Clock, Boost, Wave: A Kinematic Origin of the de Broglie Relation

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Abstract

The de Broglie relation $\lambda = h/p$ is among the most well-confirmed results in modern physics, yet the kinematic origin of the associated matter wave remains conceptually obscure. This paper presents a kinematic analysis leading to the de Broglie relation. The only assumption is the existence of a globally defined phase field with uniform rest-frame oscillation, a structure known to occur in superfluids and Bose–Einstein condensates. Under a Lorentz boost, the scalar transformation of the phase field converts temporal periodicity directly into spatial periodicity, yielding a wave number $k' = \gamma\omega_0 v/c_s^2$ and a phase velocity $v_{\text{phase}} = c_s^2/v$. No wave equation, dynamical model, or quantization procedure is required. The result follows solely from the relativity of simultaneity applied to a scalar phase field. When the characteristic frequency and invariant speed are identified as $\omega_0 = mc^2/\hbar$ and $c_s = c$, the resulting wavelength and phase velocity reproduce the standard de Broglie relations. The logical architecture inverts that of de Broglie's original construction: rather than postulating a separate clock and wave linked by phase harmony, the wave emerges as the Lorentz-transformed image of a single coherent phase clock. The phase harmony that de Broglie introduced as an independent postulate becomes a consequence of Lorentz covariance. The analysis establishes a precise kinematic correspondence without asserting that the physical vacuum is a coherent medium. Whether this correspondence reflects a deeper physical principle or a mathematical analogy remains an open question.

Keywords: de Broglie relation, matter waves, Lorentz transformation, phase coherence, superfluidity, quantum foundations

1 Introduction

The de Broglie relation $\lambda = h/p$ is among the most successful results in modern physics [1, 2]. Yet the physical origin of the associated matter wave remains conceptually obscure. Why should a periodic phenomenon give rise to a propagating wave? And why should that wave possess the apparently superluminal phase velocity c^2/v?

A suggestive clue emerges from coherent condensed-matter systems. In superfluids and Bose–Einstein condensates, the order parameter $\Psi = \sqrt{n}\, e^{i\phi}$ maintains a globally coherent phase across macroscopic distances [3, 4, 5]. The ground state may therefore provide a temporally periodic phase reference extending throughout the medium.

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This raises a simple question. What does a moving phase-sensitive detector register when traversing such a coherent phase field?

In this paper we show that the answer follows directly from Lorentz kinematics. A Lorentz transformation of a globally coherent phase field converts temporal periodicity into spatial periodicity. A moving detector therefore records a wave-like phase structure even when the medium itself remains in its unperturbed ground state.

We develop this observation into a purely kinematic analysis and no assumption is made here that such a coherent phase field corresponds to a physical medium. The comparison with de Broglie's original construction [6] reveals an exact formal correspondence between the two frameworks.

Whether this correspondence suggests a new perspective on the origin of matter-wave kinematics or a mathematical analogy remains an open question.

2 Kinematic Projection of a Phase Clock

2.1 Ideal Coherent Medium: Axiomatic Definition

We define an *ideal coherent medium* by the following four axioms.

Axiom 1: Global phase coherence. The medium admits a macroscopic order parameter

$$\Psi(\mathbf{r},t) = \sqrt{n}\, e^{i\phi(\mathbf{r},t)}, \tag{1}$$

where $\phi(\mathbf{r},t)$ is a smooth phase field defined everywhere except at isolated defect cores.

Axiom 2: Uniform ground-state oscillation. In the rest frame $\mathcal{S}$, the ground state is

$$\phi_0(\mathbf{r},t) = \omega_0 t, \tag{2}$$

where ω_0 defines a globally synchronized phase clock.

Axiom 3: Finite invariant speed. A characteristic speed c_s defines the effective causal structure of the medium.

Axiom 4: Lorentz-covariant phase field. The phase field is assumed to be a Lorentz scalar:

$$\phi'(x'^{\mu}) = \phi(x^{\mu}(x'^{\nu})). \tag{3}$$

These assumptions do not specify a microscopic model; they define only the kinematic structure required for the subsequent Lorentz analysis and the analysis below is purely kinematic.

