A Multiphysics Simulation of the Crookes Radiometer with Real-Time Interactive Visualisation

Thermal Creep, Molecular Rebound, Radiation Pressure, Convective Bulk Flow, and Electrostatic Torque in a Unified Framework

Phoenix Avila[1]

[1]Independent Research. github.com/X4lis/Crookes-Radiometer

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Abstract

We describe a multiphysics simulation of the Crookes radiometer that covers the main torque-producing mechanisms across four decades of gas pressure: thermal creep, differential molecular rebound, photon radiation pressure, buoyancy-driven convection, and electrostatic charging, together with the thermal and drag sub-models each requires. The physics is implemented in Python and coupled to a real-time interactive front-end built on Pygame, in which the user can reposition light and heat sources, adjust gas pressure, and vary electrostatic charge while watching the rotor respond. The simulation reproduces the characteristic bell-shaped RPM–pressure curve, the ability of body heat to spin the device at atmospheric pressure when placed asymmetrically, and the erratic torques associated with static charge on the glass—three effects that are rarely treated together quantitatively.

Keywords: Crookes radiometer; thermal creep; Knudsen number; molecular flow; convective torque; triboelectric charging; Python simulation.

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1 Introduction

The Crookes radiometer, invented by Sir William Crookes in 1873 (Crookes, 1874), consists of four aluminium vanes mounted on a frictionless spindle inside a partially evacuated glass bulb. Each vane is black on one face and white (or mirror-polished) on the other. When illuminated, the rotor spins with the black faces retreating from the light source. Despite its apparent simplicity, the device eluded a satisfactory theoretical explanation for decades (Maxwell, 1879; Reynolds, 1879).

The controversy has three distinct layers:

1. **Direction.** The observed rotation—black faces away from light—contradicts the naive radiation-pressure prediction (which would push the more-absorbing black face toward the light).
2. **Mechanism.** In the partial-vacuum regime the dominant force is thermal creep along the hot vane surface, not direct photon momentum.
3. **Pressure dependence.** The RPM peaks sharply in the transition-flow regime (Kn $\sim$ 1) and falls toward zero at both very low pressure (no gas to carry momentum) and atmospheric pressure (viscous damping overcomes the drive) (Wolfe et al., 2016).

At atmospheric pressure the device can be made to spin by holding a warm hand near the glass—a convective effect that has attracted comparatively little quantitative study. Similarly, rubbing the glass charges it electrostatically, producing erratic torques that have been reported anecdotally but not, to our knowledge, modelled in a unified framework.

The implementation consists of a batch simulation (`crookes_radiometer.py`) and an interactive visualiser (`crookes_interactive.py`). One aspect that required more care than expected was coupling the exterior convective plume through the glass wall to the interior gas: a naive attenuation factor produced unphysically large convective torques at low pressure, so a pressure-dependent sigmoid was added to suppress bulk flow in the rarefied regime (section 2.7.1). The physical model is developed in section 2, the numerical choices in section 3, and the interactive architecture in section 4, before results and discussion in sections 5 and 6.

2 Physical Model

2.1 Geometry and Coordinate System

The radiometer has $n_v = 4$ vanes arranged at 90° intervals on a central spindle. Each vane has width $w = 15\,\mathrm{mm}$, height $h = 15\,\mathrm{mm}$, and thickness $L = 0.5\,\mathrm{mm}$. The pivot-to-vane-centre arm length is $\ell = 12\,\mathrm{mm}$. The glass bulb has outer radius $R_b = 30\,\mathrm{mm}$.

In the 2-D top-down projection used by the interactive renderer, vane k ($k = 0,1,2,3$) has its centre at

$$\mathbf{r}_k = \ell\big(\cos(\theta + k\pi/2),\ \sin(\theta + k\pi/2)\big), \tag{1}$$

where $\theta(t)$ is the rotor angle. The black-face normal of vane k points outward along $\hat{e}_k = \big(\cos(\theta + k\pi/2), \sin(\theta + k\pi/2)\big)$.

Treating vanes as point masses, the moment of inertia is

$$I = n_v m_v \ell^2 + \tfrac{1}{2} m_s r_s^2 \approx n_v m_v \ell^2, \tag{2}$$

with $m_v = 0.3\,\mathrm{g}$ per vane and the spindle contribution negligible. For the chosen parameters, $I \approx 1.73 \times 10^{-8}\,\mathrm{kg\,m^2}$.

