Zero-Subwindow Obstructions in the Collatz Carry Equation: Why the -17 Contact Vanishes Whole but Not Locally

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Abstract

The seventeenth note proved that the -17 contact necklace in the $(7,11)$ shell is locally isolated because no proper cyclic carry-window vanishes modulo 139, with one nonautomatic step: seven explicit length-3 sums. This note shows those sums could not have vanished. In exponent coordinates, a length-3 zero-window corresponds to a point of the zero-triple curve $Z_3 = \{(x,y) : 1 + 2^x + 2^y \equiv 0 \pmod{139}\}$ — a 137-point graph, since 2 is a primitive root — while the exponent gaps available to *any* $(7,11)$ resident are confined to a letter box $x \in \{98,\ldots,102\}$, $y \in \{58,\ldots,66\}$. The five curve points over the box's x-range demand $y = 114, 78, 104, 89, 26$: *the curve and the box are disjoint* (Corridor Exclusion Lemma). Combined with the unit argument (length 1), the letter congruence (length 2), and complementarity (lengths 4–6), this yields a shell-wide theorem: *every contact in the* $(7,11)$ *shell, actual or hypothetical, is window-indecomposable and hence locally isolated*. The -17 isolation theorem becomes a corollary, its seven sums confirmations rather than checks. Calibration: random 7-term power-of-2 zero-sums modulo 139 are window-indecomposable at rate ≈ 0.900, matching the complementarity-corrected estimate $(1-1/139)^{14} \approx 0.904$ rather than the naive $(1-1/139)^{35} \approx 0.777$; and a stacked toy zero-sum with six vanishing windows shows indecomposability is not automatic. Isolation here is corridor-forced, not luck. No proof of the Collatz conjecture is claimed, and no shell beyond $(7,11)$ is resolved.

1 Introduction

The seventeenth note's isolation proof for $D_{-17} = (1,1,1,2,1,1,4)$ [1] ended in seven additions modulo 139. Additions are checks, not explanations. This note supplies the explanation: *the shell's letter corridor and the zero-triple curve are disjoint*, so the length-3 windows of every $(7,11)$ resident — not just D_{-17} — are barred from vanishing. The isolation of -17 is thereby revealed as one visible instance of a shell-wide obstruction.

2 Review: isolation via zero-subwindows

For a contact word with cyclic carry-term sequence $\tau = (\tau_0,\ldots,\tau_{L-1})$ modulo the wall $|N|$, the Window Criterion of the seventeenth note gives local isolation of the whole necklace if no proper cyclic window of τ vanishes; and since the total sum is 0, a window vanishes iff its complement does (Complementarity), so for $L = 7$ only lengths $1, 2, 3$ require argument.

3 Exponent coordinates

In $\mathbb{F}_{139}$: $3 = 2^{41}$ and $\mathrm{ord}(2) = 138$, so each term $\tau_k = 3^{6-k}2^{s_k}$ is, after the common unit 3^6, the power 2^{e_k} with $e_k = s_k - 41k \bmod 138$, and consecutive exponents differ by

$$e_{k+1} - e_k \equiv d_k - 41 \pmod{138}.$$

4 The zero-triple curve

A length-3 window vanishes iff $2^a + 2^b + 2^c \equiv 0$; normalizing by the unit 2^a,

$$1 + 2^x + 2^y \equiv 0 \pmod{139}, \qquad x = b - a,\ y = c - a.$$

Lemma 4.1 (Zero-triple curve)**.** *Since 2 is a primitive root modulo 139, for every $x \neq 69$ there is a unique $y = \log_2(-1-2^x) \bmod 138$ solving the equation, and none for $x = 69$ (where $1+2^x \equiv 0$). Hence $Z_3 = \{(x, y) : 1 + 2^x + 2^y \equiv 0\}$ is the graph of a function on $138 - 1$ points: $|Z_3| = 137$. It carries the S_3-symmetry of permuting the exponents $\{0, x, y\}$; in particular $(x, y) \in Z_3 \iff (y, x) \in Z_3 \iff (-x,\ y - x) \in Z_3 \pmod{138}$.*

5 The $(7, 11)$ letter box

Every letter of a $(7, 11)$ resident satisfies $1 \leq d_k \leq S - L + 1 = 5$ (the remaining six letters contribute at least 6). For a length-3 window starting at position a, the gaps are

$$x = d_a - 41, \qquad y - x = d_{a+1} - 41 \pmod{138},$$

so

$$x \in \{98, 99, 100, 101, 102\}, \qquad y \in \{58, 59, \ldots, 66\}.$$

This 5×5 *letter box* is in fact a superset of the realizable pairs, which further satisfy $d_a + d_{a+1} \leq 6$; checking the full box is therefore the stronger test.

6 The Corridor Exclusion Lemma

Lemma 6.1 (Corridor exclusion)**.** *The zero-triple curve Z_3 is disjoint from the $(7, 11)$ letter box. Explicitly, the curve points over the box's x-range are*

d_a	1	2	3	4	5
x	98	99	100	101	102
curve demands y	114	78	104	89	26

and none of these lies in $\{58, \ldots, 66\}$. All 25 letter pairs were also checked directly: no pair satisfies $1 + 2^{d_a-41} + 2^{d_a+d_{a+1}-82} \equiv 0 \pmod{139}$.

