THE MOTION OF AN ELECTRICAL DOUBLET.

By Leigh Page.

In a recent paper[1] it was shown that the expression obtained by Larmor for the reaction exerted on a moving mass by its own radiation is invalid because it is based on the assumption, that the effect of the damping which must accompany the emission of energy by the vibrators constituting the moving body, is negligible. The line of reasoning pursued was as follows. First the equation of motion of a single vibrating electron was derived rigorously from the electrodynamic equations, taking damping into account and including all mass reaction and radiation reaction terms of orders no higher than that of Larmor's expression, and was found to be of exactly the form, in so far as the velocity is concerned, demanded by the principle of relativity. Consequently it was concluded that the motion of a single vibrating electron is entirely in accord with this principle, and that such a particle is subject to no retardation opposed to its drift velocity. From this result it was inferred that a more complex body, such as a star, could not be subject to a reaction depending only upon its rate of radiation and its drift velocity. For a retardation which is a function of these two quantities alone must exist for all moving and radiating bodies, or for none. As it was shown rigorously to be non-existent for a single vibrating electron, it must be non-existent for a more complex radiator, such as a star. Hence Larmor's expression, which involves only the two quantities mentioned, must be invalid, and his assumption that the effect of damping can be neglected, unjustifiable.

A possible objection to this line of argument lies in the following consideration. In order that a single electron shall be accelerated and

[1] L. Page, Phys. Rev., 11, p. 376, 1918. In this paper the following errata should be noted:

Equation (12), p. 381. Denominator of second term within braces should contain the factor r.

Last equation, p. 384. For outstanding factor of left hand side read $-1/c$ instead of $-1/c^2$.

Last equation, p. 389. For coefficient of b^2 read 3/8 instead of 3/2.

Equation (29), p. 392. For coefficient of $(\partial\beta_0/\partial x)^2$ read $k^5(1 + 2\beta_0^2)$ in place of $k^5(1 + 2\beta_0)$.

Expression for x near bottom of page 392, and expressions for x and y near bottom of page 395. Second term should contain factor r.

hence radiate energy, a force must be exerted on it by some other charged particle in its vicinity. Now it might be contended, that while the equation of motion of the electron alone is entirely in accord with the principle of relativity, yet the moving doublet as a whole may suffer a retardation from the reaction of the radiation emitted. Hence it has seemed worth while to treat rigorously the problem of two charged particles, whose charges and masses are not necessarily equal, whose initial velocities are quite arbitrary, and whose subsequent motions are determined entirely by the fields produced by the two particles themselves. The analysis, which follows very simply from the equation of motion of the electron and the expansions in series of expressions for the electric and magnetic intensities which were published in the paper already referred to, includes all terms of orders no higher than the expression given by Larmor.

Let v_1 and v_2 be the velocities of the two particles, and r the line joining them. Pass a plane yz through r such that v_1 and v_2 have equal components normal to it. Then the particles may be considered as moving under the actions of each other in this plane, while the plane has a velocity $v_1 \cos \theta_1 = v_2 \cos \theta_2$ perpendicular to itself, θ_1 and θ_2 being the angles made by v_1 and v_2 respectively with the normal. Now the expression obtained by Larmor demands that the radiation emitted by the charged particles should exert a retardation on the doublet in the direction opposite to this normal velocity, whereas the principle of relativity denies the existence of any such force.

In determining the orders of the terms entering into the analysis, we shall suppose the charges on the two particles to be comparable in magnitude, and shall consider for the moment that they are rotating approximately in circles about some point in the moving plane (such as their center of gravity). Then equating the product of mass by acceleration to the external force on one of the particles we have

$$\frac{a}{r} \sim \beta^2$$

where $\beta \equiv v/c$, and a is the radius of the charged particle.

