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Review of Hydrodynamic Modeling for Wave Energy Converter Arrays

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Abstract

Wave energy's high predictability reduces energy storage needs and enhances energy security, complementing wind and solar power. To capture meaningful amounts of energy, wave energy converters (WECs) can be deployed in arrays. However, a major challenge in optimizing WEC array design is the significant computational cost of modeling the complex hydrodynamic interactions between individual WECs. Numerous methods have been proposed to reduce computation time, but their effectiveness in addressing this challenge remains unreviewed. This work presents a comprehensive overview of WEC array hydrodynamic modeling in the frequency domain under linear potential flow, comparing their limiting assumptions and computational advantages.

Keywords: wave energy converter arrays; multiple scattering; interaction theory; hydrodynamics

1 Introduction

Wave energy has the potential to play a key role in the transition to renewable energy. It is predictable [1], energy-dense [2], and highly available [3]. Additionally, wave energy can have a smoothing effect on the power production of intermittent renewable sources such as wind and solar power [4, 5, 6]. To capture meaningful amounts of energy, numerous WECs can be deployed together in an array. Arrays of WECs are advantageous as constructive hydrodynamic

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interactions and shared costs can increase energy production [7] and reduce the total cost [8, 9], respectively. However, the well-known bottleneck when optimizing WEC arrays is the computationally expensive hydrodynamic model [10]. This work reviews frequency domain methods under linear potential flow that have the most promise to overcome this computational barrier. The objective of these methods is to efficiently determine the frequency-dependent forces associated with the incident, diffracted, and radiated wave potential.

2 Computational Challenges in WEC Array Modeling

When choosing a method to model the hydrodynamics of WEC arrays, trade-offs between the computation time and fidelity must be considered. Linear potential flow in the frequency domain is the least computationally expensive method for modeling WEC arrays, but generally does not account for nonlinearities unless hybrid modeling methods are used [11]. Meanwhile, computational fluid dynamics models are the most accurate, but are far too computationally expensive to model large WEC arrays [12, 10]. While many methods exist between these two extremes, linear potential flow is often used when optimizing the design of WEC arrays since the computation cost of other methods becomes unmanageable for large numbers of WECs [10].

Still, within the area of linear potential flow in the frequency domain, computation time is problematic. Boundary element method (BEM) solvers, which can model WECs with arbitrary 3D geometries, become computationally expensive when modeling large numbers of WECs due to their computation time scaling quadratically with the number of panels [13]. This has motivated researchers to use interaction theory to reduce the computational expense. While some of the methods in this work are capable of modeling arbitrary WEC geometries, many of them are limited to only modeling axisymmetric geometries. The following sections overview these methods, highlighting their trade-offs between computation time and generality of WEC geometry.

3 Interaction Theory Modeling Methods

3.1 Overview of Multiple Scattering Methods

In this section, we discuss the multiple scattering (MS) method and its variants, which have been used to model WEC arrays. This method was originally proposed as an iterative procedure by Ohkusu in 1974 [14] and was later extended by Mavrakos and Koumoutsakos to be used for arrays of axisymmetric bodies with multiple cylindrical regions using the matched eigenfunction expansion method [15, 16]. Over the years, there have been many alterations to this method. In 1986, Kagemoto and Yue developed a noniterative version of MS that has been widely used since [17]. This was generalized by McNatt et al. (2015) so that standard BEM solvers can be used to determine the isolated body properties of each WEC in the array [18]. However, evanescent modes are neglected when using the version that is compatible with BEM. Both the iterative [19] and noniterative [9] versions have been used recently in the literature, showing that it is unclear which method is best.

