closure transform of K to be formed. In analytic language, the regularized closure construction must possess a stable value independent of cutoff, chart, angular frame, and regularization representative. This is the same logical order by which a function must be integrable before an integral can act on it. The equation does not range over arbitrary syntax.

The purpose of this paper is specific. We isolate the exact closure datum, prove the first consequences that follow from it, and recover the local analytic-geometric objects attached to a realized branch: kernel states, nonzero diagonal, normalized overlap, local potential, and the relationship between the raw kernel metric and the normalized overlap metric. The point is to set up the minimum exact-closure formalism in which classification, boundary rigidity, and physical response questions become well posed.

The relationship with the surrounding papers is structural. Logicality gives the universal criterion: admissibility and reproduction must be co-determined by the same closure datum. The present paper gives the physical DRK form of that criterion. The Fixed Point Law then classifies the realized exact branch and proves that the surviving geometry is the canonically normalized $\mathbb{C}H^2$ branch [5, 4].

2 Reproductive Co-Determination

The closure claim is not that an independently chosen geometry happens to carry a useful reproducing kernel. That would import the arena first. The claim is that the maintained reproducing kernel K, the candidate state Σ, and the closure transform T_K are co-determined. The transform is existentially constrained, not prescribed. In particular, T_K is not a specified background function and the assignment $K \mapsto T_K$ is not a prior rule. They are part of what a physical solution must realize. A candidate closes only if it admits a transform generated by K that sends an admissible state to an admissible produced state while reproducing K exactly. There are no prior prescriptions on the domain or on the function beyond this closure achievement. A candidate that requires a separately chosen domain, boundary chart, projection, regularization, or channel decomposition has not closed as an object-level closure state.

This is the premise that must be understood before the equation is read. DRK closure is not $T = \mathrm{Cn}(T)$, which closes a theory under a preassigned consequence relation. It is not a domain with rules imposed on top. It is a reproductive closure test: only states that admit a kernel-generated closure transform are admitted, and only transforms that reproduce the same kernel into further admissible states count as physical closure. Whether a solution exists, and whether it is unique, is downstream of this premise; the premise itself replaces external domain/rule matching by reproductive co-determination.

Equivalently, the equation below should be read as an admission rule rather than as a function already living in a background universe. In the ordinary order one writes

domain first, rule second, compatibility checked afterward.

In reproductive closure the order is reversed:

$$K \rightsquigarrow T_K, \qquad \Sigma \text{ is admitted only when } T_K(\Sigma) = (K, \Sigma') \text{ with } K \subsetneq \Sigma, \quad K \subsetneq \Sigma'.$$

The transform is therefore not a freely chosen map. It is the closure rule a candidate must be able to generate in order to count as admitted at all. A solution determines both the admissible domain and the generated transform. This is the fixed-point content of the datum: K generates a rule, the rule preserves K, and the produced state is admissible over the same kernel without being exhausted by K.

3 Primitive Exact Closure

Definition 3.1 (DRK closure datum). *A DRK closure datum for a maintained reproducing kernel K is the pair-valued closure form*

$$T_K(\Sigma) = (K, \Sigma'), \qquad K \subsetneq \Sigma, \quad K \subsetneq \Sigma'.$$

Here Σ is the whole candidate physical state, K is the co-determined reproduced physical basis, T_K is the closure transform over that basis, and Σ' is the produced physical state. There is no pointwise admission relation hidden underneath the datum. The candidate state Σ is tested as a whole, and neither the candidate state nor the produced state is exhausted by the reproduced basis.

Definition 3.2 (Admissibility over K). *A candidate physical state Σ is admissible over the reproduced kernel K, written $\mathrm{Adm}_K(\Sigma)$, when*

$$\mathrm{Adm}_K(\Sigma) \iff \Sigma \in \mathrm{Dom}(T_K) \quad \textit{and} \quad K \subsetneq \Sigma.$$

Admissibility is the input condition for causal standing in the branch: the candidate is a closure-produced state for T_K, and it properly carries the reproduced kernel. Thus $\mathrm{Dom}(T_K)$ is not an independently stipulated background domain. It records closure-produced status: $\Sigma \in \mathrm{Dom}(T_K)$ means that Σ has been produced by T_K and is available as an input for a further closure event.