Hence if the force exerted by one of the particles on the other is taken to be of the zeroth order, inspection of the equation of motion (32) in the previous paper shows the terms to be of the following orders,

$$\frac{e^2 f}{6\pi a c^2} \text{ of zeroth order,}$$

$$\frac{e^2 f^2 \beta}{2\pi c^4} \text{ of fifth order,}$$

$$\frac{e^2\dot{f}}{6\pi c^3} \text{ of third order,}$$

$$\frac{e^2\ddot{f}a}{9\pi c^4} \text{ of sixth order,}$$

$$\frac{e^2\dddot{f}a^2}{18\pi c^5} \text{ of ninth order.}$$

As Larmor's expression is of the fifth order, only the first three terms in the equation of motion need be retained. As a matter of fact two more terms of this equation were evaluated than actually required to disprove Larmor's result. If we make use of the following notation,

$$k \equiv \frac{1}{\sqrt{1-\beta^2}}, \quad A \equiv \frac{e^2}{4\pi r^2}, \quad B \equiv \frac{e_1 e_2}{4\pi r^2}, \quad m_x \equiv \frac{x}{r}, \quad \gamma_x \equiv \frac{f_x r}{c^2}, \quad \delta_x \equiv \frac{\dot{f}_x r^2}{c^3},$$

where the subscript 1 refers to the particle a in the figure and the subscript 2 to b, the component (32) in the direction of motion of the reaction exerted on a by its own field may be written

$$K_{1_v} = A_1 k_1^3 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_v} + \tfrac{2}{3}k_1\delta_{1_v} + 2k_1^3\gamma_{1_v}^2\beta_1 \cdots \right]$$

and that (41) at right angles to the direction of motion

$$K_{1_n} = A_1 k_1^3 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_n}(1-\beta_1^2) + \tfrac{2}{3}k_1\delta_{1_n}(1-\beta_1^2) + 2k_1^3\gamma_{1_v}\gamma_{1_n}\beta_1(1-\beta_1^2)\cdots \right].$$

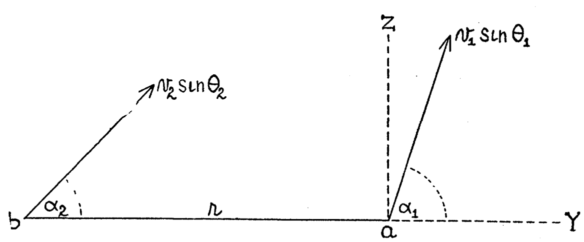

Fig. 1.

Hence if the x axis is taken perpendicular to the paper and upward a simple calculation gives

$$K_{1_x} = A_1 k_1^3 \beta^2 \cos^2\theta \tan\theta_1 \left[-\frac{2}{3}\frac{r}{a_1}(\gamma_{1_y}\cos\alpha_1 + \gamma_{1_z}\sin\alpha_1) + \tfrac{2}{3}(\delta_{1_y}\cos\alpha_1 + \delta_{1_z}\sin\alpha_1)\cdots \right] \quad (1)$$

to the fifth order, and

$$K_{1_y} = A_1k_1^3\left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_y}(1-\beta_1^2+\beta_1^2\sin^2\theta_1\cos^2\alpha_1)\right.$$
$$\left.-\frac{2}{3}\frac{r}{a_1}\gamma_{1_z}\beta_1^2\sin^2\theta_1\sin\alpha_1\cos\alpha_1+\tfrac{2}{3}\delta_{1_y}\cdots\right] \quad (2)$$
$$K_{1_z} = A_1k_1^3\left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_z}(1-\beta_1^2+\beta_1^2\sin^2\theta_1\sin^2\alpha_1)\right.$$
$$\left.-\frac{2}{3}\frac{r}{a_1}\gamma_{1_y}\beta_1^2\sin^2\theta_1\sin\alpha_1\cos\alpha_1+\tfrac{2}{3}\delta_{1_z}\cdots\right] \quad (3)$$

to the third order, which is as far as required for the subsequent work.

From the expressions (23) and (24) of the previous paper we obtain for the force exerted by b on a

