An Alternating Four-Block Classification in the Collatz Carry Equation (DRAFT)

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Abstract

We study the accelerated odd-only Collatz carry equation. For a valuation word D of length L and total valuation S, *wall contact* means the divisibility $N_{L,S} = 2^S - 3^L \mid C_L(D)$, where $C_L(D)$ is the carry sum; wall contacts are exactly the words realized by integer cycles of the accelerated odd map. We classify the alternating four-block family $C(1/3)^a\,(4)^b\,C(1/6)^c\,(3)^d$, $a, b, c, d \geq 1$, over all 18 internal phase choices of the composite blocks $C(1/3)$ and $C(1/6)$. The family has sign pattern $(-,+,-,+)$ relative to the critical slope $\alpha = \log_2 3$, making it the first alternating four-block test of the finite-block obstruction program. The proof combines (i) the four-block telescoping identity, (ii) cyclic rotation invariance of wall contact, (iii) a rotated 2-adic lever proved at full strength $2^{S_X n_X + 1}$ with an empty degenerate-lift branch, (iv) a corrected spread comparison with error at most $|S - L\alpha|$, (v) a top-anchored mass bound $V^{\uparrow} \leq 47$, (vi) a certified continued-fraction ladder for $\log_2 3$ whose load-bearing rung is verified by exact integer comparisons, and (vii) an exact finite computation. The finite box collapses to $a \leq 13$, $b \leq 13$, $c \leq 6$, $d \leq 32$, and a padded exact computation checked 1,474,560 candidates with zero wall contacts. Consequently the family admits no wall contacts: no integer cycle of the accelerated odd Collatz map has a valuation word of this specific alternating four-block form. This note does not prove the Collatz conjecture, nor does it classify any class of families beyond the one treated; it demonstrates that the first tested alternating $k = 4$ family is fully obstructed.

1 Introduction

Cycles of the $3x+1$ map correspond, in the accelerated odd-only formulation, to integer solutions of a divisibility condition between an exponential *wall* $2^S - 3^L$ and a structured *carry sum* determined by the sequence of 2-adic valuations along the orbit (see Lagarias [1], Terras [2]). Classical results obstruct specific shapes of valuation words: Steiner [3] excluded 1-cycles, and Simons–de Weger [4] excluded m-cycles for all $m \leq 68$ (later extended), in both cases by combining structural identities with effective lower bounds for linear forms in $\log 2$ and $\log 3$.

The present note carries out a classification of a different shape: a *fixed finite-block family* in which the valuation word is a concatenation of four explicit blocks raised to arbitrary powers, with return slopes alternating in sign about the critical slope $\alpha = \log_2 3$. The alternating sign pattern is the structurally interesting case: ascending and descending runs can nearly cancel, so no single global monotonicity argument applies, and the obstruction must instead be extracted locally from each block by rotation. The mechanism is a *rotated* 2*-adic lever*: any hypothetical wall contact, rotated so that a block run leads the word, is forced into a rigid 2-adic congruence class determined by the block's own rational cycle value, and the integrality of the contact then forces the contact value to be exponentially large in the run length. Balancing this against an upper bound from a spread comparison and an effective lower bound on $|S - L\alpha|$ yields a small finite box, which is then checked exactly.

We emphasize the scope. This note classifies *one explicit family.* It does not prove the Collatz conjecture; it does not classify all four-block families, nor all finite-block families; and it does not address blocks with slopes near the critical line or valuation words of positive entropy. The contribution is a complete, conservative treatment of the first alternating four-block test case, with every input either proved, certified by exact integer computation, or explicitly flagged as quoted.

2 Setup and notation

Definition 2.1. A *valuation word* is a finite sequence $D = (d_0, \dots, d_{L-1})$ of integers $d_i \geq 1$. Write $s_j = \sum_{i<j} d_i$ (so $s_0 = 0$), $S = s_L$, and

$$N_{L,S} \;=\; 2^S - 3^L, \qquad C_L(D) \;=\; \sum_{j=0}^{L-1} 3^{\,L-1-j}\, 2^{\,s_j}.$$

Wall contact means $N_{L,S} \mid C_L(D)$; equivalently $C_L(D) = m\, N_{L,S}$ for some $m \in \mathbb{Z}$, called the *contact value.*

Wall contacts are precisely the valuation words of cycles: if m is an odd integer whose accelerated orbit $x \mapsto (3x+1)/2^{v_2(3x+1)}$ returns to m after L odd steps with valuations $(d_0, \dots, d_{L-1})$, then $C_L(D) = m\, N_{L,S}$, and conversely each contact value is the seed of a rational (and, when integral, an integer) cycle. Note that $C_L(D)$ is always odd (the $j = 0$ term 3^{L-1} is the unique odd term) and $N_{L,S}$ is odd; hence every contact value m is odd.

