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XVIII. Solution of two geometrical problems

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those situated beneath participating in this motion*. The observation of our learned countryman is quite naturally explained by attributing to the ascending sap the transport of the corrosive poison, a transport which, in this case, would take place in the direction from below upwards. But how to account for the apparent transmission of the effects of the chloroform in the contrary direction, from above downwards? Might the descending sap more peculiarly have the property of transmitting the narcotic effects of this singular compound from one part of the sensitive plant to the other; or might there exist in this plant some special organ susceptible of being affected by certain vegetable poisons in a manner analogous to the nervous system of animals? Notwithstanding the interesting investigations of Dutrochet and other physiologists, there still prevails too much obscurity on this subject to hazard an opinion. But in any case the fact is singular, and appears to me to merit the attention of persons accustomed to engage in questions of this nature.

Experiments of the same kind, made on the contractility of the sensitive plant with rectified æther, have furnished me with results nearly similar to the preceding; with this difference, however, that whilst one drop of chloroform placed on the common petiole of a leaf situated at the extremity of a branch of a sensitive plant suffices to cause most of the other leaves situated beneath on the same branch to close, æther in general produces an effect only on the leaf itself with which it is put in contact. The next leaves have generally appeared to me not affected. I must however add, that my experiments with æther having been made after the others, and at a time of year when the sensitiveness of the plant already began to diminish, it is possible that the intensity of the effects produced may have thereby been affected.

XVIII. *Solution of two Geometrical Problems.* By JAMES COCKLE, *Esq., M.A., of Trinity College, Cambridge, and Barrister-at-Law of the Middle Temple*†.

THE following solutions are effected by what may be termed a Uniaxal or an Imaginary Geometry. The equations of the problems are formed and treated as if the points which constitute the data and quæsita were in the same straight line. The sketch here given of such a geometry is necessarily short and confessedly imperfect. And yet, perhaps, it will be found

* DeCandolle, *Physiologie Végétale*, vol. ii. p. 866.
† Communicated by the Author.

sufficient for my purpose,—which is, to show the interpretability of impossible quantity.

DEFINITION. In the equation

$$v = A + iB + jC \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

let

$$A^2 + B^2 + C^2 = a^2;$$

then I propose to call (1.) a *virtual* solution of the equation

$$v = a.$$

RULE. To solve a problem by the imaginary geometry, let its conditions be expressed by the independent relations $U=0$, and $V=0$; form the equation

$$U + mV = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

where m is a disposable multiplier: then, if a solution of (2.) is a virtual solution of

$$V = 0,$$

the thing required is done*.

PROBLEM I. Find three points equidistant from each other.

Let A be one of the points. Draw AB equal to any quantity a, and in any direction, and let B be another of the points. Let C be the third point. Then, since C is equidistant from A and B, we have $AC \times CB = AC^2$; and also, since A is equidistant from B and C, we obtain $AC^2 = AB^2$. These equations will, on putting $AC = x$, be expressed algebraically as follows:—

$$x(a-x) = x^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

$$x^2 = a^2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

add (3.) and (4.) and we obtain

$$x(a-x) = a^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

and hence

$$x = \frac{a}{2} \pm \frac{a\sqrt{-3}}{2},$$

$$= \frac{a}{2} \pm i\frac{a\sqrt{3}}{2};$$

and, the solution of (5.) being a virtual solution of (4.), the problem is solved. ABC is, of course, an equilateral triangle.

PROBLEM II. Find four points equidistant from each other.

Complete the rhombus ACDB. Then D is equidistant from B and C. And, by symbolical geometry, we have $AD = AB + BD$. But, we must also have, since A is equi-

* Observations on this Rule and its grounds are reserved for another opportunity.

distant from B and D, AD=AB. Let AD=z; then, we may express the two last conditions by

$$z=a+x, \quad \ldots \ldots \ldots \quad (6.)$$

and

$$z=a; \quad \ldots \ldots \ldots \ldots \quad (7.)$$

add (6.) and (7.) and we obtain, on reducing,

$$z=a+\frac{x}{2}, \quad \ldots \ldots \ldots \quad (8.)$$

or

$$z-\frac{5a}{4}-i\frac{a\sqrt{3}}{4}=0. \quad \ldots \ldots \quad (9.)$$

Now, in its present form, the last equation does not furnish us with any solution of the problem. But it may be rendered available by decomposing it into two congeneric surd equations and selecting the impossible congener. For, the left-hand side of (9.) may be resolved into two factors, both of which are included in the expression

$$\pm\sqrt{Z}+\sqrt{z-\frac{5a}{4}-i\frac{a\sqrt{3}}{4}+Z} \quad \ldots \quad (10.)$$

Assume that

$$z-\frac{5a}{4}-i\frac{a\sqrt{3}}{4}+Z=jZ, \quad \ldots \quad (11.)$$

then one of the values of (10.) takes the form

$$(1+\sqrt{j})\sqrt{Z},$$

and, consequently, vanishes. This, then, is the solution of (9.) which we are in quest of, and, further, this is a solution which must not be neglected supposing that we admit impossible quantities into algebra. We must now consider D as situate out of the plane of ABC.

But, the question occurs, which of the infinite number of values of Z are we to select? The answer is, that which satisfies the condition* that the orthographical projection of D on the plane of ABC shall be the centre of the circle inscribed in ABC. But, this condition gives,

$$-\frac{5a}{4}+Z=-\frac{a}{2},$$

whence,

$$Z=\frac{3a}{4}.$$

* The condition in question is, perhaps, the one best adapted to the problem before us; the determination of Z under its most general aspect will be discussed on a fitting occasion.

Let

$$A=\frac{a}{2},\quad B=\frac{a\sqrt{3}}{4},\quad C=(Z=)\frac{3a}{4},$$

then we have

$$z=A+iB+jC; \quad . \quad . \quad . \quad . \quad . \quad (12.)$$

and, since (12.) is a virtual solution of (7.), the problem is solved. ABCD is a regular tetrahedron.

SCHOLIUM. These solutions may be readily verified. The form, which imaginary geometry will finally take, may possibly be very different from that exhibited here, but I have endeavoured to show *à priori* that, under certain limitations, j indicates perpendicularity to a plane. As to those limitations the reader is referred to pp. 44, 45 of this volume. The geometrical illustration given at the latter of those pages will be more correct if we suppose the small sphere to be moved, parallel to itself and perpendicular to its axis, until its pole is at a distance unity from its former position. The reader who is interested in the subject of the impossible quantity j is referred to my papers at pp. 435–439 of the last, and pp. 37–47 of the present volume of this Journal. For distinctness of reference I have in this paper used i and j instead of α and β. I have not thought it necessary to refer to the case where k (or γ) enters into a geometrical problem, as it was beyond my present object.

Note. The value of $w\mu'^2+2y\nu'^2$ (*supra*, p. 47) should be

$$w'w''+x'x''+y'y''+z'z''.$$

The omission of accents has occasioned the error.

2 Church-Yard Court, Temple,
January 16, 1849.

Correction. Supra, p. 42, note*, line 6,
for "and then n" *read* when the index.

XIX. *Proceedings of Learned Societies.*

CAMBRIDGE PHILOSOPHICAL SOCIETY.

[Continued from vol. xxxiii. p. 394.]

Nov. 13, 1848. SECOND Memoir on the Fundamental Antithesis of Philosophy. By W. Whewell, D.D.

This memoir is a continuation of a former one in which the antithesis of thoughts and things, of ideas and facts, of subjective and objective, were shown to be at bottom the same antithesis, and to be a fundamental antithesis, the union of the two elements entering into all knowledge, and their separation being the test of all philo-