$$K_{1_x}' = e_1[\beta_1\sin\theta_1(H_z\cos\alpha_1-H_y\sin\alpha_1)]$$
$$= Bk_2^3\beta^2\cos^2\theta\tan\theta_1[\cos\alpha_1\,([1-\beta_2^2-\tfrac{2}{3}\beta_2^2\sin^2\theta_2\cos^2\alpha_2$$
$$-\gamma_{2_y}+\tfrac{2}{3}\delta_{2_y})-\sin\alpha_1\,(\tfrac{1}{2}\gamma_{2_z}-\tfrac{2}{3}\delta_{2_z})\cdots] \quad (4)$$
$$K_{1_y}' = e_1[E_y+\beta_1(H_x\sin\theta_1\sin\alpha_1-H_z\cos\theta_1)]$$
$$= Bk_2^3\,[1-\beta_2^2-\tfrac{3}{2}\beta_2^2\sin^2\theta_2\cos^2\alpha_2$$
$$-\beta_1\beta_2(\sin\theta_1\sin\theta_2\sin\alpha_1\sin\alpha_2+\cos\theta_1\cos\theta_2)$$
$$-\gamma_{2_y}+\tfrac{2}{3}\delta_{2_y}\cdots], \quad (5)$$
$$K_{1_z}' = e_1[E_z+\beta_1(H_y\cos\theta_1-H_x\sin\theta_1\cos\alpha_1)]$$
$$= Bk_2^3\,[\beta_1\beta_2(\sin\theta_1\sin\theta_2\cos\alpha_1\sin\alpha_2)-\tfrac{1}{2}\gamma_{2_z}+\tfrac{2}{3}\delta_{2_z}\cdots]. \quad (6)$$

Now the total force on the particle a in the plane of vibration must be zero. Hence

$$K_{1_y}'+K_{1_y}=0,$$
$$K_{1_z}'+K_{1_z}=0.$$

Therefore,

$$A_1k_1^3\left(\frac{2}{3}\frac{r}{a_1}\gamma_{1_y}-\tfrac{2}{3}\delta_{1_y}\right)$$
$$= Bk_2^3\,[1-\beta_2^2-\tfrac{3}{2}\beta_2^2\sin^2\theta_2\cos^2\alpha_2+\beta_1^2-\beta_1^2\sin^2\theta_1\cos^2\alpha_1$$
$$-\beta_1\beta_2\,(\sin\theta_1\sin\theta_2\sin\alpha_1\sin\alpha_2+\cos\theta_1\cos\theta_2)-\gamma_{2_y}+\tfrac{2}{3}\delta_{2_y}\cdots],$$
$$A_1k_1^3\left(\frac{2}{3}\frac{r}{a_1}\gamma_{1_z}-\tfrac{2}{3}\delta_{1_z}\right) = Bk_2^3\,[\beta_1\beta_2\sin\theta_1\sin\theta_2\cos\alpha_1\sin\alpha_2$$
$$-\beta_1^2\sin^2\theta_1\sin\alpha_1\cos\alpha_1-\tfrac{1}{2}\gamma_{2_z}+\tfrac{2}{3}\delta_{2_z}\cdots].$$

Remembering that $\beta_1 \cos\theta_1 = \beta_2 \cos\theta_2$,

$$A_1 k_1^3 \left[\frac{2}{3}\frac{r}{a_1}(\gamma_{1_y}\cos\alpha_1 + \gamma_{1_z}\sin\alpha_1) - \tfrac{2}{3}(\delta_{1_y}\cos\alpha_1 + \delta_{1_z}\sin\alpha_1) \right]$$

$$= Bk_2^3 [\cos\alpha_1(1 - \beta_2^2 - \tfrac{3}{2}\beta_2^2 \sin^2\theta_2 \cos^2\alpha_2 - \gamma_{2_y} + \tfrac{2}{3}\delta_{2_y})$$

$$- \sin\alpha_1(\tfrac{1}{2}\gamma_{2_z} - \tfrac{2}{3}\delta_{2_z})],$$

from which

$$K_{1_x}' = A_1 k_1^3 \beta^2 \cos^2\theta \tan\theta_1 \left[\frac{2}{3}\frac{r}{a_1}(\gamma_{1_y}\cos\alpha_1 + \gamma_{1_z}\sin\alpha_1) - \tfrac{2}{3}(\delta_{1_y}\cos\alpha_1 + \delta_{1_z}\sin\alpha_1)\cdots \right], \quad (7)$$

whence

$$K_{1_x}' + K_{1_x} = 0.$$

The resultant force normal to the plane of vibration is zero for each particle, and hence zero for the doublet. In fact the analysis shows that the retardation K_x on one of the particles due to the reaction of its own field is exactly annulled by the accelerating force K_x' produced by the magnetic field of the other particle.