Define the *return path*

$$R_j \;=\; s_j - j\alpha, \qquad \alpha = \log_2 3,$$

so that $R_0 = 0$ and $R_L = S - L\alpha$. Then directly from the definitions,

$$C_L(D) \;=\; 3^{\,L-1} \sum_{j=0}^{L-1} 2^{R_j}, \qquad N_{L,S} \;=\; 3^L\bigl(2^{R_L} - 1\bigr), \qquad Q(D) \;:=\; \frac{C_L(D)}{N_{L,S}} \;=\; \frac{1}{3}\,\frac{\sum_j 2^{R_j}}{2^{R_L} - 1}.$$

Wall contact means $Q(D) \in \mathbb{Z}$.

3 The family

We study the four-block family

$$D_{a,b,c,d} \;=\; A^a B^b C^c E^d, \qquad a, b, c, d \geq 1,$$

where, to avoid a clash with the generic word symbol D, the fourth block is denoted $E = (3)$. The blocks are

$$A = C(1/3), \qquad B = (4), \qquad C = C(1/6), \qquad E = (3),$$

with $C(1/3)$ any composition of 4 into 3 positive parts and $C(1/6)$ any composition of 7 into 6 positive parts; the wall data (q_X, S_X) are forced, and the internal arrangement (the *phase*) is a free parameter swept below. For each block put $q_X = |X|$, $S_X = \sum X$, $\delta_X = 3^{q_X} - 2^{S_X}$, and the return slope $R_X = S_X - q_X\alpha$.

Block	phases	q_X	S_X	δ_X	R_X	sign	$C(X)$ over phases
$A = C(1/3)$	$(1,1,2),(1,2,1),(2,1,1)$	3	4	11	$4-3\alpha \approx -0.7549$	$-$	19, 23, 29
$B = (4)$	(4)	1	4	-13	$4-\alpha \approx +2.4150$	$+$	1
$C = C(1/6)$	$(1^5, 2)$-shuffles	6	7	601	$7-6\alpha \approx -2.5098$	$-$	$1087, 925, 817, 745, 697, 665$
$E = (3)$	(3)	1	3	-5	$3-\alpha \approx +1.4150$	$+$	1

Table 1: Block data. There are $3\cdot 1\cdot 6\cdot 1 = 18$ phase tuples. The sign pattern of (R_A, R_B, R_C, R_E) is $(-,+,-,+)$.

Lemma 3.1 (Coprimality of carry and wall, all phases)**.** *For every block $X \in \{A, B, C, E\}$ and every phase,* $\gcd\big(C(X), \delta_X\big) = 1$*; in particular $\delta_X \nmid C(X)$.*

Proof. Finite check: $11 \nmid \{19, 23, 29\}$; $13 \nmid 1$; $601 \nmid \{1087, 925, 817, 745, 697, 665\}$; $5 \nmid 1$. Since $11, 13, 601, 5$ are prime, coprimality follows. □

Lemma 3.1 is what will empty the degenerate-lift branch in Section 7.

4 The four-block telescoping identity

For a block X with phase ϕ, write $R_j(X_\phi)$, $0 \le j < q_X$, for the internal return path of the block ($R_0 = 0$), and set

$$A_X^{(\phi)} \;=\; \sum_{j=0}^{q_X-1} 2^{R_j(X_\phi)}, \qquad G_X \;=\; \frac{1}{3}\,\frac{A_X^{(\phi)}}{2^{R_X}-1}.$$

A direct computation gives $G_X = C(X)/(2^{S_X} - 3^{q_X}) = -C(X)/\delta_X$: the quantity G_X is the block's own rational cycle value. Write

$$x_A = 2^{aR_A}, \quad x_B = 2^{bR_B}, \quad x_C = 2^{cR_C}, \quad x_E = 2^{dR_E}.$$

Proposition 4.1 (Telescoping identity)**.** *For $D = A^a B^b C^c E^d$,*

$$Q(D) \;=\; \frac{G_A\,(x_A-1) \;+\; G_B\,x_A(x_B-1) \;+\; G_C\,x_Ax_B(x_C-1) \;+\; G_E\,x_Ax_Bx_C(x_E-1)}{x_Ax_Bx_Cx_E - 1}.$$

Wall contact for D means precisely that this quantity is an integer.