2.2 Radiative Heating and Vane Temperatures

Each face of a vane is in radiative and conductive equilibrium. Denoting the black-face and white-face temperatures as T_b and T_w, the steady-state energy balance per unit area is

$$\alpha_b G - \varepsilon_b \sigma T_b^4 - \frac{k_v}{L}(T_b - T_w) - h_{\mathrm{fm}}(T_b - T_g) = 0, \tag{3}$$

$$\alpha_w G - \varepsilon_w \sigma T_w^4 + \frac{k_v}{L}(T_b - T_w) - h_{\mathrm{fm}}(T_w - T_g) = 0, \tag{4}$$

where G is the incident irradiance ($\mathrm{W\,m^{-2}}$), $\alpha_{b,w}$ and $\varepsilon_{b,w}$ are the absorptivity and emissivity of each face, $\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$ is the Stefan–Boltzmann constant, $k_v = 15\,\mathrm{W\,m^{-1}\,K^{-1}}$ is the vane thermal conductivity, and h_{fm} is the free-molecular heat-transfer coefficient.

In the free-molecular regime the molecular flux $\Gamma = n\bar{v}/4$ dominates. Following Kennard (1938):

$$h_{\mathrm{fm}} = \alpha_{\mathrm{acc}} \frac{\gamma + 1}{\gamma - 1} \frac{k_B}{4(M/N_A)} \rho\, \bar{v}, \tag{5}$$

where $\alpha_{\mathrm{acc}} = 0.9$ is the thermal accommodation coefficient, $\gamma = 1.4$ for air, ρ is the gas density, and $\bar{v} = \sqrt{8k_B T_g/(\pi m)}$ is the mean molecular speed.

Equations (3)–(4) are solved by Newton–Raphson iteration with Jacobian

$$J = \begin{pmatrix} -4\varepsilon_b \sigma T_b^3 - c_k - h_{\mathrm{fm}} & c_k \\ c_k & -4\varepsilon_w \sigma T_w^3 - c_k - h_{\mathrm{fm}} \end{pmatrix}, \tag{6}$$

where $c_k = k_v/L$. Typical converged values at $G = 800\,\mathrm{W\,m^{-2}}$, $P = 10\,\mathrm{Pa}$, and $T_g = 293.15\,\mathrm{K}$ are $T_b \approx 296.0\,\mathrm{K}$ and $T_w \approx 293.2\,\mathrm{K}$, giving $\Delta T \approx 2.8\,\mathrm{K}$.

2.3 Thermal Creep (Radiometric) Force

The dominant drive mechanism in the transition-flow regime is thermal creep. A temperature gradient ∇T along a solid surface drives a tangential gas flow—the *creep flow*—that in turn exerts a reaction force on the vane edge. The phenomenon was analysed by Maxwell (1879) and Knudsen (1909) and placed on a rigorous kinetic-theory footing by Maxwell (1879) and Knudsen; it is sometimes called *thermophoretic edge flow* or the *radiometric force* in the modern literature.

The creep force on the two long edges of one vane (edge area $A_e = 2hL$) is

$$F_{\text{creep}} = C_{\text{tc}} \frac{\mu}{T_{\text{mean}}} \frac{\Delta T}{w} A_e \frac{R_g T_{\text{mean}}}{M}, \tag{7}$$

where $C_{\text{tc}} = 1.147$ is the free-molecular thermal-creep coefficient, μ is the dynamic viscosity of air (Sutherland's law), $\Delta T = T_b - T_w$, w is the vane width (characteristic gradient length), R_g is the universal gas constant, and M is the molar mass of air.

The force is multiplied by the Loyalka interpolation factor (Loyalka, 1975)

$$f_{Kn} = \frac{Kn}{1+Kn}, \tag{8}$$

with the Knudsen number $Kn = \lambda/d_{\text{gap}}$, where $\lambda = k_B T_g/(\sqrt{2}\pi d_{\text{air}}^2 P)$ is the mean free path ($d_{\text{air}} = 3.7$ Å) and $d_{\text{gap}} = 3$ mm is the vane-edge gap. This factor ensures $F_{\text{creep}} \to 0$ in the viscous continuum limit ($Kn \to 0$). The torque contribution from all four vanes is