Definition 3.3 (Reproduction over K). *A closure event is reproductive over K, written $\mathrm{Rep}_K(T_K; \Sigma)$, when the candidate is admissible and the closure output reproduces K while producing an admissible state over the same kernel:*

$$\mathrm{Rep}_K(T_K; \Sigma) \iff \mathrm{Adm}_K(\Sigma) \quad \textit{and} \quad T_K(\Sigma) = (K, \Sigma') \quad \textit{with} \quad \mathrm{Adm}_K(\Sigma').$$

Reproduction is the closure condition for causality: the output must preserve the reproduced kernel as K and remain admissible over K. A transform reproduces over K only insofar as its asserted closure events satisfy $\mathrm{Rep}_K(T_K; \Sigma)$.

Proposition 3.4 (Admissibility follows from the datum). *Any state on which a DRK closure datum for K is asserted is admissible for that closure transform in the primitive sense: it carries K as a proper substate and lies in the generated realized domain $\mathrm{Dom}(T_K)$ of the closure transform of K.*

Proof. The datum contains the strict inclusion $K \subsetneq \Sigma$, so the input state already carries the maintained kernel. It also asserts a closure event for T_K, so the input has survived whatever tests are needed for the closure transform of K to be formed. Those inputs are exactly the closure-produced domain $\mathrm{Dom}(T_K)$. □

Proposition 3.5 (Causal closure requires reproduction). *A DRK closure event is causal over K only when it satisfies $\mathrm{Rep}_K(T_K; \Sigma)$. Thus closure must reproduce the maintained kernel exactly and produce a state that remains admissible over the same kernel.*

Proof. Admissibility gives the input side of causal standing: an input state over K must carry K properly and lie in the generated domain of T_K. Reproduction gives the first output component K, and physical closure requires that the produced state remain admissible over the same kernel. If the produced state is not again admissible over K, then the transform has not produced physical closure from physical closure; it has produced a formal output outside the same causal branch. □

Proposition 3.6 (Nondegenerate closure). *A DRK closure datum is nondegenerate because the produced physical state is not exhausted by K:*

$$K \subsetneq \Sigma'.$$

Proof. The first component of $T_K(\Sigma) = (K, \Sigma')$ is the maintained kernel. The second component is the produced physical state. The strict containment $K \subsetneq \Sigma'$ says that the produced state is not exhausted by the kernel. If the second component collapsed to K, the event would record kernel-level idempotency, not nondegenerate physical closure. □

Proposition 3.7 (Ordinary reproduction is below exact closure). *The identity that a reproducing kernel reproduces in its Hilbert space is not the DRK closure datum. Ordinary reproducing-kernel idempotency records only the kernel component. It does not supply the pair-valued closure datum*

$$T_K(\Sigma) = (K, \Sigma'), \qquad K \subsetneq \Sigma, \quad K \subsetneq \Sigma',$$

nor does it supply a realized closed state carrying content beyond the maintained kernel.

Proof. Ordinary reproduction is a Hilbert-space identity for the kernel component. Exact nondegenerate reproduction is a statement about the closure transform of K on an admissible candidate state carrying K properly: it returns the pair (K, Σ'), requires $K \subsetneq \Sigma$ and $K \subsetneq \Sigma'$, and requires the produced state to remain admissible over the same kernel. These are additional components of the DRK closure datum, so ordinary reproduction is strictly below exact closure. □

Proposition 3.8 (Basis-exhausted closure is degenerate). *A purported closure event of the form*

$$T_K(\Sigma) = (K, K)$$

does not satisfy the DRK closure datum, because the datum requires

$$K \subsetneq \Sigma'.$$

Proof. The first component of closure is the maintained kernel K. The second component must be a closed state that properly contains that kernel. If the second component is exhausted by K, then the strict containment $K \subsetneq \Sigma'$ fails. Such an event may record a kernel-level idempotency, but it does not produce a nondegenerate closed state. □

Proposition 3.9 (Nonprojective exactness and basis drift). *Exact DRK closure requires the reproduced basis to be exactly K. If the first component of closure is a drifted basis $\widetilde{K}$ not proved to be the same admissible representative inside the same datum, then reproduction fails. A scalar residue is the special projective subtype*

$$T_K = cK, \qquad c \neq 1.$$

Thus an unabsorbed scalar residue is a projective branch, not an exact nonprojective DRK closure branch.