In both methods, the general form of the scattered or radiated wave potential outside a body is

$$\phi^{\mathrm{S,R}}(r,\theta,z) = \frac{\cosh k_0(z+h)}{\cosh k_0 h} \sum_{m=-M}^{M} A_{0m} H_m^{(1)}(k_0 r) e^{\mathrm{i}m\theta} + \sum_{\ell=1}^{L} \cos k_\ell(z+h) \sum_{m=-M}^{M} A_{\ell m} K_m(k_\ell r) e^{\mathrm{i}m\theta} \tag{1}$$

where r, θ, and z are cylindrical coordinates; h is the water depth; A_{0m} and $A_{\ell m}$ are unknown scattered/radiated wave coefficients for the propagating and evanescent wave modes, respectively; $H_m^{(1)}$ is an mth order Hankel function of the first kind; K_m is an mth order modified Bessel function of the second kind; and k_0 and k_ℓ are wave numbers found from dispersion relations [17]. Eq. 1 contains a total of $(2M+1)(L+1)$ unknown coefficients, A_{0m} and $A_{\ell m}$.

For the iterative MS method, the full interaction problem is discretized into orders of scattering. For each order of scattering, a system of linear equations with $(2M+1)(L+1)$ unknowns must be solved at each WEC to relate the incident wave potential to the scattered wave potential in Eq. 1. As higher orders of scattering are considered, the method approaches the exact solution.

For the noniterative MS method, the full interaction is accounted for all at once. A diffraction transfer matrix is defined that maps the incident wave coefficients to the scattered wave coefficients for each WEC in the array. By relating the incident wave coefficients for one WEC to the scattered wave coefficients from all other WECs, a system of equations with $N(2M+1)(L+1)$ unknowns can be written, where N is the number of WECs in the array. This method is exact up to the order of truncation of the series in Eq. 1.

3.2 Comparative Analysis of Iterative vs. Noniterative Approaches

Table 1 shows the computations and restrictions of each version of MS when determining the hydrodynamic forces for the array. When finding the excitation force, all bodies are modeled as being fixed, and forces from the incident

Table 1: Comparison of iterative and noniterative MS methods. M, L, N, S, and Q are the highest order Bessel function in the series representation, the number of evanescent wave modes, the number of WECs in the array, the highest order of scattering, and the number of degrees-of-freedom of the array, respectively.

	Iterative MS [14, 15, 16]	**Noniterative MS [17, 18]**
Excitation Force Computations	Solve for $(2M+1)(L+1)$ unknowns $N \times S$ times	Solve for $N(2M+1)(L+1)$ unknowns once
Radiation Force Computations	Solve for $(2M+1)(L+1)$ unknowns $Q \times N \times S$ times	Solve for $N(2M+1)(L+1)$ unknowns Q times
Restrictions on Geometry	Axisymmetric bodies	Any geometry if evanescent waves are neglected. Otherwise, axisymmetric bodies.

and diffracted waves are superimposed for each body. For the radiation force, all Q degrees-of-freedom of the array are modeled, and the forces on each body from the radiated waves are found.

The iterative version of MS solves a smaller system of equations than the noniterative version does. However, the iterative version must solve this system of equations $N \times S$ times more than the noniterative version. Both methods require solving a system of linear equations Q more times to obtain the radiation force than to obtain the excitation force. Thus, solving for the radiation force will be the majority of the total runtime.

Note that the computations listed in the noniterative MS column are solely for solving the interaction problem once the individual body properties, such as the diffraction transfer matrices, are already known. However, solving the interaction problem dominates the total runtime [20].

Assuming the computation time scales cubically with the number of unknowns, as is typical for dense system matrices solved directly [21], and that each WEC has at least one degree-of-freedom ($Q \geq N$), the iterative and noniterative MS methods scale $\propto N^2 S\,[(2M+1)(L+1)]^3$ and $\propto N^4\,[(2M+1)(L+1)]^3$, respectively. The iterative method scales quadratically with the number of WECs, while the noniterative method scales quartically. For scattering orders $S < N^2$ with the same M and L, the iterative method is faster. Note that in some cases, the structure of the system matrix can allow for the computation time to scale sub-cubically with the number of unknowns [21].

Nevertheless, the noniterative version of MS has a clear advantage over the iterative version: it can model any arbitrary 3D WEC geometry since it is compatible with traditional BEM solvers. This is thanks to the cylindrical surface method developed in Ref. [22]. Meanwhile, the iterative version can only model axisymmetric bodies. It seems possible for the cylindrical surface method to also be used with iterative MS, but this has yet to be explored.