$$\tau_{\text{creep}} = n_v F_{\text{creep}} \ell. \tag{9}$$

2.4 Differential Molecular Rebound Pressure

In the free-molecular regime, molecules impinging on the hot black face depart with higher thermal speeds than those from the cooler white face. The differential normal pressure on a single vane face is

$$\Delta p_{\text{mol}} = \frac{nm}{4}\left(\bar{v}_b^2 - \bar{v}_w^2\right) = \frac{P}{2}\left(\sqrt{\frac{T_b}{T_w}} - 1\right), \tag{10}$$

where $n = P/(k_B T_{\text{mean}})$. The net force on one vane is $F_{\text{mol}} = \Delta p_{\text{mol}} A_v$, attenuated by the same f_{Kn} factor. This contribution is typically 5–10% of the creep force at $Kn \sim 1$ but becomes dominant in the deep free-molecular regime ($Kn \gg 1$).

2.5 Photon Radiation Pressure

An absorbing surface of area A illuminated with irradiance G receives radiation pressure $P_{\text{rad}} = G/c$ per unit area; a perfectly reflecting surface receives $2G/c$. For the two vane faces:

$$F_{\text{rad},b} = \alpha_b G A_v / c, \tag{11}$$
$$F_{\text{rad},w} = (2-\alpha_w) G A_v / c. \tag{12}$$

The net radiation-pressure force is $F_{\text{rad}} = F_{\text{rad},w} - F_{\text{rad},b}$. With $\alpha_b = 0.95$ and $\alpha_w = 0.15$ this acts in the same direction as thermal creep (black retreating), but is roughly four orders of magnitude smaller.

2.6 Rotational Dynamics and Viscous Drag

The equation of motion for the rotor is

$$I\dot{\omega} = \tau_{\text{drive}} - b\,\omega, \tag{13}$$

where $\tau_{\text{drive}} = \tau_{\text{creep}} + \tau_{\text{mol}} + \tau_{\text{rad}} + \tau_{\text{conv}} + \tau_{\text{elec}}$ and b is the rotational drag coefficient.

Free-molecular drag. In the free-molecular regime each molecule that strikes the vane carries away angular momentum proportional to $m\bar{v}\omega r^2$. The resulting drag coefficient per vane is

$$b_{\text{fm}}^{\text{one}} = \alpha_{\text{acc}} \frac{P}{k_B T_g} m \bar{v} A_v \ell^2, \tag{14}$$

scaled by f_{Kn} to suppress it in the continuum limit.

Continuum (Stokes) drag. In the viscous regime shear stress over the vane face contributes

$$b_{\text{cont}}^{\text{one}} = \frac{\mu A_v \ell^2}{d_{\text{clear}}}, \tag{15}$$

where $d_{\text{clear}} \approx R_b - (\ell + w/2) \approx 12$ mm is the clearance between the vane tip and the bulb wall. A bearing friction term $b_{\text{bearing}} = 5 \times 10^{-10}$ N m s models the PTFE needle pivot. The total drag coefficient is

$$b = n_v\left(b_{\text{fm}}^{\text{one}} f_{Kn} + b_{\text{cont}}^{\text{one}}\right) + b_{\text{bearing}}. \tag{16}$$

2.7 Buoyancy-Driven Convective Torque

A heat source of power Q (e.g. a human hand at ≈ 309 K in a 293 K room, radiating $Q \approx 1.5$ W) produces a buoyant plume. At horizontal distance r_s from the source, the local temperature excess is approximated as

$$\Delta T_{\text{ext}} = \frac{Q}{4\pi k_{\text{air}} r_s}, \tag{17}$$

from which the vertical plume speed scales as

$$U_{\text{vert}} = \sqrt{g\beta\,\Delta T_{\text{ext}}\,H_b}, \tag{18}$$

with $g = 9.81\,\text{m}\,\text{s}^{-2}$, $\beta = 1/T_g$, and $H_b = 60$ mm the bulb height. The horizontal wind component wrapping around the bulb is taken as $U_h = 0.45\,U_{\text{vert}}$.

2.7.1 Glass boundary-layer attenuation.

The glass shell presents two sequential attenuations before the exterior plume can couple to the interior gas dynamics.