Proof. Reproduction over K requires

$$T_K(\Sigma) = (K, \Sigma')$$

with the produced state admissible over the same kernel. If the first component is $\widetilde{K}$ rather than K, then the transform has not reproduced the basis under which the input had causal standing unless $\widetilde{K}$ is itself proved to be the same admissible representative inside the datum. A residual scalar multiple cK is the projective case of this failure. If the scalar part is absorbed into the closed state Σ', it contributes to the produced state and leaves the maintained kernel exactly K. If it is not absorbed, it remains as an independent scalar multiple of the kernel component, so the kernel-level output is $T_K = cK$. The case $c = 1$ has no residual scalar; a nontrivial unabsorbed scalar residue therefore has $c \neq 1$, which is exactly the projective branch. □

4 Realized Integral Closure

The displayed closure datum is abstract. In analytic realizations the regularized family $T_{K,\varepsilon}$ is the candidate construction. It does not define the realized transform T_K until the normalized residue exists and is independent of admissible representatives. Once that stable value is proved, T_K is the realized closure value and $\mathrm{Dom}(T_K)$ is the generated domain on which that value has causal standing. The raw closure integral may be singular near a boundary, or may require a cutoff before its stable residue can be extracted. Exact closure therefore requires a well-posed limiting value, not necessarily an absolutely convergent uncut integral.

Notation 4.1 (Regularized closure representatives). *Let $T_{K,\varepsilon}$ denote a regularized representative of the closure construction for K, where $\varepsilon > 0$ is a cutoff parameter. A realized closure value is written*

$$\mathrm{Res}_K(\Sigma) = \lim_{\varepsilon \to 0^+} N_\varepsilon^{-1} T_{K,\varepsilon}(\Sigma),$$

when the normalized limit exists and is independent of admissible changes of cutoff, boundary chart, angular frame, and regularization representative. This notation does not assert T_K in advance. It records the candidate family whose stable residue is required before T_K can be read as a realized closure transform.

Proposition 4.2 (Exact closure generates its integral domain). *If*

$$T_K(\Sigma) = (K, \Sigma')$$

is asserted as a realized exact closure equation, then the regularized closure construction has a stable value $\mathrm{Res}_K(\Sigma)$, independent of the admissible representatives used to compute it. Consequently integral well-posedness is generated by exact closure itself: it is not a geometric domain imported before the equation.

Proof. The regularized family $T_{K,\varepsilon}$ is the candidate construction from which the realized transform is to be extracted. The equation asserts a single realized value of T_K at Σ. If no normalized limit existed, there would be no realized value $T_K(\Sigma)$. If the value changed under an admissible cutoff, chart, angular frame, or regularization representative, then the same input state would produce more than one closure value. Either case contradicts the asserted exact equation. Therefore the integral representatives must determine a stable value before T_K is read as a realized closure transform, and the states for which they do so are exactly the generated domain on which T_K is causal. □

Proposition 4.3 (Integral realization of admissibility)**.** *A realized exact closure datum makes Σ admissible for the integral closure transform of K. It satisfies both:*

1. $K \subsetneq \Sigma$*;*
2. *the cutoff closure construction for Σ has a stable, representative-independent realized value.*

Proof. The first item is a field of the primitive datum. The second item follows from exact closure generating its integral domain. Together these are the primitive admissibility conditions for this realized branch: the input carries the maintained kernel, and the closure construction can actually act on it. □

Theorem 4.4 (Exact closure only evolves admissible states)**.** *If*

$$T_K(\Sigma) = (K, \Sigma')$$

is a realized exact closure equation, then Σ is admissible for K. Equivalently,

$$\mathrm{Adm}_K(\Sigma).$$

Proof. The realized exact closure datum contains an input state, a maintained-kernel representative $K \subsetneq \Sigma$, and a stable value of the closure construction. Therefore the input state carries the kernel and survives the integral well-posedness test needed for $T_K(\Sigma)$ to be asserted. This is precisely $\mathrm{Adm}_K(\Sigma)$. □

Remark 4.5 (Surviving the integral)**.** *Integral well-posedness does not mean that every formal expression has an ordinary uncut integral. It means that the closure construction has a canonical residue or realized value selected from its regularized representatives. A state that cannot produce such a value is not an input for the realized T_K.*

5 Regularity Forced by Closure

The first analytic consequences are not a classification of geometry. They are the minimum regularity consequences needed for the maintained kernel to be read inside a closed state.

Lemma 5.1 (Closure-admissible kernel states)**.** *Let $T_K(\Sigma) = (K, \Sigma')$ be a realized exact closure branch. For each realized point z, the kernel section*

$$k_z(\cdot) = K(\cdot, z)$$

defines a nonzero finite state in the closed state Σ', and its normalization is independent of the cutoff, chart, angular frame, and regularization used to compute the closure value.