7 Shell-wide nonvanishing and the Shell Isolation Theorem

Theorem 7.1 (Shell-wide window nonvanishing)**.** *In the* $(7, 11)$ *shell, no resident has a vanishing cyclic window of length* 1*,* 2*, or* 3 *modulo* 139*: length* 1 *because terms are units; length* 2 *because the letter congruence would require* $d_k \equiv 110 \pmod{138}$*, impossible for* $d_k \leq 5$*; length* 3 *by Lemma 6.1. Consequently* every contact in $(7, 11)$ is window-indecomposable*: for a contact, lengths* $4, 5, 6$ *are complements of* $3, 2, 1$*.*

Theorem 7.2 (Shell Isolation Theorem for $(7, 11)$)**.** *Every contact resident of the* $(7, 11)$ *shell, actual or hypothetical, is locally isolated in the one-unit move graph.*

Proof. Theorem 7.1 plus the Window Criterion of the seventeenth note. □

Corollary 7.3 (-17, as corollary)**.** *The* -17 *contact necklace — the shell's unique primitive nondegenerate contact — is locally isolated.*

The proof of Theorem 7.2 never mentions D_{-17}. Its term sequence is $(34, 69, 46, 77, 10, 53, 128)$; in the exponent normalization of the seventeenth note — equivalently, after dividing the raw τ-window sums by the common unit $3^6 \equiv 34 \pmod{139}$ — its seven length-3 window sums are

$$33,\ 22,\ 8,\ 45,\ 116,\ 84,\ 109 \pmod{139}.$$

The corresponding raw τ-window sums are

$$10,\ 53,\ 133,\ 1,\ 52,\ 76,\ 92 \pmod{139},$$

and the two lists differ by multiplication by 34, so nonvanishing is identical in either normalization. These sums are now confirmations of the corridor exclusion, not independent checks; and a full-shell sweep of all 210 residents finds, as the theorem requires, zero vanishing length-3 windows.

8 Window-indecomposability

Definition 8.1. A contact zero-sum $\sum_{k=0}^{L-1} \tau_k \equiv 0 \pmod{|N|}$ is *window-indecomposable* if no proper cyclic window sum vanishes.

By the seventeenth note, local isolation is equivalent to window-indecomposability modulo the move-legality caveat (a vanishing window fails to produce a neighbor only when both boundary letters equal 1). The -17 zero-sum is window-indecomposable; by Theorem 7.1, any $(7, 11)$ contact would be.

9 Random baseline: typical, not automatic

Among 40,000 random 7-term power-of-2 zero-sums modulo 139 (six exponents uniform, the seventh solved), the window-indecomposable fraction is ≈ 0.900. The naive estimate $(1 - 1/139)^{35} \approx 0.777$ over all 35 potentially-vanishing windows is wrong for a structural reason: for zero-sums, complementarity pairs the windows and lengths $1, 6$ never vanish, leaving about 14 effective checks, and

$$(1 - 1/139)^{14} \approx 0.904,$$

matching the observation. Indecomposability is therefore *typical but not automatic*: chance alone would have made the -17 isolation likely; the corridor makes it certain.

10 The +1-power contacts: shallow isolation

The three imprimitive contacts decompose structurally: $(2,2)$ on $|N| = 7$ and $(2,2,2)$ on $|N| = 37$ are isolated by the unit/complement argument alone (essential window lengths: $\{1\}$); $(2,2,2,2)$ on $|N| = 175$ requires exactly one genuine check, length 2: with $\tau = (27,36,48,64)$, the four sums $63,84,112,91 \pmod{175}$ are nonzero. Their isolation is structurally shallow compared with -17's, which needed the full curve-versus-corridor mechanism.

11 A decomposable toy: indecomposability is not automatic

The exponent tuple $(0,5,48,0,5,48)$ — two stacked copies of the zero-triple $1 + 2^5 + 2^{48} \equiv 0$ — is a 6-term zero-sum with *six* vanishing length-3 cyclic windows. Decomposable zero-sums exist freely in exponent space; were such a configuration realized by a contact word with a legal boundary, it would have a contact neighbor. Primitive contact therefore does not automatically imply local isolation: in $(7,11)$ the letter corridor is what excludes the fatal exponent patterns.

12 The next general problem

The $(7,11)$ shell is resolved by a curve-versus-corridor exclusion. The natural successor question is whether analogous exclusions hold in larger shells — especially $(17,27)$, where the essential lengths run to 8 and the curves become higher-dimensional zero-sum varieties. The future target: classify whether the relevant zero-window varieties intersect the shell letter corridors at essential lengths.

13 Tier ledger

- **Tier 1** zero-triple curve (Lem. 4.1); letter-box formalism; Corridor Exclusion (Lem. 6.1); shell-wide nonvanishing (Thm. 7.1); Shell Isolation Theorem (Thm. 7.2); the -17 corollary; +1-power anatomy; the decomposable toy.
- **Tier 3** random baseline 0.900; corrected estimate 0.904; typicality interpretation.
- **Tier 4** general primitive-contact isolation; corridor exclusions in larger shells; zero-window classification for arbitrary walls.

14 Limitations

No proof of the Collatz conjecture; one nondegenerate shell is treated, and the theorem proves shell-wide isolation only for contacts of $(7,11)$; window-indecomposability is not automatic in general; the random baselines are heuristic diagnostics; larger shells remain open.

15 Conclusion

The -17 contact vanishes whole because its shell permits no smaller way to vanish: length 1 is excluded by units, length 2 by a letter congruence, and length 3 by the disjointness of the zero-triple curve from the shell's letter corridor. Every $(7, 11)$ contact is locally isolated, and the known -17 necklace is the visible instance of a shell-wide obstruction.

References

[1] E. De Jesús, *Structured Sectors of the Collatz Carry Equation: Christoffel Towers, 2-Adic Windows, and Shell Residents — Series Notes*, Zenodo, 2026. `https://doi.org/10.5281/zenodo.20557441`

[2] J. C. Lagarias, *The* $3x + 1$ *problem and its generalizations*, Amer. Math. Monthly **92** (1985), 3–23.