Proof. The return path of D traverses the four runs in order. Within the run X^n entering at height z, the i-th copy starts at $z + iR_X$ and contributes $2^{z+iR_X}A_X$ to $\sum_j 2^{R_j}$; summing the geometric series over $0 \le i < n$ gives $2^z A_X\,(2^{nR_X} - 1)/(2^{R_X} - 1)$. The entry heights are 0, aR_A, $aR_A + bR_B$, $aR_A + bR_B + cR_C$, i.e. the prefix products $1, x_A, x_Ax_B, x_Ax_Bx_C$ as powers of 2. Summing the four runs, dividing by $3(2^{R_L} - 1)$ with $2^{R_L} = x_Ax_Bx_Cx_E$, and inserting the definition of G_X yields the display. □

Clearing denominators in Proposition 4.1 produces the *five-monomial contact equation*: after multiplication by $\prod_X \delta_X$, the contact condition is a vanishing combination of five monomials in powers of 2 and 3 — the 3^L-corner, three run junctions, and the 2^S-corner. We use this form only as a computational cross-check (Section 11); the proofs below work with the integral identities directly.

5 Rotation invariance

For $0 \le t \le L$ let $\sigma^t D = (d_t, \ldots, d_{L-1}, d_0, \ldots, d_{t-1})$.

Lemma 5.1 (Rotation identity)**.** *For every word D and every $0 \le t \le L$,*

$$2^{s_t} C_L(\sigma^t D) \;=\; 3^t C_L(D) \;+\; N_{L,S}\, C_t(D), \qquad C_t(D) = \sum_{j=0}^{t-1} 3^{t-1-j} 2^{s_j}.$$

Consequently, since $\gcd(2^{s_t}, N_{L,S}) = \gcd(3^t, N_{L,S}) = 1$, wall contact is invariant under cyclic rotation, and the rotated contact values $m_t = C_L(\sigma^t D)/N_{L,S}$ are integers for all t as soon as one of them is.

Proof. For $t = 1$: $C_L(\sigma D) = \sum_{j=0}^{L-1} 3^{L-1-j} 2^{s_{j+1}-d_0}$, so $2^{d_0} C_L(\sigma D) = \sum_{i=1}^{L} 3^{L-i} 2^{s_i} = 3\big(C_L(D) - 3^{L-1}\big) + 2^S = 3\, C_L(D) + N_{L,S}$, with $C_1 = 1$. The general case follows by composing rotations; the carry satisfies $C_{t+1} = 3\, C_t + 2^{s_t}$, which telescopes to the stated sum. □

Lemma 5.1 allows any block run X^n of $D_{a,b,c,d}$ to be rotated to the front of the word without affecting the existence of a contact. We exploit each of the four rotations in turn.

6 The rotated 2-adic lever

This is the central mechanism. Let $W = X^n Y$ be a cyclic rotation of a family word beginning with the full run of the block X, where Y is the concatenation of the remaining three runs in cyclic order (Y is nonempty since $a, b, c, d \ge 1$). Write $q_X = |X|$, $S_X = \sum X$, $\delta_X = 3^{q_X} - 2^{S_X}$, $|Y| = L - q_X n$, $S_Y = S - S_X n \ge 1$, and, assuming wall contact, $m_W = C(W)/N$ with $N = N_{L,S}$. (Here and below we abbreviate $C = C_L$, suppressing the length subscript.)

Lemma 6.1 (Concatenation and repetition)**.** *For any words U, V,*

$$C(UV) \;=\; 3^{|V|}\, C(U) \;+\; 2^{S_U}\, C(V),$$

and for any block X and $n \ge 1$,

$$C(X^n) \;=\; C(X)\, \frac{3^{q_X n} - 2^{S_X n}}{\delta_X} \qquad (\textit{exact integer division; } \delta_X \textit{ odd}).$$

Proof. The first identity is immediate from the definition: terms $j < |U|$ acquire the factor $3^{|V|}$, terms $j \ge |U|$ the factor 2^{S_U}. The second follows by induction from the first with $U = X$, $V = X^{n-1}$, summing the geometric series $\sum_{i<n} 3^{q_X(n-1-i)} 2^{S_X i}$. □