Proof. The first component of the exact closed output is the maintained kernel K. Thus the sections k_z are not auxiliary probes; they are state coordinates through which the preserved kernel component is read inside Σ'. If k_z failed to be finite, the cutoff construction would not produce a finite normalized state at z. If $k_z = 0$, the point z would carry no maintained-kernel state and could not occur as a realized point of the closed branch. If the normalization of k_z depended on the chosen cutoff, boundary chart, angular frame, or regularization, then the same closed state would depend on an auxiliary representative. That contradicts realized exact closure. □

Proposition 5.2 (Closure forces nonzero diagonal)**.** *On a realized exact closure branch, the distinguished kernel has nonzero diagonal:*

$$K(z, z) \neq 0.$$

Proof. In the reproducing realization determined by the maintained kernel, the diagonal records the self-pairing of the kernel state:

$$K(z, z) = \langle k_z, k_z \rangle.$$

By the closure-admissibility lemma, k_z is a nonzero finite state of the closed branch. Therefore the diagonal self-pairing cannot vanish on the realized branch. If it did, the normalized closure state at z would be undefined or zero, and T_K could not return K as a maintained kernel on the realized state space. □

Proposition 5.3 (Kernel potential is locally defined)**.** *On a realized exact closure branch, nonzero diagonal gives a local logarithmic potential*

$$\Phi_K(z) = \log K(z, z),$$

after choosing a closure-admissible logarithmic branch.

Proof. The preceding proposition gives $K(z, z) \neq 0$ on the realized branch. Therefore, locally, the diagonal admits a logarithmic branch. A branch choice is admissible exactly when it is compatible with the same closure representatives used to compute $T_K(\Sigma)$. Thus Φ_K is a local potential associated with the realized closure branch, not an external background potential. □

Remark 5.4 (Nonzero, not positivity)**.** *The closure datum forces nonvanishing of the diagonal needed to form local normalizations and logarithmic potentials. Positivity, when present, belongs to a positive Hilbert representative or a positive Hermitian metric branch. It is not part of the primitive exact-closure statement.*

6 Raw Kernel and Normalized Overlap

The raw kernel and the normalized overlap have different roles. The raw kernel K is the maintained object in the exact closure law. The normalized overlap removes diagonal amplitude and records relative section comparison.

Notation 6.1 (Normalized overlap)**.** *On a local closure-admissible square-root branch of the nonzero diagonal, define*

$$\widehat{\Phi}(A, B) = \frac{K(A, B)}{\sqrt{K(A, A)K(B, B)}}.$$

Proposition 6.2 (Diagonal normalization)**.** *For every realized point A,*

$$\widehat{\Phi}(A, A) = 1.$$

Proof. Substituting $B = A$ into the definition gives

$$\widehat{\Phi}(A, A) = \frac{K(A, A)}{\sqrt{K(A, A)K(A, A)}}.$$

On the chosen closure-admissible square-root branch, the denominator is $K(A, A)$. Since the diagonal is nonzero, the quotient is 1. □

Proposition 6.3 (Hilbert representative bound)**.** *If the realized closure branch is represented by a positive Hilbert inner product for which*

$$K(A, B) = \langle k_B, k_A \rangle, \qquad K(A, A) = \langle k_A, k_A \rangle,$$

then

$$|\widehat{\Phi}(A, B)| \leq 1.$$

Proof. The Cauchy-Schwarz inequality gives

$$|K(A, B)|^2 = |\langle k_B, k_A \rangle|^2 \leq \langle k_A, k_A \rangle \langle k_B, k_B \rangle = K(A, A)K(B, B).$$

Dividing by the nonzero diagonal product gives the stated bound. □

Notation 6.4 (Contrast and phase)**.** *On a simply connected local patch where $\widehat{\Phi}(A, \cdot) \neq 0$, choose a logarithmic branch and write*

$$\log \widehat{\Phi}(A, \cdot) = -d_A + iS_A.$$

Here $d_A = -\log |\widehat{\Phi}(A, \cdot)|$ is the contrast function and S_A is the local phase, defined modulo the logarithmic branch.

Lemma 6.5 (Holomorphic phase-contrast relation)**.** *If $\widehat{\Phi}(A, \cdot)$ is holomorphic and nonvanishing on a simply connected coordinate patch, then the real and imaginary parts satisfy the Cauchy-Riemann relation*

$$\nabla S_A = J \nabla d_A,$$

up to the sign convention used for the complex structure J.