1. **Exterior boundary layer.** Over the bulb surface of radius R_b an exterior velocity U_{ext} is reduced by a Blasius-type factor

$$f_{\text{BL}} = 1 - \frac{1}{\sqrt{Re_{\text{ext}}}}, \quad Re_{\text{ext}} = \frac{\rho_{\text{atm}} U_{\text{ext}} R_b}{\mu}. \tag{19}$$

At typical plume speeds $U_{\text{ext}} \approx 0.05$–$0.15\,\text{m}\,\text{s}^{-1}$ this gives $f_{\text{BL}} \approx 0.88$–$0.94$.

2. **Conductive coupling through the glass.** The glass wall of thickness $t_g = 2\,\text{mm}$ and conductivity $k_g = 1.2\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1}$ conducts a heat flux $q'' = k_g \Delta T_{\text{ext}}/t_g$ to the interior surface, raising the inner wall temperature by $\Delta T_{\text{int}} = q''/h_{\text{int}}$, where h_{int} combines free-molecular and Churchill–Chu natural-convection coefficients in parallel. The resulting interior buoyant velocity is

$$U_{\text{int}} = \sqrt{g\beta\,\Delta T_{\text{int}}\,L_c}, \quad L_c = 2R_b. \tag{20}$$

3. **Pressure sigmoid.** At very low interior pressure the gas is too rarefied for bulk convective flow; this is captured by

$$\eta_P = \frac{P}{P + P_{1/2}}, \quad P_{1/2} = 500\,\text{Pa}. \tag{21}$$

The combined glass attenuation factor is

$$f_{\text{glass}} = \text{clamp}\left(f_{\text{BL}} \cdot \frac{U_{\text{int}}}{U_{\text{ext}}} \cdot \eta_P,\ 0,\ 1\right), \tag{22}$$

and the effective velocity acting on the interior vanes is $U_{\text{eff}} = U_{\text{ext}}\, f_{\text{glass}}$.

2.7.2 Vane drag in crossflow.

Each vane is treated as a flat plate in crossflow. The black face (rough paint) has drag coefficient $C_{D,b} = 1.30$ and the white face (polished aluminium) has $C_{D,w} = 0.80$. The differential drag on the two faces of vane k produces a net torque

$$\tau_{\text{conv},k} = \left(C_{D,b} - C_{D,w}\right) \tfrac{1}{2} \rho\, U_{\text{eff}}^2 A_v\, \ell \cdot \text{sgn}(\hat{U} \times \hat{n}_k), \tag{23}$$

summed over all four vanes with appropriate projections.

2.8 Electrostatic Torque

Handling or rubbing the glass bulb deposits triboelectric charge Q on the outer surface in a spatially concentrated patch (Schein, 2007). The aluminium vane faces ($\sigma \sim 10^7\,\text{S}\,\text{m}^{-1}$) respond on microsecond timescales via induced image charges, while the black-paint faces ($\sigma \sim 10^{-6}\,\text{S}\,\text{m}^{-1}$) do not: the RC relaxation time $\tau = \varepsilon_0/\sigma_{\text{paint}} \sim 10^4\,\text{s}$ far exceeds any relevant mechanical timescale. The net electrostatic force is therefore the image-charge attraction on the aluminium face only.

Modelling the charge deposit as a single point charge Q at azimuthal angle ϕ_{patch} on the glass surface, the Coulomb field at vane k is

$$E_k = \frac{Q}{4\pi\varepsilon_0 d_k^2}, \tag{24}$$

where d_k is the distance from the patch to the vane centre. The induced charge on the aluminium face is $q_{\text{ind},k} = \varepsilon_0 E_k A_v$, and the resulting attractive force has a tangential component contributing

$$\tau_{\text{elec},k} = q_{\text{ind},k}\, E_k\, \ell\, \frac{d_{y,k}\cos\phi_k - d_{x,k}\sin\phi_k}{d_k}, \tag{25}$$

where $\phi_k = \theta + k\pi/2$. Summing over all four vanes gives the total torque; because the four vanes are not equidistant from the off-centre patch, the sum is generically non-zero and varies sinusoidally with θ. The torque scales as Q^2; at $Q = 1\,\text{nC}$ the amplitude is $\approx 5\,\text{nN}\,\text{m}$, and at $Q = 5\,\text{nC}$ it rises to $\approx 120\,\text{nN}\,\text{m}$, making it the dominant mechanism at those charge levels.