Theorem 6.2 (Rotated 2-adic lever, full strength)**.** *Define the integral lever quantity*

$$\mathfrak{G}_X(W) \;=\; \delta_X\, C(W) \;+\; C(X)\, N.$$

Then, unconditionally (for every family word, contact or not),

$$\mathfrak{G}_X(W) \;=\; 2^{\,S_X n}\,\mathfrak{h}, \qquad \mathfrak{h} \;=\; C(X)\,2^{\,S_Y} \;+\; \delta_X\,C(Y) \;-\; 3^{|Y|}\,C(X),$$

and $\mathfrak{h}$ is even. Hence

$$2^{\,S_X n+1} \;\Big|\; \delta_X\, C(W) + C(X)\, N,$$

and under wall contact $C(W) = m_W N$ with N odd,

$$2^{\,S_X n+1} \;\Big|\; \delta_X\, m_W + C(X).$$

Proof. By Lemma 6.1,

$$\delta_X\, C(W) = 3^{|Y|} C(X)\big(3^{q_X n} - 2^{S_X n}\big) + \delta_X\, 2^{S_X n} C(Y) = 3^L C(X) - 2^{S_X n}\big(3^{|Y|} C(X) - \delta_X C(Y)\big),$$

using $3^{|Y|}3^{q_X n} = 3^L$. Adding $C(X)N = C(X)2^S - C(X)3^L$, the 3^L terms cancel and $2^S = 2^{S_X n}\,2^{S_Y}$ factors out, giving $\mathfrak{G}_X(W) = 2^{S_X n}\mathfrak{h}$ with $\mathfrak{h}$ as displayed. For the parity of $\mathfrak{h}$: $S_Y \geq 1$ makes $C(X)2^{S_Y}$ even, while δ_X, $C(Y)$, $C(X)$, and $3^{|Y|}$ are all odd (every carry sum is odd, every δ_X is odd), so $\delta_X C(Y) - 3^{|Y|}C(X) \equiv 1 - 1 \equiv 0 \pmod 2$. The divisibility for m_W follows because $v_2(\mathfrak{G}_X) = v_2(\delta_X m_W + C(X))$ when N is odd. □

Remark 6.3. Equivalently, since δ_X is a 2-adic unit, every contact of the rotated word satisfies $m_W \equiv -C(X)\,\delta_X^{-1} \pmod{2^{\,S_X n+1}}$: the contact value is forced to agree, to 2-adic depth $S_X n + 1$, with the block's own rational cycle value $-C(X)/\delta_X = C(X)/(2^{S_X} - 3^{q_X}) = G_X$. The "+1" arises from a parity cancellation between the two odd interior coefficients and is generically sharp (Section 11).

Specializing Theorem 6.2 to the four anchors (writing m_A for the contact value of the rotation with A^a leading, etc.):

$$2^{\,4a+1} \mid 11\, m_A + C(A), \qquad 2^{\,4b+1} \mid 13\, m_B - 1, \qquad 2^{\,7c+1} \mid 601\, m_C + C(C), \qquad 2^{\,3d+1} \mid 5\, m_E^{\flat} - 1,$$

where $m_E^{\flat}$ denotes the contact value of the rotation led by the run of the block $E = (3)$ (the signs absorb $\delta_B = -13$, $\delta_E = -5$).

7 The degenerate-lift branch is empty

A congruence modulo $2^{S_X n+1}$ does not by itself force $|m_W|$ to be large: the residue class could contain a small integer. The following dichotomy shows the question is rigid.

Proposition 7.1 (Degenerate-lift dichotomy)**.** *Fix a block X and suppose $W = X^n Y$ is a wall contact.*

1. *If $\delta_X \mid C(X)$, then $-C(X)/\delta_X \in \mathbb{Z}$ is the value of an integer cycle whose word is X itself, and the congruence class of Theorem 6.2 contains this one bounded lift for every n.*

2. If $\delta_X \nmid C(X)$, then $\delta_X m_W + C(X) \neq 0$, and since it is divisible by $2^{S_X n+1}$,

$$|m_W| \;\geq\; \frac{2^{\,S_X n+1} - |C(X)|}{|\delta_X|}.$$

In particular the least absolute residue grows at the full exponential rate; there are no bounded subsequences and no sporadic small lifts.

Proof. (1) is immediate. For (2): if $\delta_X m_W + C(X) = 0$ then $m_W = -C(X)/\delta_X \notin \mathbb{Z}$, a contradiction; a nonzero multiple of $2^{S_X n+1}$ has absolute value at least $2^{S_X n+1}$; solve for $|m_W|$. □

By Lemma 3.1, case (2) holds for all four blocks and all 18 phases. *The degenerate branch is empty for this family*, and

$$\log_2 |m_X| \;\geq\; S_X n_X - \log_2 |\delta_X| \qquad \text{once } 2^{\,S_X n_X} \geq |C(X)|,$$

i.e. from $n_X \geq n_0(X)$ with $n_0 = (2, 1, 2, 1)$ for (A, B, C, E); smaller n_X lie trivially inside the final box. The per-block lever constants are $\lambda_X := \log_2 |\delta_X| = (\log_2 11, \log_2 13, \log_2 601, \log_2 5) \approx (3.459,\ 3.700,\ 9.231,\ 2.322)$.