Proof. The logarithm of a nonvanishing holomorphic function is holomorphic on the chosen patch. Its real and imaginary parts are harmonic conjugates. Written in geometric form, the Cauchy-Riemann equations give the displayed relation, with only the global sign depending on the convention for J. □

7 Kernel Potential and Metric Identity

The kernel potential supplies the local Hermitian form associated with the raw kernel:

$$\omega_K = i\partial\bar{\partial}\Phi_K = i\partial\bar{\partial} \log K(z, z).$$

Because it is an $i\partial\bar{\partial}$-form, it is closed. When the realized branch supplies nondegeneracy and the relevant positivity or fixed signature, this form is the local Kähler or pseudo-Kähler form used by the closure integral. The following identity does not require a global classification; it is a local Hessian identity.

Proposition 7.1 (Closed Hermitian form). *The form*

$$\omega_K = i\partial\bar{\partial}\log K(z,z)$$

is closed wherever the local potential $\log K(z,z)$ *is defined.*

Proof. Since $d = \partial + \bar{\partial}$ and $\partial^2 = \bar{\partial}^2 = 0$,

$$d\omega_K = i(\partial + \bar{\partial})\partial\bar{\partial}\log K(z,z) = 0.$$

Thus ω_K is closed. This is a local differential identity attached to the closure-generated potential. □

Notation 7.2 (Raw and normalized Hessian forms). *Write*

$$g^K_{\mu\bar{\nu}} = \partial_\mu\partial_{\bar{\nu}}\log K(z,z).$$

Fix w_0 *and, on a coordinate neighborhood where* $K(z,w_0) \neq 0$, *define the normalized overlap Hessian by*

$$g^{\widehat{\Phi}}_{\mu\bar{\nu}}(w_0) := -\partial_\mu\partial_{\bar{\nu}}\log|\widehat{\Phi}(z,w_0)|\big|_{z=w_0}.$$

Theorem 7.3 (Normalized metric identity). *Assume* $K(\cdot,w_0)$ *is holomorphic and nonvanishing near* w_0. *Then*

$$g^{\widehat{\Phi}}_{\mu\bar{\nu}}(w_0) = \frac{1}{2}g^K_{\mu\bar{\nu}}(w_0).$$

Proof. By definition,

$$\widehat{\Phi}(z,w_0) = \frac{K(z,w_0)}{\sqrt{K(z,z)K(w_0,w_0)}}.$$

Taking logarithmic absolute values gives

$$\log|\widehat{\Phi}(z,w_0)| = \log|K(z,w_0)| - \frac{1}{2}\log K(z,z) - \frac{1}{2}\log K(w_0,w_0).$$

The final term is independent of z. Since $K(\cdot,w_0)$ is holomorphic and nonvanishing near w_0, $\log|K(z,w_0)|$ is pluriharmonic there, so its mixed complex Hessian vanishes. Therefore

$$-\partial_\mu\partial_{\bar{\nu}}\log|\widehat{\Phi}(z,w_0)|\big|_{z=w_0} = \frac{1}{2}\partial_\mu\partial_{\bar{\nu}}\log K(z,z)\big|_{z=w_0}.$$

This is the stated identity. □

Remark 7.4 (Two objects, two roles). *The raw kernel* K *generates the potential and carries exact closure. The normalized overlap* $\widehat{\Phi}$ *compares kernel states after diagonal amplitude has been removed. The identity*

$$g_{\widehat{\Phi}} = \frac{1}{2}g_K$$

does not collapse these objects. It records their canonical local relation.

8 State and Transition Admissibility

The pair-valued closure law also separates state admissibility from transition admissibility. A candidate state can be syntactically describable without being an input for T_K. A candidate output can be describable without being the closed state produced from a proposed input by the same maintained kernel.

Proposition 8.1 (Transition admissibility criterion)**.** *For a fixed maintained kernel K, an ordered pair of states (Σ_0, Σ_1) is a causal transition only if*

$$\mathrm{Adm}_K(\Sigma_0), \qquad T_K(\Sigma_0) = (K, \Sigma_1), \qquad \mathrm{Adm}_K(\Sigma_1).$$

Proof. The DRK closure law is pair-valued and preserves the maintained kernel. A transition is causal for this branch only when its input lies in the generated domain and the proposed output is the second component actually returned by T_K from that input while remaining admissible over the same kernel. Otherwise the pair either lacks an admissible input, lacks an admissible produced state, or is not the transition produced by the same kernel-preserving closure law. □