In order to illustrate the fact that a correct solution of the problem is impossible unless damping is taken into account, consider two particles which have equal radii and equal and opposite charges. At the instant considered let them have velocities of which the components in the plane of vibration are equal and opposite, and at right angles to the line joining them. As before, the plane of their motion will be supposed to have a velocity $v \cos\theta$ along the normal. Then

$$A_1 = -B,\ \beta_1 = \beta_2,\ \gamma_1 = -\gamma_2 \equiv \gamma,\ \delta_1 = -\delta_2 \equiv \delta,\ \alpha_1 = \alpha_2 - \pi = \pi/2.$$

Hence

$$K_{1_x} = Ak^3\beta^2 \sin\theta\cos\theta \left[-\frac{2}{3}\frac{r}{a}\gamma_z + \tfrac{2}{3}\delta_z \cdots \right],$$

$$K_{1_y} = Ak^3 \left[-\frac{2}{3}\frac{r}{a}\gamma_y(1 - \beta^2) + \tfrac{2}{3}\delta_y \cdots \right],$$

$$K_{1_z} = Ak^3 \left[-\frac{2}{3}\frac{r}{a}\gamma_z(1 - \beta^2\cos^2\theta) + \tfrac{2}{3}\delta_z \cdots \right],$$

$$K_{1_x}' = -Ak^3\beta^2 \sin\theta\cos\theta[\tfrac{1}{2}\gamma_z - \tfrac{2}{3}\delta_z \cdots],$$

$$K_{1_y}' = -Ak^3[1 - 2\beta^2\cos^2\theta + \gamma_y - \tfrac{2}{3}\delta_y \cdots],$$

$$K_{1_z}' = -Ak^3[\tfrac{1}{2}\gamma_z - \tfrac{2}{3}\delta_z \cdots].$$

Therefore

$$K_{1_x}' + K_{1_x} = -Ak^3\beta^2 \sin\theta \cos\theta \left[\frac{2}{3}\frac{r}{a}\gamma_z + \tfrac{1}{2}\gamma_z - \tfrac{4}{3}\delta_z \cdots \right] \tag{8}$$

is the retarding force on one of the particles in the direction perpendicular to the plane of the rotation.

Suppose we neglect damping. Then the particles will move in a circle about a point half way between them, and

$$\gamma_y = -2\beta^2 \sin^2\theta, \qquad \gamma_z = 0, \qquad \delta_y = 0, \qquad \delta_z = -4\beta^3 \sin^3\theta,$$

whence

$$\begin{aligned} K_{1_x}' + K_{1_x} &= -Ak^3\beta^2 \sin\theta \cos\theta[\tfrac{16}{3}\beta^3 \sin^3\theta] \\ &= -\frac{4e^2}{3\pi r^2}\beta^5 \sin^4\theta \cos\theta \\ &= -\frac{e^2 f^2}{3\pi c^4}\beta \cos\theta, \end{aligned}$$

and for the two particles the retarding force is

$$K_x = -\frac{2e^2 f^2}{3\pi c^4}\beta \cos\theta.$$

Now the rate of radiation from the doublet is

$$R = \frac{2e^2 f^2}{3\pi c^3}.$$

Therefore,

$$K_x = -\frac{1}{c^2}Rv_x,$$

which is just Larmor's expression for the retardation.

But in order to satisfy the equation of motion in the plane of rotation,

$$K_{1_y}' + K_{1_y} = 0,$$
$$K_{1_z}' + K_{1_z} = 0,$$

or

$$\gamma_y = -\frac{3}{2}\frac{a}{r}(1 - \beta_2),$$

$$\gamma_z = -\frac{3}{2}\frac{a}{r}(-\tfrac{4}{3}\delta_z),$$

showing that undamped motion is inconsistent with the electrodynamic equations. Substituting this value of γ_z in (8) we see that the total force normal to the plane of vibration is zero for each charged particle individually, and hence for the doublet as a whole.

It may be of interest to discuss the case where the mass of one of the particles is not entirely electromagnetic. For instance, consider a single electron and a positive nucleus rotating about their center of gravity. The experiments of Bucherer, Neumann, and others have proved that the electron's mass is entirely electromagnetic, but that of the nucleus may be in part mechanical, *i. e.*, a constant, independent of the velocity. The problem is then no longer purely a matter of electrodynamics, and the close connection between the relativity transformations and the electrodynamic equations can not be advanced as showing that there can be no retardation from the reaction of the radiation. If the electron is at a in the figure and the nucleus at b, and if the velocity components in the plane of vibration are taken opposite and perpendicular to the line joining the particles, and of such a magnitude as to make the motion approximately circular, we have for the electron