Remark 7.2. The dichotomy is calibrated by the known cycles: the integer-cycle words (1), (2) satisfy $\delta_X \mid C(X)$ and give $\mathfrak{G}_X \equiv 0$ identically on $(1)^n$, $(2)^n$ — the degenerate lift *is* the known cycle. The hypothesis "no block of the family is itself an integer-cycle word" is exactly what part (2) requires.

8 Corrected spread comparison and mass bound

The lever gives a *lower* bound on each rotated contact value. The matching upper bound comes from comparing rotations.

Lemma 8.1 (Window identity)**.** *Extend s periodically by $s_{j+L} = s_j + S$, so $R_{j+L} = R_j + R_L$. Then for every t,*

$$3\,(2^{R_L} - 1)\, m_t \;=\; 2^{-R_t}\, \Sigma_t, \qquad \Sigma_t = \sum_{j=t}^{t+L-1} 2^{R_j}.$$

Proof. $C(\sigma^t D) = 3^{L-1} \sum_{j=0}^{L-1} 2^{R_{t+j} - R_t}$ directly from the definition with the periodic extension; divide by $N = 3^L(2^{R_L} - 1)$. □

Proposition 8.2 (Spread comparison)**.** *For all $0 \leq t, t' < L$,*

$$\frac{|m_t|}{|m_{t'}|} \;=\; 2^{\,R_{t'} - R_t + \varepsilon}, \qquad |\varepsilon| \leq |R_L|.$$

In particular, inside the critical core $|R_L| \leq 1$ the rotation-to-rotation comparison costs at most one bit.

Proof. By Lemma 8.1, $|m_t|/|m_{t'}| = 2^{R_{t'} - R_t}\, \Sigma_t / \Sigma_{t'}$. Writing $\Sigma_t = \Sigma_0 + (2^{R_L} - 1) P_t$ with $P_t = \sum_{j<t} 2^{R_j} \in (0, \Sigma_0)$ shows Σ_t / Σ_0 lies strictly between $\min(1, 2^{R_L})$ and $\max(1, 2^{R_L})$ for every t, whence $\Sigma_t / \Sigma_{t'} \in (2^{-|R_L|}, 2^{|R_L|})$. □

Let $t^* = \arg\max_t R_t$, $m_\star = m_{t^*}$ (the minimal-magnitude representative, up to $2^{|R_L|}$, by Proposition 8.2), and define the *top-anchored mass*

$$V^{\uparrow}(D) = \sum_{j=0}^{L-1} 2^{-(R_{\max}-R_j)}, \qquad R_{\max} = \max_j R_j.$$

By Lemma 8.1,

$$1 \;\le\; 3\left|2^{R_L}-1\right| |m_\star| \;\le\; 2^{|R_L|}\, V^{\uparrow}(D). \tag{1}$$

Proposition 8.3 (Mass bound)**.** *For every $(a,b,c,d) \in \mathbb{Z}^4_{\ge 1}$ and every phase tuple, $V^{\uparrow}(D_{a,b,c,d}) \le 47$.*

Proof sketch. Rotate the window to start at $R_{\max}$. One period splits into the four block-runs, at most one of which is cut into two pieces. Within a run of X^n entering at height $z \le R_{\max}$, the start of copy i sits at $z + iR_X$ and every index inside copy i has $R_j \le z + iR_X + h_X$, where $h_X \le S_X - q_X$ is the maximal internal prefix rise (word-free bound, since all valuations are ≥ 1). Summing the geometric tails, each run piece contributes at most $q_X 2^{h_X}/(1-2^{-|R_X|})$, and the cut run at most twice that. With the slopes of Table 1, $(1-2^{-|R_X|})^{-1} = (2.454, 1.231, 1.213, 1.600)$ and $h_A, h_C \le 1$, $h_B = h_E = 0$, the total is at most $14.73 + 1.23 + 14.56 + 1.60 + 14.73 < 47$. □

The bound is deliberately loose; the observed values are $V^{\uparrow} \approx 3.55$, stable across the family. Looseness costs only $\log_2 47 < 5.6$ bits in the exponent comparison below.

9 The Diophantine ladder

Set

$$\Delta_2 = S - L\alpha = R_L, \qquad \Lambda = S\log 2 - L\log 3 = \Delta_2 \log 2,$$

so $|\Delta_2| = |\Lambda|/\log 2$, and write $\|L\alpha\| = \min_{S\in\mathbb{Z}} |S - L\alpha|$. The chain of Section 10 consumes an effective lower bound on $|\Delta_2|$. Two inputs are used, with sharply separated roles.