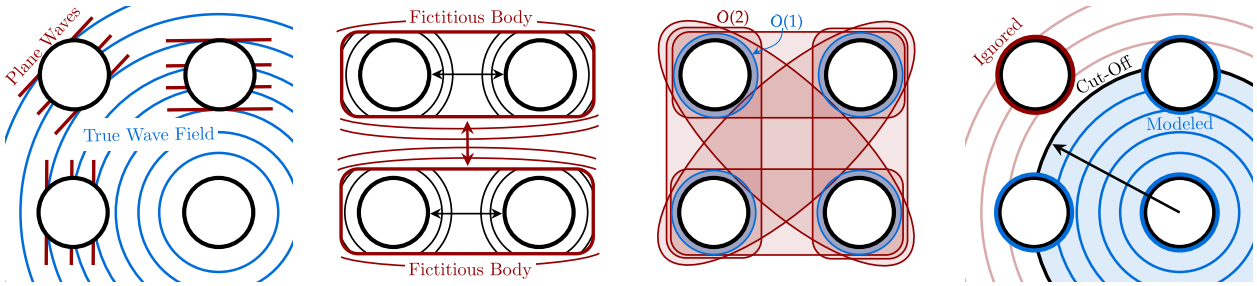
4 Simplifications and Enhancements to Interaction Theory

There are numerous ways of altering the interaction theory in Sec. 3 to reduce the total runtime. Table 2 shows a comparison of some of these methods. The CPU time is once again assumed to scale cubically with the number of unknowns. Additionally, the version of MS that each version has been used with is listed.

The point absorber (PA) approximation, plane wave (PW) approximation, and interaction distance cut-off (IDC) method are all based on assuming large separation distances between WECs. For the PA approximation, each WEC is assumed to be a weak scatterer - a body whose scattered waves do not influence any other body [23, 24]. The excitation force on each WEC is the same as if each WEC were in isolation, while the waves radiated by each WEC are unaffected by the presence of all other WECs. However, all WECs can experience a force due to the radiated waves [25]. This approximation is equivalent to the iterative MS method when $S = 1$. The PW approximation, shown in Fig. 1a, refers to treating all waves incident on a WEC as being in the form of a plane wave – waves with infinitely long, parallel wave crests [26]. As shown by Table 2, the computational savings come from the number of unknowns no longer depending on M and L. This method was modified to include a correction term, which adds one more unknown at each WEC [27]. The IDC method exploits the weak coupling between widely spaced WECs by ignoring

Table 2: Comparison of interaction theory simplifications. G is the number of groups of WECs acting as fictitious bodies in the HI method. N_b is the largest number of WECs modeled at once for the MBE method. E is the scaling of the CPU time with the size of the sparse system matrix in the IDC method. r and d are the radius and separation distances of cylinders.

	PA [23, 24, 25]	**PW [26, 27]**	**HI [28, 29, 29]**	**MBE [19, 30, 31]**	**GHR [32]**	**IDC [20]**
Excitation Force CPU Time	$\propto N[(2M+1)(L+1)]^3$	$\propto [N(N-1)]^3$	$\propto [N(2M+1)/G]^3 + [G(2M+1)]^3$	$\propto \frac{N!}{N_\mathrm{b}!(N-N_\mathrm{b})!} N_\mathrm{b} S[(2M+1)(L+1)]^3$	No significant computations required	$\propto [N(2M+1)(L+1)]^E$
Radiation Force CPU Time	$\propto QN[(2M+1)(L+1)]^3$	$\propto Q[N(N-1)]^3$	$\propto Q[[N(2M+1)/G]^3 + [G(2M+1)]^3]$	$\propto \frac{N!}{N_\mathrm{b}!(N-N_\mathrm{b})!} N_\mathrm{b} QS[(2M+1)(L+1)]^3$	Unchanged	$\propto Q[N(2M+1)(L+1)]^E$
Valid For	$k_0 r < 0.8$ [33]	$k_0 d > 2$ [33]	All cases	Sufficient N_b	All cases	Sufficient d
Version of MS	Iterative ($S = 1$)	Noniterative	Noniterative	Iterative	Noniterative	Noniterative



(a) PW approximation. (b) HI method. (c) MBE method. (d) IDC method.