Theorem 8.2 (State failure and transition failure are distinct)**.** *Relative to a nontrivial realized branch T_K, there are two different ways a counterfactual can fail:*

1. *a candidate Γ is an inadmissible state when $\neg\,\mathrm{Adm}_K(\Gamma)$;*
2. *a candidate pair (Σ_0, Γ_1) is an inadmissible transition when $\mathrm{Adm}_K(\Sigma_0)$ but not*

$$T_K(\Sigma_0) = (K, \Gamma_1) \quad \textit{with} \quad \mathrm{Adm}_K(\Gamma_1).$$

In the second case the failure is not that the initial state is meaningless; the failure is that the proposed output is not produced by the same kernel-preserving closure law as an admissible produced state.

Proof. The first claim follows from generated admissibility: a state that fails Adm_K has not survived the maintained-kernel and integral well-posedness tests required for T_K to act on it. For the second claim, assume $\mathrm{Adm}_K(\Sigma_0)$. Exact closure returns a specific pair

$$T_K(\Sigma_0) = (K, \Sigma').$$

The proposed transition to Γ_1 is admissible exactly when $\Gamma_1 = \Sigma'$ as the closed component of that same transform and $\mathrm{Adm}_K(\Gamma_1)$. If the proposal changes the maintained kernel, removes state data encoded by the input, inserts a closed component not returned by T_K, or produces a state not admissible over the same kernel, then the pair fails transition admissibility even though the initial state was admissible. □

Proposition 8.3 (Partial counterfactual collapse)**.** *A physical counterfactual has causal standing only when it specifies a whole candidate state or a whole candidate transition. A partial edit of a closure-coupled state is not a physical state; it is an admissibility failure.*

Proof. The physical closure law acts on the whole state Σ, not on an isolated verbal component of that state. A counterfactual therefore has causal standing only after the whole candidate state or whole candidate transition has been specified. If only one component of a closure-coupled state is edited while the remaining closure data are required to retain their old causal role, then the candidate has not survived Adm_K. It is a partial formal edit, not a state in the generated domain of T_K. □

Example 8.4 (Sun-disappearance counterfactual)**.** *The question "what happens if the Sun disappears?" is not yet a physical candidate. One must ask what whole-state claim is being made. If the mass-source label is removed while the old curvature, residue, and orbital closure data are retained, then the Earth can still appear to follow the old curved-state trajectory because the closure data of the old state have not actually been removed. If the Sun and the closure data determined with it are collapsed together into a no-Sun state, then the resulting candidate is a different whole state, and the Earth is already in the corresponding no-Sun trajectory. There is no paradox in either whole-state reading. The paradox appears only when the counterfactual removes one component of* Σ *while demanding that other closure-coupled components retain their old causal role. That partial edit has not survived admissibility.*

Example 8.5 (The empty-state counterfactual)**.** *Relative to any nontrivial realized branch, "nothing exists" is not a physical state inside the branch. It removes the maintained kernel, the state carrying that kernel, and the closure representatives from which the realized value is extracted. It therefore fails state admissibility for that branch.*

9 Outlook: Exact Closure as a Geometric Principle

The point of the DRK formalism is not that a geometry has already been chosen. It is that geometry becomes a closure question. A maintained kernel K, a candidate state Σ, and a closure transform T_K are co-determined by the same datum. Exact closure preserves K, and the closed output Σ' carries state content beyond the kernel:

$$T_K(\Sigma) = (K, \Sigma'), \qquad K \subsetneq \Sigma, \quad K \subsetneq \Sigma'.$$

This is the minimum nondegenerate and nonprojective structure. It is already enough to distinguish ordinary RKHS reproduction from exact closed-state reproduction, to generate admissibility, to require reproductive closure over the same kernel, to require integral well-posedness, and to recover the local kernel potential and normalized overlap geometry.

Several further directions are natural. One may classify realized branches that satisfy stronger global hypotheses. One may study scalar residues and boundary response channels produced by the regularized closure integral. One may ask when the local Hermitian form generated by $\log K(z,z)$ extends to a complete global geometry. One may also use the state/transition distinction to formulate physical counterfactuals without allowing arbitrary syntactic descriptions to masquerade as admissible states.

The foundational lesson is simple: the kernel is not placed inside a finished arena. The domain in which the kernel has causal standing is generated by the exact closure event itself. In a DRK realization, geometry is recovered as the residue of exact admissible reproduction. Logicality gives the universal criterion, DRK gives its physical closure form, and the Fixed Point Law classifies the unique realized exact branch.

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