$$K_{1_x} = A_1 k_1^3 \beta^2 \cos^2\theta \tan\theta_1 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_z} + \tfrac{2}{3}\delta_{1_z}\cdots \right],$$

$$K_{1_y} = A_1 k_1^3 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_y}(1-\beta_1^2) + \tfrac{2}{3}\delta_{1_y}\cdots \right],$$

$$K_{1_z} = A_1 k_1^3 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_z}(1-\beta_1^2\cos^2\theta_1) + \tfrac{2}{3}\delta_{1_z}\cdots \right],$$

$$K_{1_x}' = -Bk_2^3\beta^2\cos^2\theta\tan\theta_1\left[-\tfrac{1}{2}\gamma_{2_z} + \tfrac{2}{3}\delta_{2_z}\cdots\right],$$

$$K_{1_y}' = -Bk_2^3\left[1-\beta_2^2+\beta_1\beta_2(\sin\theta_1\sin\theta_2-\cos\theta_1\cos\theta_2)-\gamma_{2_y}+\tfrac{2}{3}\delta_{2_y}\cdots\right],$$

$$K_{1_z}' = -Bk_2^3\left[-\tfrac{1}{2}\gamma_{2_z}+\tfrac{2}{3}\delta_{2_z}\cdots\right],$$

and for the nucleus

$$K_{2_x} = A_2 k_2^3 \beta^2 \cos^2\theta \tan\theta_2 \left[\frac{2}{3}\frac{r}{a_2}\gamma_{2_z} - \tfrac{2}{3}\delta_{2_z}\cdots \right],$$

$$K_{2_y} = -M_2\gamma_{2_y} + A_2 k_2^3 \left[-\frac{2}{3}\frac{r}{a_2}\gamma_{2_y}(1-\beta_2^2) + \tfrac{2}{3}\delta_{2_y}\cdots \right],$$

$$K_{2_z} = -M_2\gamma_{2_z} + A_2 k_2^3 \left[-\frac{2}{3}\frac{r}{a_2}\gamma_{2_z}(1-\beta^2\cos^2\theta_2) + \tfrac{2}{3}\delta_{2_z}\cdots \right],$$

$$K_{2_x}' = -Bk_1^3\beta^2\cos^2\theta\tan\theta_2\left[\tfrac{1}{2}\gamma_{1_z} - \tfrac{2}{3}\delta_{1_z}\cdots\right],$$

$$K_{2_y}' = -Bk_1^3\left[-1+\beta_1^2-\beta_1\beta_2(\sin\theta_1\sin\theta_2-\cos\theta_1\cos\theta_2)-\gamma_{1_y}+\tfrac{2}{3}\delta_{1_y}\cdots\right],$$

$$K_{2_z}' = -Bk_1^3\left[-\tfrac{1}{2}\gamma_{1_z}+\tfrac{2}{3}\delta_{1_z}\cdots\right],$$

where

$$M_2 \equiv \frac{c^2}{r} m_2,$$

and m_2 is the mechanical mass of the nucleus.

Put

$$k_2^2 M_2 + \tfrac{2}{3} A_2 k_2^3 \frac{r}{a_2} = \tfrac{2}{3} A_2 k_2^3 \frac{r}{b_2}.$$

Then, as the force on each particle in the plane of the vibration must vanish,

$$A_1 k_1^3 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_y} + \tfrac{2}{3}\delta_{1_y} \right]$$
$$= Bk_2^3 [1 - \beta_2^2 + \beta_1^2 - \beta_2^2 \cos^2\theta_2 + \beta_1\beta_2 \sin\theta_1 \sin\theta_2 - \gamma_{2_y} + \tfrac{2}{3}\delta_{2_y}],$$

$$A_1 k_1^3 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_z} + \tfrac{2}{3}\delta_{1_z} \right] = Bk_2^3 [-\tfrac{1}{2}\gamma_{2z} + \tfrac{2}{3}\delta_{2z}],$$

$$A_2 k_2^3 \left[-\frac{2}{3}\frac{r}{b_2}\gamma_{2_y} + \tfrac{2}{3}\delta_{2_y} \right]$$
$$= Bk_1^3 [-1 + \beta_1^2 - \beta_2^2 + \beta_1^2 \cos^2\theta_1 - \beta_1\beta_2 \sin\theta_1 \sin\theta_2 - \gamma_{1_y} + \tfrac{2}{3}\delta_{1_y}],$$

$$A_2 k_2^3 \left[-\frac{2}{3}\frac{r}{b_2}\gamma_{2_z} + \tfrac{2}{3}\delta_{2_z} \right] = Bk_1^3 [-\tfrac{1}{2}\gamma_{1z} + \tfrac{2}{3}\delta_{1z}].$$