The patch position ϕ_{patch} undergoes a slow Gaussian random walk ($\sigma = 3°$ per simulation step) to model gradual charge redistribution as the warm glass equilibrates. A multiplicative noise term ($\sigma = 25\%$ of $|\tau_{\text{elec}}|$) captures the finite spatial extent and non-uniformity of the charge deposit, reproducing the qualitative "quivering" of a charged radiometer.

3 Numerical Methods

3.1 Thermal Solver

The Newton–Raphson iteration for (T_b, T_w) is run for up to 30 iterations with convergence criterion $|\delta T_b| + |\delta T_w| < 10^{-5}\,\text{K}$. Temperatures are clamped to $[0.5\,T_g,\ 8\,T_g]$ to guard against divergence. A short-circuit early exit is employed when $G_b = G_w$ and $G < 10^{-9}\,\text{W}\,\text{m}^{-2}$ (both faces at ambient; no Newton iteration needed).

3.2 Exact Analytical Integrator

The equation of motion (13) with constant drive torque is a first-order linear ODE with exact solution

$$\omega(t + \Delta t) = \omega(t)\, e^{-b\Delta t/I} + \frac{\tau_{\text{drive}}}{b}\left(1 - e^{-b\Delta t/I}\right). \tag{26}$$

This integrator is unconditionally stable for any Δt, unlike the RK4 scheme used in the batch simulation, which requires $\Delta t < 2.79\,I/b$ for stability. In the interactive simulation, where Δt can be scaled by the user's simulation-speed factor (up to 512×), this stability guarantee is essential.

3.3 Sub-Stepping and Simulation Speed Scaling

In interactive mode, real-time wall-clock steps Δt_{real} are first capped at 50 ms (to handle pauses and focus loss), then multiplied by the user-selected speed factor $s \in [0.25, 512]$. The total is divided into sub-steps of at most 20 ms of simulated time:

$$n_{\text{sub}} = \left\lceil \frac{s\,\Delta t_{\text{real}}}{0.02} \right\rceil, \qquad \Delta t_{\text{sub}} = \frac{s\,\Delta t_{\text{real}}}{n_{\text{sub}}}.$$

3.4 Batch Simulation: RK4

The batch simulation in `crookes_radiometer.py` employs a classical 4th-order Runge–Kutta scheme with fixed step $\Delta t = 1\,\text{ms}$:

$$\omega_{n+1} = \omega_n + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4), \quad k_i = \frac{\tau_{\text{net}}(\omega_i)}{I}. \tag{27}$$

3.5 Pressure Sweep

The terminal angular velocity under a fixed drive torque τ_{drive} is the steady-state of (13):

$$\omega_\infty = \frac{\tau_{\text{drive}}(P)}{b(P)}. \tag{28}$$

Both numerator and denominator depend on pressure; (28) is evaluated at 60 logarithmically spaced pressures from 0.01 Pa to 10^4 Pa without running the time integration.

4 Interactive Simulation

4.1 Architecture

The interactive front-end is a single-threaded Pygame application (`crookes_interactive.py`) following a classic game loop. All simulation state is held in a singleton `State` object `S`, which stores: rotor angle θ and angular velocity ω; light source pixel coordinates and irradiance; up to four heat-spot positions and powers; gas pressure and ambient temperature; ring-buffer history arrays (length 400) for plotting; and the electrostatic charge index.

Algorithm 1 Interactive simulation main loop.

```
Initialise display (1400 × 860 px), fonts, physics state S
Initialise particle cloud (N_p = 180 gas molecules)
loop
    Δt_real ← elapsed wall-clock time
    Process user events (drag, scroll, key)
    PHYSICSTICK(Δt_real)                          ▷ Eq. (26)
    STEPPARTICLES
    DRAW(screen)
    Blit to display; tick clock at 60 fps
end loop
```

4.2 User Interactions

Table 1 summarises the interactive controls.

4.3 Rendering Pipeline

The rendering pipeline draws layers in the following order:

1. Background panels and title text.
2. Light toggle button with state-dependent glow.
3. Animated photon ray bundle from the draggable sun icon.
4. Convection streamlines (exterior cyan; interior muted cyan-green), with opacity proportional to U_{ext} and f_{glass} respectively.
5. Electrostatic radial field lines with outward (positive) or inward (negative) arrowheads.
6. Pulsing heat-spot icons with inverse-intensity glow.
7. Glass bulb (glow, fill, rim, specular highlight).
8. Gas-particle cloud ($N_p = 180$ bouncing discs).
9. Spindle, arms, and vane polygons (black/white faces).
10. Thermal-creep force arrows on vane edges.
11. Rotation arc with arrowhead proportional to $|\omega|$.
12. Dashboard: RPM gauge, ω trace, torque bars, temperature gradient, parameter table.
13. Controls overlay and status bar.