Figure 1: Illustrations of modifications to interaction theory.

WECs that are farther than a specified distance from a particular WEC [20], as shown in Fig. 1d. More than a 50 % reduction in the computation time can be achieved since the square matrix that must be inverted in the noniterative MS method becomes sparse [20].

The hierarchical interaction (HI) and many-body expansion (MBE) methods are based on modeling groups of WECs in the array. The HI method uses noniterative MS and divides the array into groups of WECs, as shown in Fig. 1b, which are referred to as fictitious bodies in Ref. [29]. Instead of finding the diffraction transfer matrix for each WEC individually, the diffraction transfer matrix for each fictitious body is found [28, 29]. As shown by Table 2, the computation time has a complicated relationship with the number of fictitious bodies G. Furthermore, groups of fictitious bodies can be treated as a larger fictitious body, and this process can be repeated multiple times for different levels of fictitious bodies. The MBE method instead considers all unique combinations of WECs for groups of WECs containing up to N_b WECs, as shown in Fig. 1c for $N = 4$ and $N_\mathrm{b} = 2$. Contributions to the added mass, radiation damping, and excitation force from each group of WEC are added in a series. This method has been used to train kriging [19] and artificial neural network models [30, 31]. The computational advantage of these methods comes from reducing the number of WECs modeled at once.

The generalized Haskind relation (GHR) allows the excitation force vector to be represented in terms of just the incident and radiated wave potential [32]. This completely removes the need to solve the boundary value problem associated with the scattered potential.

5 Discussion

While it may be difficult to say which method is best for all cases, some advantages are clear. Noniterative MS should be used if the order of scattering required to obtain an accurate solution with iterative MS is too large ($S > N^2$) or arbitrary WEC geometries are being modeled. Otherwise, the iterative MS will provide a faster computation time since it scales more gradually with the number of WECs.
If simplifications and enhancements to the interaction theory are used, the GHR is the best method for simplifying the computations for the excitation force and comes with no drawbacks. The PA, PW, and IDC methods are good options to consider to reduce the computation time for determining the radiation force when WECs are widely spaced. Ref. [33] compared the PA and PW approximations to the iterative MS method and found that the PA approximation is better for the long-wave regime ($k_0 r < 0.8$) while the PW approximation is better for the short-wave regime ($k_0 d > 2$), for cylindrical bodies with radius r separated by a distance d. However, the trade-off between accuracy and computation time can be chosen for the IDC method, while it is fixed for the PA and PW methods. Additionally, the PW approximation may not lead to computation savings when modeling large arrays since its computation time scales more rapidly with the number of WECs than the PA and IDC methods, as shown in Table 2. The HI method is able to simplify the computations without introducing significant errors, but the best choice of the number of fictitious bodies G and the number of levels is unclear. Finally, the MBE method gives the user flexibility to change the number of WECs N_b to adjust the trade-off between the accuracy and computation time. However, all combinations of WECs containing N_b or fewer WECs are modeled, regardless of their separation distance. That is, even weakly coupled WECs are modeled simultaneously.

6 Conclusion

This review paper highlights methods for finding the hydrodynamic forces on WEC arrays in the frequency domain under the assumption of linear potential flow. The iterative and noniterative MS methods are two ways of solving for the hydrodynamic forces. The noniterative version allows users to model arbitrary WEC geometries, while the computation time for the iterative version scales more gradually with the number of WECs. Various modifications to the MS interaction theory have also been outlined. It is clear that the GHR is the best method for simplifying the excitation force computations. To simplify the radiation force computations, the PA, PW, and IDC methods are good options to consider when WECs are widely spaced. However, the trade-off between accuracy and computation time can be chosen for the IDC method, while it is fixed for the PA and PW methods. Meanwhile, the uncertainty in the best parameter settings for the HI method makes its advantages unclear. Additionally, the MBE method gives users the flexibility to decide on the trade-off between accuracy and computation time, but groups of widely spaced WECs may be modeled unnecessarily.
Areas of future work include the following:

1. The iterative version of MS should be modified to be compatible with traditional BEM solvers, using the cylindrical surface method [22], since it would enable more flexibility in the geometry of WECs that can be optimized while still ensuring the computation time scales better with the number of WECs.
2. The MBE method can significantly reduce the total computation time since it reduces the number of WECs modeled simultaneously. However, groups of WECs are modeled regardless of their separation distance. A modified version of the MBE may be able to exploit the interaction's dependence on separation distance better if the series expansion depends on the separation distance, or contributions from neighboring WECs are included for lower-order approximations.
3. While the PA, PW, and IDC methods are the most fitting when WECs are widely separated, it is less clear how the methods in Sec. 4 compare for cases when WECs are close together. In the context of design optimization, small separation distances are of interest since the cost of the array can be reduced by sharing mooring and transmission lines. A more rigorous comparison between the methods in Sec. 4 is required to properly decide which is best in the closely spaced scenario.

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References

[1] J Fernandez-Chozas, Hans Christian Soerensen, and JP Kofoed. "Predictability and variability of wave and wind: wave and wind forecasting and diversified energy systems in the Danish North Sea". In: (2013).

[2] Michael E McCormick. *Ocean wave energy conversion*. Courier Corporation, 2013.

[3] Iraide López et al. "Review of wave energy technologies and the necessary power-equipment". In: *Renewable and Sustainable Energy Reviews* 27 (2013), pp. 413–434. ISSN: 1364-0321. DOI: https://doi.org/10.1016/j.rser.2013.07.009. URL: https://www.sciencedirect.com/science/article/pii/S1364032113004541.

[4] Francesco Fusco, Gary Nolan, and John V. Ringwood. "Variability reduction through optimal combination of wind/wave resources – An Irish case study". In: *Energy* 35.1 (2010), pp. 314–325. ISSN: 0360-5442. DOI: https://doi.org/10.1016/j.energy.2009.09.023. URL: https://www.sciencedirect.com/science/article/pii/S0360544209004095.

[5] Joakim Widén et al. "Variability assessment and forecasting of renewables: A review for solar, wind, wave and tidal resources". In: *Renewable and Sustainable Energy Reviews* 44 (2015), pp. 356–375. ISSN: 1364-0321. DOI: https://doi.org/10.1016/j.rser.2014.12.019. URL: https://www.sciencedirect.com/science/article/pii/S1364032114010715.

[6] Christina Kalogeri et al. "Assessing the European offshore wind and wave energy resource for combined exploitation". In: *Renewable Energy* 101 (2017), pp. 244–264. ISSN: 0960-1481. DOI: https://doi.org/10.1016/j.renene.2016.08.010. URL: https://www.sciencedirect.com/science/article/pii/S096014811630711X.

[7] Weixing Chen et al. "Design of the wave energy converter array to achieve constructive effects". In: *Ocean Engineering* 124 (2016), pp. 13–20. ISSN: 0029-8018. DOI: https://doi.org/10.1016/j.oceaneng.2016.07.044. URL: https://www.sciencedirect.com/science/article/pii/S0029801816302906.

[8] Vincent S Neary et al. "Methodology for design and economic analysis of marine energy conversion (MEC) technologies". In: *Sandia National Laboratories* (2014). DOI: 10.13140/RG.2.2.10201.95846.

[9] Yerai Peña-Sanchez et al. "Control co-design for wave energy farms: Optimisation of array layout and mooring configuration in a realistic wave climate". In: *Renewable Energy* 227 (2024), p. 120506. ISSN: 0960-1481. DOI: https://doi.org/10.1016/j.renene.2024.120506. URL: https://www.sciencedirect.com/science/article/pii/S0960148124005718.

[10] Matt Folley et al. "A review of numerical modelling of wave energy converter arrays". In: *International Conference on Offshore Mechanics and Arctic Engineering*. Vol. 44946. American Society of Mechanical Engineers. 2012, pp. 535–545. DOI: https://doi.org/10.1115/OMAE2012-83807.