So, for the electron

$$K_{1_x}' + K_{1_x} = 0$$

as before. For the nucleus, however,

$$K_{2_x}' + K_{2_x} = -A_2 k_2^3 \beta^2 \cos^2\theta \tan\theta_2 \tfrac{2}{3} \gamma_{2_z} \left(\frac{r}{b_2} - \frac{r}{a_2} \right)$$
$$= -\beta^2 \cos^2\theta \tan\theta_2 k_2^2 M_2 \gamma_{2_z}. \quad (9)$$

Let us take the charge on the nucleus equal to that on the electron. Then $A_2 = B$, and

$$\gamma_{2_z} = \frac{b_2}{r}(\delta_{2_z} - \delta_{1_z})$$
$$= \frac{b_2}{r}\left(\frac{\gamma_{2_y}^2}{\beta_2 \sin\theta_2} + \frac{\gamma_{1_y}^2}{\beta_1 \sin\theta_1} \right).$$

But, for approximately circular motion

$$\frac{\gamma_{2_y}}{b_2} = \frac{\gamma_{1_y}}{a_1}, \qquad \frac{\beta_2 \sin\theta_2}{b_2} = \frac{\beta_1 \sin\theta_1}{a_1}.$$

Hence

$$K_x = -\beta \cos\theta\, b_2 m_2 \frac{f_{2_y}^2}{c^2}\left(1 + \frac{a_1}{b_2} \right),$$

or, if we denote the electromagnetic mass of the electron by m_1' and that of the nucleus by m_2'

$$K_x = -\frac{\beta}{c}\cos\theta\cdot\frac{e^2 f_{2_y}{}^2}{6\pi c^3}\frac{\left(1+\dfrac{m_2+m_2'}{m_1'}\right)}{\left(1+\dfrac{m_2'}{m_2}\right)},$$

Now the rate of radiation from the doublet is

$$R = \frac{e^2}{6\pi c^3}(f_{2_y}{}^2 + 2f_{1_y}f_{2_y} + f_{1_y}{}^2)$$

$$= \frac{e^2 f_{2_y}{}^2}{6\pi c^3}\left(1+\frac{m_2+m_2'}{m_1'}\right)^2.$$

Therefore

$$K_x = -\frac{1}{c^2}Rv_x\frac{1}{\left(1+\dfrac{m_2'}{m_2}\right)\left(1+\dfrac{m_2+m_2'}{m_1'}\right)}. \qquad (10)$$

This expression vanishes if the mass of the nucleus is entirely electromagnetic. If, however, its mass is assumed to be practically all mechanical, and eighteen hundred times the electromagnetic mass of the electron,

$$K_x = -\frac{1}{1800}\frac{1}{c^2}Rv_x,$$

which is a small fraction of Larmor's expression. In fact the above analysis shows that his result would be true only if the entire mass of the vibrating system were mechanical. If its mass is entirely electromagnetic, as has been assumed in this and the preceding paper, there is no retardation, and the principle of relativity is confirmed. For a doublet consisting of an electromagnetic electron rotating about a mechanical nucleus there is a retardation, which, however, is a very small part of that calculated by Larmor.

Summary.

(*a*) The problem of two charged particles with electromagnetic masses, moving under the action of each other's fields in a plane which has a drift velocity along its normal, has been treated rigorously, and no retardation found to exist as a result of the radiation emitted.

(*b*) Larmor's expression for the retardation due to the reaction of the radiation from the moving doublet has been shown to be a consequence of his failure to take into account the effect of damping. The assumption that damping is negligible leads to a contradiction with the electrodynamic equations. When damping is taken into consideration this inconsistency disappears, and the retarding force vanishes.

(*c*) Damping may be neglected, and hence Larmor's expression holds, only when the mass of the vibrating system is assumed to be entirely mechanical (not a function of the velocity). If the radiating doublet consists of an electron of electromagnetic mass and a positive nucleus of a mechanical mass eighteen hundred times as great, there is a retardation amounting to one eighteen hundredth part of that calculated by Larmor.

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June 1, 1918.