9.1 Certified continued-fraction bounds (Tier 1)

The inequality $\alpha \gtrless p/q$ is equivalent to the exact integer comparison $3^q \gtrless 2^p$. Two such comparisons, at the deep convergents of α, certify the enclosure

$$\alpha \;\in\; \left(\tfrac{176251}{111202},\ \tfrac{301994}{190537}\right), \qquad \text{width } 4.72\times 10^{-11}. \tag{2}$$

(The convergent denominators of α are $1, 1, 2, 5, 12, 41, 53, 306, 665, 15601, 31867, \ldots$; how the bracketing fractions are found is irrelevant to rigor, since the two inequalities are certified directly.) Given (2), for each L the quantity $L\alpha$ lies in an exact rational interval of width $< 10^{-6}$ over the ranges below, and the distance of that interval to the nearest integer is an exact rational lower bound for $\|L\alpha\|$. An exhaustive scan in exact rational arithmetic — no floating point anywhere — certifies:

$$1 \le L < 306: \quad \|L\alpha\| \;\ge\; 3.0125357\times 10^{-3}, \quad \text{attained at } (L,S) = (53, 84); \tag{3}$$

$$1 \le L < 665: \quad \|L\alpha\| \;\ge\; 1.4747792\times 10^{-3}, \quad \text{attained at } (L,S) = (306, 485); \tag{4}$$

$$1 \le L < 15601: \quad \|L\alpha\| \;\ge\; 6.2948508\times 10^{-5}, \quad \text{attained at } (L,S) = (665, 1054). \tag{5}$$

Since α is irrational ($2^S \ne 3^L$ by unique factorization), exact equality $\Delta_2 = 0$ is impossible for every pair.

9.2 The Simons–de Weger/Rhin tail (Tier 2)

Simons and de Weger [4], using Rhin's effective irrationality measure for $(1, \log 2, \log 3)$ [5], deploy the explicit lower bound

$$|\Lambda| \;>\; \exp\bigl\{-13.3\,(0.46057 + \log K)\bigr\}, \tag{6}$$

where K is the coefficient of $\log 3$; in the present notation $K = L$, so

$$|\Delta_2| \;>\; \frac{1}{\log 2}\,\exp\bigl\{-13.3\,(0.46057 + \log L)\bigr\}, \qquad \text{i.e.} \qquad -\log_2|\Delta_2| \;<\; 13.3\log_2 L + 8.308.$$

We record two honest caveats. First, Rhin's inequality (8) in [5] yields the stronger exponent 7.616, but only for $H = \max(|u_1|, |u_2|)$ "sufficiently large", with no explicit threshold in the accessible statements; we therefore do not use it. Second, the constants 13.3 and 0.46057 in (6) are taken from the published deployment by Simons–de Weger; they should be verified against the printed Acta Arithmetica text before journal submission. Both caveats are inconsequential for the structure of the proof because (6) is invoked only for $L \geq 15601$, where the chain demands a lower bound that is exponentially weaker than the polynomial bound supplied; any explicit two-logarithm lower bound (e.g. Laurent–Mignotte–Nesterenko with explicit constants) could substitute.

9.3 The ladder

Write $\mathrm{rhs}(G)$ for the budget of the exponent comparison (Section 10) when $-\log_2|\Delta_2| \leq G$, and $L_{\mathrm{box}}(G)$ for the resulting bound on L. The values, computed in Section 10, are:

$$L_{\mathrm{box}}(8.3748) \leq 115, \qquad L_{\mathrm{box}}(9.4053) \leq 121, \qquad L_{\mathrm{box}}(13.9555) \leq 145.$$

Suppose $D_{a,b,c,d}$ is a wall contact with $|R_L| \leq 1$ (the *critical core*; the complementary case is handled in Proposition 10.1). Then:

- if $306 \leq L < 665$, rung (4) forces $L \leq 121 < 306$ — contradiction;
- if $665 \leq L < 15601$, rung (5) forces $L \leq 145 < 665$ — contradiction;
- if $L \geq 15601$, the tail (6) forces $L \leq L_{\mathrm{box}}\bigl(13.3\log_2 L + 8.308\bigr)$, which equals 1111 at $L = 15601$ and grows only logarithmically (one checks $L_{\mathrm{box}} \leq 73\log_2 L + 123 < L$ for all $L \geq 15601$) — contradiction.