2.2 A Phase Detector in Uniform Motion

Consider a detector moving with constant velocity v along the z direction. The Lorentz transformation is

$$t = \gamma\left(t' + \frac{vz'}{c_s^2}\right), \qquad z = \gamma(z' + vt'), \tag{4}$$

with

$$\gamma = \frac{1}{\sqrt{1 - v^2/c_s^2}}. \tag{5}$$

In the rest frame of the medium,

$$\phi = \omega_0 t. \tag{6}$$

Since the phase is a Lorentz scalar,

$$\phi'(z', t') = \omega_0 \gamma \left(t' + \frac{v z'}{c_s^2} \right). \tag{7}$$

This may be written in the plane-wave form

$$\phi'(z', t') = \omega' t' - k' z', \tag{8}$$

with

$$\omega' = \gamma \omega_0, \qquad k' = -\gamma \omega_0 \frac{v}{c_s^2}. \tag{9}$$

A phase field that is purely temporal in its rest frame therefore acquires a spatial wave number under a Lorentz boost.

2.3 From Temporal Periodicity to Spatial Periodicity

In the rest frame of the medium, the phase field

$$\phi = \omega_0 t \tag{10}$$

contains no spatial structure.

After a Lorentz boost, the same field becomes

$$\phi' = \omega' t' - k' z', \tag{11}$$

and acquires a spatial gradient.

The origin of this wave-like structure is the relativity of simultaneity. Lorentz transformations mix space and time, so a globally synchronized temporal oscillation is perceived as a spatially modulated phase pattern by a moving observer.

Symbolically,

$$\text{temporal periodicity} \xrightarrow{\text{Lorentz boost}} \text{spatial periodicity}. \tag{12}$$

The resulting wave is not a physical excitation of the medium. It is a kinematic projection of a globally coherent phase field.

2.4 Superluminal Phase Velocity

The phase velocity associated with the projected wave is

$$v_{\text{phase}} = \frac{\omega'}{|k'|} = \frac{c_s^2}{v}. \tag{13}$$

For any detector speed $v < c_s$, one has

$$v_{\text{phase}} > c_s. \tag{14}$$

This superluminal value does not represent the propagation of matter, energy, or information. It describes the apparent motion of constant-phase surfaces generated by Lorentz-transformed simultaneity slices.

The superluminality is therefore geometric rather than dynamical.

2.5 Embedded Phase Structures

The same kinematic mechanism applies to non-uniform phase configurations.

As an illustration, consider a quantized vortex,

$$\phi = \omega_0 t + n\varphi, \tag{15}$$

where n is the winding number and φ is the azimuthal angle.

For a vortex line parallel to the boost direction, the phase profile is unchanged.

For a vortex line perpendicular to the boost, circular phase contours become elliptical, contracted by a factor of $1/\gamma$ along the direction of motion and unchanged in the perpendicular direction. While the geometric profile is frame-dependent, whereas the topological charge

$$\oint \nabla\phi \cdot d\mathbf{l} = 2\pi n \tag{16}$$

remains invariant.

2.6 Summary

A globally coherent phase clock,

$$\phi = \omega_0 t, \tag{17}$$

acquires a spatial wave number

$$k' = \gamma\omega_0 \frac{v}{c_s^2} \tag{18}$$

under a Lorentz boost.

The resulting phase velocity,

$$v_{\text{phase}} = \frac{c_s^2}{v}, \tag{19}$$

is superluminal but non-causal, arising from the geometry of simultaneity rather than from physical transport.

These results follow solely from Lorentz kinematics and do not depend on any dynamical model of the medium. The next section compares this kinematic structure with de Broglie's original construction.

3 de Broglie's Construction and Its Kinematic Counterpart

3.1 de Broglie's Original Construction

In his 1923–1924 work, de Broglie proposed that a material particle possesses an intrinsic periodic phenomenon with rest-frame frequency

$$\omega_0 = \frac{mc^2}{\hbar}. \tag{20}$$

He further introduced an accompanying wave and required that the two remain phase-synchronized, a condition he termed *l'harmonie des phases*. From this construction he obtained

$$\lambda = \frac{h}{p}, \qquad v_{\text{phase}} = \frac{c^2}{v}. \tag{21}$$

The remarkable success of the de Broglie relation established matter-wave kinematics as a cornerstone of modern physics. However, the physical relation between the internal clock and the external wave remained postulated rather than derived.

3.2 Direct Kinematic Correspondence

The result obtained in Section II is

$$k' = \gamma\omega_0 \frac{v}{c_s^2}, \qquad v_{\text{phase}} = \frac{c_s^2}{v}. \tag{22}$$

Under the identifications

$$c_s \to c, \qquad \omega_0 \to \frac{mc^2}{\hbar}, \tag{23}$$

one immediately obtains

$$k' = \frac{\gamma m v}{\hbar} = \frac{p}{\hbar}, \tag{24}$$

and therefore

$$\lambda = \frac{2\pi}{k'} = \frac{h}{p}. \tag{25}$$

Likewise,

$$v_{\text{phase}} = \frac{c^2}{v}. \tag{26}$$

The mathematical correspondence is exact.