4.4 Gas Particle Cloud

The $N_p = 180$ particles are initialised uniformly within the glass sphere using a cube-root radial transform to ensure volumetric uniformity. At each frame, each particle receives a random velocity kick whose magnitude scales as $1 + 1.8 \cdot T_{\text{norm}}$, where $T_{\text{norm}} = \min(1, (T_b - T_g)/10\,\text{K})$. Particles are reflected elastically from the sphere boundary. This particle cloud is purely cosmetic—it does not feed back into the torque calculation—but provides an intuitive visual indicator of the thermal agitation level.

5 Results

5.1 Torque Breakdown and Steady-State RPM

Table 2 shows the computed torque contributions and resulting steady-state RPM under the default parameters.

5.2 RPM–Pressure Curve

The terminal RPM as a function of gas pressure follows the characteristic bell shape documented experimentally since Reynolds (1879):

- $P \lesssim 0.1\,\text{Pa}$ ($Kn \gg 1$): molecular rebound dominates but gas density is too low for significant creep flow; both drive and drag are vanishingly small.
- $1\,\text{Pa} \lesssim P \lesssim 100\,\text{Pa}$ ($Kn \sim 1$): optimal regime; thermal creep is maximal relative to viscous drag. Peak RPM $\approx$ 9–12 in our model.
- $P \gtrsim 1000\,\text{Pa}$ ($Kn \ll 1$): viscous damping ($b \propto \mu$, pressure-independent) dominates; drive torque $\tau_{\text{creep}} \propto f_{Kn} \to 0$; RPM $\to 0$.

Remark 1. At atmospheric pressure ($P \approx 101\,325\,\text{Pa}$) the thermal-creep mechanism is completely suppressed ($f_{Kn} \approx 5 \times 10^{-5}$), yet the convective mechanism allows a spin of $\approx$ 2–3 RPM when a warm hand is held 25 cm from the bulb with two asymmetrically placed heat spots. This arises from the differential drag $C_{D,b} - C_{D,w} = 0.50$ between the black and white vane faces.

5.3 Convective Torque: Spot Placement Sensitivity

Table 3 compares three heat-spot configurations at atmospheric pressure ($P = 101\,325\,\mathrm{Pa}$), each with two spots of power $Q \approx 1.5\,\mathrm{W}$ (skin temperature at 91 °F in a 68 °F room).

The strong sensitivity to angular placement is a direct consequence of the non-uniform $\cos(\phi_k - \phi_{\text{source}})$ projection in (23): diametrically opposite sources contribute torques that cancel by symmetry, while a 90° asymmetry is optimal.

5.4 Electrostatic Effects

At $Q = 1\,\mathrm{nC}$ the image-charge torque amplitude is $\tau_{\text{elec}} \approx 5\,\mathrm{nN\,m}$, comparable to the thermal creep torque at $P = 10\,\mathrm{Pa}$. At $Q = 5\,\mathrm{nC}$ the amplitude rises to $\approx 120\,\mathrm{nN\,m}$, making electrostatics the dominant mechanism. The 25% multiplicative noise on the patch shape causes the rotor to accelerate and decelerate erratically, in qualitative agreement with the classical observation that a charged radiometer "quivers and wobbles" rather than spinning steadily (Crookes, 1874).

5.5 Transient Spin-Up

Starting from rest at $P = 10\,\mathrm{Pa}$ with $G = 800\,\mathrm{W\,m^{-2}}$, the simulation reaches 95% of ω_∞ in approximately $t_{95} = 3I/b \approx 8.4\,\mathrm{s}$ (the effective time constant of the linear damping system), in agreement with the analytical result $t_{95} = -3I\ln(0.05)/b$.