Hence every core contact has $L < 306$, and by rung (3),

$$|\Delta_2| \;\geq\; 3.0125357 \times 10^{-3}, \qquad \text{equivalently} \qquad -\log_2|\Delta_2| \;\leq\; 8.3748. \tag{7}$$

10 Finite reduction

Fix an anchor X with run exponent n_X, and let $\mathrm{osc}_X = R_{\max} - R_{t_X} \geq 0$ be the height deficit of the anchor's rotation start. Chaining the lever (Section 7), the spread comparison (Proposition 8.2), and the mass bound (1) with $|2^{R_L} - 1| \geq \frac{\ln 2}{2}|R_L|$ for $|R_L| \leq 1$:

$$S_X n_X - \lambda_X \;\leq\; \log_2|m_X| \;\leq\; \mathrm{osc}_X + |R_L| + \log_2|m_\star| \;\leq\; \mathrm{osc}_X - \log_2|\Delta_2| + \log_2 V_0 + 3w + O(1),$$

with $V_0 = 47$ and core width $w = 1$. With the budget $r = -\log_2|\Delta_2| + \log_2 V_0 + 3w$, the per-anchor inequalities read $S_X n_X - \mathrm{osc}_X \le r + \lambda_X$. The oscillation terms are governed by the block-level path heights (0, $-0.7549a$, $H := -0.7549a + 2.4150b$, $\approx R_L - 1.4150d$, R_L, up to $O(1)$ intra-block corrections), and the system closes in two stages: first $b \le (r + \lambda_B)/\alpha$ and (in the dominant phase) the descending anchors A, C carry gaps $4a - (2.4150b - 0.7549a)_+$ and $7c - (0.7549a - 2.4150b)_+$; substituting the stage-one bound b^* then bounds $a \le (r + \lambda_A + 2.4150\, b^*)/4$, $d \le (r + \lambda_E + 2.4150\, b^*)/\alpha$, and finally $c \le (r + \lambda_C + 0.7549\, a^*)/7$.

With the certified gap (7), $r = 8.3748 + \log_2 47 + 3 = 16.93$, and the closure yields the raw bounds $(12.96, 13.02, 5.13, 31.98)$ for (a, b, c, d); rounding outward:

$$\boxed{a \le 13, \qquad b \le 13, \qquad c \le 6, \qquad d \le 32, \qquad L \le 115, \qquad S \le 236.} \tag{8}$$

(The same closure run with the rung-2 and rung-3 gaps produces the values L_{box} quoted in Section 9.3.)

Proposition 10.1 (Non-core closures)**.** *Suppose $D_{a,b,c,d}$ is a wall contact with $|R_L| \ge 1$. Then the window identity gives $\Sigma_t \le L\, 2^{R_{\max} + (R_L)_+}$ and $|2^{R_L} - 1| \ge \frac{1}{2}$, hence $\log_2|m_X| \le \mathrm{osc}_X + \log_2 L + 1$ for every anchor, and the same two-stage closure with the budget $\log_2 L + 1$ in place of $-\log_2|\Delta_2| + \log_2 V_0 + 3w$ yields fixed points $L^* \approx 52$ (positive walls, $R_L \ge 1$) and $L^* \approx 59$ (negative walls, $R_L \le -1$; here the E-anchor oscillation absorbs $|R_L|$, which is in turn bounded by $0.7549a + 2.5098c$ and the system still closes). Both resulting boxes are contained in $\{a \le 8,\ b \le 10,\ c \le 3,\ d \le 15\}$, strictly inside the padded computation box below.*

Every chain inequality above carries $O(1)$ intra-block phase corrections of at most $\max_X(S_X - q_X) + w = 2$ bits, shifting each threshold by at most $2/g_X \le 1.3$ units. The exact computation therefore runs over a *padded* box,

$$1 \le a \le 16, \qquad 1 \le b \le 16, \qquad 1 \le c \le 8, \qquad 1 \le d \le 40, \tag{9}$$

which strictly contains the core box (8), both non-core boxes, and (for belt-and-braces) the largest ladder-rung box, with a margin of at least three units in every variable over (8).

11 Exact computation

The finite check enumerates all of (9) over all 18 phase tuples — *no critical-core prefilter is applied*, so every value of Δ_2 is admitted:

$$16 \cdot 16 \cdot 8 \cdot 40 \cdot 18 \;=\; 1{,}474{,}560 \quad \text{candidates.}$$

For each candidate: compute (L, S); assemble $C(D)$ exactly by the four-run closed form of Lemma 6.1 with nested prefactors (the exact integer division by each wall δ_X is itself a per-candidate consistency check); compute $N = 2^S - 3^L$; and test exactly whether $N \mid C(D)$, in arbitrary-precision integer arithmetic throughout.

candidates enumerated	1,474,560
wall contacts found	0
runtime (single core, exact bigints)	1.5 s
minimum $\lvert\Delta_2\rvert$ in the box	$0.0030125\ldots$ at $(a,b,c,d) = (1,7,7,1)$

The logged box minimum of $|\Delta_2|$ occurs at $(a,b,c,d) = (1,7,7,1)$, which realizes $(L,S) = (53,84)$ — exactly the certified worst pair of rung (3). This is an independent cross-check of the certified continued-fraction table against the live enumeration: the family itself attains the Diophantine extremum.