3.3 An Inversion of Logic

Despite their identical mathematical structure, the logical architectures of the two constructions are different.

In de Broglie's formulation, an internal clock and an external wave are introduced as distinct entities and connected through a phase-harmony condition,

$$\text{clock} + \text{wave} \xrightarrow{\text{phase harmony}} \text{de Broglie relation.} \tag{27}$$

In the present framework, only a single object is introduced: a globally coherent phase field. The wave emerges directly from the Lorentz transformation of the phase clock,

$$\text{clock} \xrightarrow{\text{boost}} \text{wave.} \tag{28}$$

The phase harmony that appears as a postulate in de Broglie's construction becomes a consequence of the scalar nature of the phase field.

The result does not replace de Broglie's argument. Rather, it provides a kinematic realization of the same mathematical structure in which the relation between clock and wave arises from Lorentz geometry alone.

4 Discussion

4.1 Penrose and the Geometry of Appearance

The central mechanism identified in this work is the relativity of simultaneity. A globally synchronized temporal oscillation appears as a spatial wave because different inertial observers slice the same phase field differently.

In this sense, the de Broglie-like wave derived here is not a physical excitation of the medium but a geometric manifestation of observer slicing. The wave arises from how a moving observer intersects a pre-existing coherent phase structure.

This perspective bears a conceptual resemblance to the work of Penrose on relativistic appearance. In the celebrated Penrose–Terrell effect, a rapidly moving object does not appear Lorentz-contracted in a direct visual sense. Instead, the observed image is determined by the intersection of the observer's past light cone with the object's world tube [7, 8]. The apparent structure is therefore a geometric projection rather than a direct representation of the object's instantaneous state.

The situation considered here is analogous. A moving phase detector does not directly access the intrinsic phase structure of the medium. Rather, it records the intersection of its simultaneity hypersurfaces with a globally coherent phase field. The resulting wave is therefore an appearance generated by spacetime geometry.

The connection is conceptual rather than technical. No element of twistor theory or Penrose's broader programme is required. The common theme is simply that physically observable structures may emerge from the geometry of spacetime slicing itself.

4.2 Relation to Volovik's Programme

The present work is motivated by an experimental fact: superfluids and Bose–Einstein condensates possess global phase coherence across macroscopic distances. This raises a natural question: if such a coherent phase field is subjected to a Lorentz boost, what kinematic structure emerges?

The idea that condensed-matter systems with global phase coherence can illuminate the structure of the quantum vacuum has been developed extensively by Volovik [9]. In Volovik's programme, the vacuum is treated as a condensed-matter-like medium, and effective relativistic kinematics—together with gauge fields and gravity—emerge as low-energy consequences.

The present analysis shares with Volovik's programme the minimal premise that global phase coherence, when combined with Lorentz symmetry, generates nontrivial kinematic structure. It differs in scope: no assumption is made about the microscopic identity of the vacuum, nor about the emergent nature of spacetime. The only claim is that *if* a globally coherent phase field exists, *then* a Lorentz boost necessarily produces a wave whose kinematic parameters coincide with those of a de Broglie wave.

Related geometric considerations concerning compact phase defects and the characteristic scale $r_* = \hbar/mc$ are explored elsewhere [10, 11].

5 Conclusion

We have shown that a globally coherent phase field with uniform rest-frame oscillation acquires a spatial wave number under a Lorentz boost,

$$k' = \gamma\omega_0 \frac{v}{c_s^2}, \tag{29}$$

as a direct consequence of the scalar transformation of the phase field. No wave equation, dynamical assumption, or quantization procedure is required.

When the characteristic frequency and invariant speed are identified as

$$\omega_0 \to \frac{mc^2}{\hbar}, \qquad c_s \to c, \tag{30}$$

the resulting wavelength and phase velocity coincide exactly with those of de Broglie's matter wave. The mathematical correspondence is therefore exact.

The difference is logical rather than mathematical. In de Broglie's original construction, an internal clock and an external wave are linked by a phase-harmony postulate. In the present framework, the wave emerges as the Lorentz-transformed image of a single coherent phase field. The harmony of phases becomes a consequence of Lorentz covariance rather than an independent assumption.

The present analysis does not establish that the physical vacuum is a coherent medium, nor does it provide a dynamical derivation of quantum mechanics. It establishes a precise kinematic correspondence: the de Broglie wave structure can be recovered from the Lorentz transformation of a globally coherent phase clock. Whether this correspondence represents a deeper physical principle or a mathematical analogy remains an open question. The purpose of the present work is not to resolve that question, but to formulate it in a precise kinematic form.

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