6 Discussion

6.1 Model Assumptions and Limitations

The thermal model is one-dimensional: a single temperature is assigned to each vane face, coupled to the gas through a mean-field free-molecular coefficient. Passian et al. (2003) showed that edge temperature gradients can be significantly sharper than a bulk average suggests, so the creep force calculated here likely underestimates the true value at moderate Knudsen numbers. The Loyalka interpolation (Eq. (8)) is a convenient rational fit across the transition regime but is not derived from the Boltzmann equation; a BGK or DSMC calculation would give a more reliable pressure dependence (Loyalka, 1975).

The electrostatic sub-model is the most speculative part of the work. Triboelectric charging produces spatially irregular deposits, not the concentrated single patch assumed here, and the image-charge formula $q_{\text{ind}} = \varepsilon_0 EA$ treats the vane face as a grounded conductor in a uniform field—an approximation that overestimates induced charge near the edges. A boundary-element calculation of the actual charge distribution on a finite plate near a spherical shell would be straightforward but was not carried out here. The predicted torque scale (~ 5–$120\,\mathrm{nN\,m}$ at 1–5 nC) is physically motivated and agrees in order of magnitude with informal observations of a charged radiometer, but should not be taken as a quantitative prediction.

Finally, the vane temperature is solved assuming the rotor is stationary. This is well justified at the speeds observed here ($\omega \ll c_s/\ell \sim 3\times10^4\,\mathrm{rad\,s^{-1}}$), but rotation-induced gas redistribution could matter for a device spinning considerably faster.

6.2 Comparison with Prior Work

Our thermal-creep implementation reproduces the $\Delta T/P$ scaling expected from kinetic theory. The pressure-sweep curve is qualitatively consistent with the experimental and simulation results of Wolfe et al. (2016) (horizontal-vane radiometer, experiment and DSMC). The convective torque mechanism has been discussed qualitatively by Crookes (1874) and referenced in popular-science treatments, but to our knowledge the glass boundary-layer attenuation model presented here (Eqs. (17)–(22)) represents a novel quantitative formulation.

6.3 Extensions

Natural extensions of this work include:

- Replacing the Loyalka interpolation with a full BGK-kernel DSMC simulation for the transition-regime torques.
- A finite-element 3-D thermal solver to resolve hot-edge temperature gradients.
- A vortex-panel aerodynamic model to improve the convective torque calculation around the sphere.
- GPU-accelerated particle simulation for a physically accurate gas cloud replacing the decorative particles.

7 Conclusion

The simulation captures the essential behaviour of the Crookes radiometer across a wide pressure range. Thermal creep dominates in the transition regime ($Kn \sim 1$), accounting for roughly 92% of the drive torque at the default pressure of 10 Pa, while convection takes over as the pressure approaches atmospheric and thermal creep is suppressed. The electrostatic sub-model is the most speculative part of the work, but even a rough monopole treatment produces the qualitative "quivering" that makes a charged radiometer visually distinctive.

The interactive visualiser proved to be a useful debugging tool during development—watching the torque bars respond in real time made it easy to spot sign errors and missing Knudsen-number factors. Whether it will serve

Table 1. Interactive controls in `crookes_interactive.py`.

Input	Effect
Drag sun icon	Moves the collimated light-beam source; irradiance on each vane updates via cosine projection and inverse-square heat-spot law.
Drag heat spot (`HeatSrc`)	Repositions an independent buoyant heat source; convective torque recomputed in real time.
Scroll wheel (on bulb)	Changes gas pressure by $\times 1.15$ or $\times 0.87$ per notch, sweeping the Knudsen regime.
`+` / `-`	Irradiance $\pm 50\,\mathrm{W\,m^{-2}}$ (range: 0–3000 $\mathrm{W\,m^{-2}}$).
`L` / light button	Toggles light source on/off.
`A` / `D`	Adds / removes a heat spot (max. 4).
`E`	Cycles electrostatic charge: none $\to +1\,\mathrm{nC} \to +5\,\mathrm{nC} \to -1\,\mathrm{nC}$.
`[` / `]`	Simulation speed $\nabla \cdot 2$ / $\times 2$ ($0.25\times$–$512\times$).
`Space`	Pause / resume.
`R`	Full reset to default parameters.

Table 2. Torque breakdown at default parameters: $P = 10\,\mathrm{Pa}$, $G = 800\,\mathrm{W\,m^{-2}}$, $T_g = 293.15\,\mathrm{K}$, no heat spots, no charge.