Validation suite (all passed, attached to the run).

1. concatenation identity (Lemma 6.1), 300 random word pairs, exact;
2. repetition formula, 300 random (block, $n \le 8$) pairs, exact divisibility by δ_X each time;
3. rotated 2-adic lever $v_2\big(\delta_X C(W) + C(X)N\big) \ge S_X n_X + 1$ on 300 random box members $\times$ all four anchors; observed excess valuations $v_2 - S_X n_X \in \{1,2\}$, confirming the "+1" and its generic sharpness;
4. rotation invariance of (non)contact on 100 random box members $\times$ 5 rotations each, plus all rotations of the contact words $(1,2)^3$ and the -17 word;
5. five-monomial run assembly equals the direct carry sum, 100 random members, exact;
6. known-cycle calibrations: $(2) \mapsto 1$, $(1) \mapsto -1$, $(1,2) \mapsto -5$, $(2,1) \mapsto -7$, and the word $(1,1,1,2,1,1,4) \mapsto -17$ of the -17 cycle.

12 Main theorem

Theorem 12.1 (Alternating four-block classification)**.** *For every $a,b,c,d \ge 1$ and every one of the 18 phase tuples of the family*

$$D_{a,b,c,d} \;=\; C(1/3)^a\,(4)^b\,C(1/6)^c\,(3)^d,$$

the wall divisibility

$$2^S - 3^L \;\big|\; C_L(D_{a,b,c,d})$$

does not hold. Equivalently, the family admits no wall contacts. Consequently, no integer cycle of the accelerated odd Collatz map has a valuation word of this specific alternating four-block form.

Proof. Suppose a contact exists. If $|R_L| \le 1$, the ladder of Section 9.3 forces $L < 306$ and the gap (7); the finite reduction of Section 10 then places the contact in the box (8). If $|R_L| \ge 1$, Proposition 10.1 places it in a smaller box. All these boxes lie inside the padded box (9), which the exact computation of Section 11 exhausted with zero contacts. □

We state plainly what Theorem 12.1 is not: it is not a proof of the Collatz conjecture, and it is not a classification of any class of families beyond the single family treated. The inputs are itemized in the ledger below.

13 Tier ledger

Following the epistemic convention of the broader program, we classify every ingredient.

Tier 1 (proved or certified by exact finite computation). The telescoping identity (Proposition 4.1); rotation invariance (Lemma 5.1); the rotated 2-adic lever at full +1 strength (Theorem 6.2); emptiness of the degenerate-lift branch (Proposition 7.1 with Lemma 3.1); the corrected spread comparison (Proposition 8.2); the mass bound $V^{\uparrow} \leq 47$ (Proposition 8.3); the certified continued-fraction enclosure (2) and rung table (3)–(5) (exact integer comparisons and exact rational scans); and the exact finite computation of Section 11 with its validation suite.

Tier 2 (quoted effective input). The Simons–de Weger/Rhin tail bound (6), used only for $L \geq 15601$ and with exponential slack. *Action item:* verify the constants 13.3 and 0.46057 against the printed Acta Arithmetica text [4] before submission; any explicit two-logarithm bound may substitute.

Tier 3 (reduced scans). None remaining; earlier partial scans are subsumed by the full run.

Tier 4 (conjectural context). The general fixed-k finite-block obstruction conjecture, and the near-critical / positive-entropy barrier discussed below.

14 Limitations and next frontier

This note classifies one family only. It does not classify all alternating four-block families; it does not classify all $k = 4$ families; it does not handle arbitrary finite-block families; it does not handle block slopes approaching the critical line ($R_X \to 0$, where the constants of Proposition 8.3 degenerate like $|R_X|^{-1}$ and the lever thresholds inflate); and it does not handle unbounded block count or positive-entropy valuation words, where the method's constants compound with the number of blocks. These are structural boundaries of the technique, not oversights.

The natural next steps, in increasing order of difficulty, are: (1) broader alternating $k = 4$ sweeps over other block quadruples with $|R_X|$ bounded away from 0; (2) a general rotated-lever finite-reduction theorem for fixed finite-block families with slopes bounded away from the critical line, with the multi-stage closure constants made explicit; (3) near-critical block accumulation, beginning with the convergent blocks $(q, S) = (12, 19), (53, 84), \ldots$; and (4) the irregular positive-entropy sector, which the present method does not approach.

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