Mechanism	Torque (nN·m)	Fraction (%)
Thermal creep	+27.4	92.1
Molecular rebound	+2.1	7.1
Radiation pressure	+0.3	1.0
Convective (no spots)	0.0	0.0
Electrostatic (off)	0.0	0.0
Viscous drag	−29.8	—
Net (at ω_∞)	≈ 0	—
ω_∞	$0.92\,\mathrm{rad\,s^{-1}}$	
RPM	8.8	

Table 3. Convective torque for three heat-spot configurations at atmospheric pressure ($P = 101\,325\,\mathrm{Pa}$). τ_{conv} is the cycle-averaged net torque; RPM is the resulting steady-state rotation (drag limited).

Configuration	τ_{conv} (nN·m)	RPM
Below + left (default)	+0.60	≈ 2.7
Symmetric (above + below)	≈ 0	≈ 0
Single spot (below)	+0.31	≈ 1.4

as an educational tool for others is harder to judge, but the code is publicly available at the repository listed on the title page.

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A Physical Constants and Default Parameters

Table 4. Physical constants and simulation default parameters.

Symbol	Quantity	Value	Unit
k_B	Boltzmann constant	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
σ_{SB}	Stefan–Boltzmann constant	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
n_v	Number of vanes	4	—
w	Vane width	15	mm
h	Vane height	15	mm
L	Vane thickness	0.5	mm
ℓ	Arm length	12	mm
m_v	Vane mass	0.3	g
α_b	Black face absorptivity	0.95	—
α_w	White face absorptivity	0.15	—
ε_b	Black face emissivity	0.95	—
ε_w	White face emissivity	0.10	—
k_v	Vane thermal conductivity	15	$\mathrm{W\,m^{-1}\,K^{-1}}$
G_0	Default irradiance	800	$\mathrm{W\,m^{-2}}$
P_0	Default gas pressure	10	Pa
T_g	Ambient temperature	293.15	K
d_{gap}	Vane edge gap	3	mm
R_b	Glass bulb radius	30	mm
C_{tc}	Thermal-creep coefficient	1.147	—
α_{acc}	Accommodation coefficient	0.9	—
d_{air}	Air molecule diameter	3.7×10^{-10}	m
k_g	Glass thermal conductivity	1.2	$\mathrm{W\,m^{-1}\,K^{-1}}$
t_g	Glass wall thickness	2	mm
$P_{1/2}$	Pressure sigmoid half-point	500	Pa
$C_{D,b}$	Black vane drag coefficient	1.30	—
$C_{D,w}$	White vane drag coefficient	0.80	—
b_{bearing}	Pivot bearing friction	5×10^{-10}	$\mathrm{N\,m\,s}$
N_p	Particle cloud size	180	—

B Derivation of the Glass Attenuation Factor

Starting from the exterior plume temperature excess ΔT_{ext} at the bulb surface, the conductive heat flux through the glass wall is

$$q'' = \frac{k_g}{t_g}\,\Delta T_{\mathrm{ext}}. \tag{29}$$

This flux heats the inner wall, setting up an interior surface temperature excess above the gas temperature of

$$\Delta T_{\mathrm{int}} = \frac{q''}{h_{\mathrm{int}}} = \frac{k_g\,\Delta T_{\mathrm{ext}}}{t_g\,h_{\mathrm{int}}}, \tag{30}$$

where $h_{\mathrm{int}} = h_{\mathrm{fm}} + h_{\mathrm{nc}}$ combines free-molecular heat transfer (Eq. (5)) with the Churchill–Chu natural-convection coefficient:

$$h_{\mathrm{nc}} = \frac{k_{\mathrm{air}}}{L_c} \begin{cases} 0.59\,Ra_{L_c}^{1/4} & 10^4 < Ra < 10^9, \\ 1 & \text{otherwise}, \end{cases} \tag{31}$$

with $L_c = 2R_b$ and $Ra_{L_c} = g\beta\,\Delta T_{\mathrm{int}}\,L_c^3/(\nu\,\alpha_d)$.

The interior buoyant velocity is then $U_{\mathrm{int}} = \sqrt{g\beta\,\Delta T_{\mathrm{int}}\,L_c}$, and the velocity ratio $U_{\mathrm{int}}/U_{\mathrm{ext}}$ gives the conductive coupling efficiency. Multiplying by the exterior boundary-layer factor f_{BL} and the pressure sigmoid η_P yields Eq. (22).

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