[11] Louis Papillon, Ronan Costello, and John V Ringwood. "Boundary element and integral methods in potential flow theory: A review with a focus on wave energy applications". In: *Journal of Ocean Engineering and Marine Energy* 6.3 (2020), pp. 303–337. DOI: 10.1007/s40722-020-00175-7. URL: https://link.springer.com/article/10.1007/s40722-020-00175-7.

[12] Emmanuel B. Agamloh, Alan K. Wallace, and Annette von Jouanne. "Application of fluid–structure interaction simulation of an ocean wave energy extraction device". In: *Renewable Energy* 33.4 (2008), pp. 748–757. ISSN: 0960-1481. DOI: https://doi.org/10.1016/j.renene.2007.04.010. URL: https://www.sciencedirect.com/science/article/pii/S096014810700119X.

[13] Matthieu Ancellin. "On the Accuracy of Linear Boundary-Element-Method Sea-Keeping Codes". In: *19e Journées de l'Hydrodynamique*. 2024.

[14] M Ohkusu. "Hydrodynamic forces on multiple cylinders in waves". In: *Proceedings of international symposium on the dynamics of marine vehicles and structures in waves, 1974*. Institute of Mechanical Engineers. 1974.

[15] S.A. Mavrakos and P. Koumoutsakos. "Hydrodynamic interaction among vertical axisymmetric bodies restrained in waves". In: *Applied Ocean Research* 9.3 (1987), pp. 128–140. ISSN: 0141-1187. DOI: https://doi.org/10.1016/0141-1187(87)90017-4. URL: https://www.sciencedirect.com/science/article/pii/0141118787900174.

[16] S.A. Mavrakos. "Hydrodynamic coefficients for groups of interacting vertical axisymmetric bodies". In: *Ocean Engineering* 18.5 (1991), pp. 485–515. ISSN: 0029-8018. DOI: https://doi.org/10.1016/0029-8018(91)90027-N. URL: https://www.sciencedirect.com/science/article/pii/002980189190027N.

[17] Hiroshi Kagemoto and Dick K. P. Yue. "Interactions among multiple three-dimensional bodies in water waves: an exact algebraic method". In: *Journal of Fluid Mechanics* 166 (1986), pp. 189–209. DOI: 10.1017/S0022112086000101.

[18] J. Cameron McNatt, Vengatesan Venugopal, and David Forehand. "A novel method for deriving the diffraction transfer matrix and its application to multi-body interactions in water waves". In: *Ocean Engineering* 94 (2015), pp. 173–185. ISSN: 0029-8018. DOI: https://doi.org/10.1016/j.oceaneng.2014.11.029. URL: https://www.sciencedirect.com/science/article/pii/S0029801814004417.

[19] Jize Zhang, Alexandros A. Taflanidis, and Jeffrey T. Scruggs. "Surrogate modeling of hydrodynamic forces between multiple floating bodies through a hierarchical interaction decomposition". In: *Journal of Computational Physics* 408 (2020), p. 109298. ISSN: 0021-9991. DOI: https://doi.org/10.1016/j.jcp.2020.109298. URL: https://www.sciencedirect.com/science/article/pii/S0021999120300723.

[20] Aurélien Babarit. "Impact of long separating distances on the energy production of two interacting wave energy converters". In: *Ocean Engineering* 37.8 (2010), pp. 718–729. ISSN: 0029-8018. DOI: https://doi.org/10.1016/j.oceaneng.2010.02.002. URL: https://www.sciencedirect.com/science/article/pii/S0029801810000387.

[21] Lloyd N Trefethen and David Bau. *Numerical linear algebra*. SIAM, 2022.

[22] J. Cameron McNatt. "Cylindrical linear water waves and their application to the wave-body problem". PhD thesis. University of Edinburgh, 2016. URL: https://era.ed.ac.uk/handle/1842/20378.

[23] Johannes Falnes. "Radiation impedance matrix and optimum power absorption for interacting oscillators in surface waves". In: *Applied Ocean Research* 2.2 (1980), pp. 75–80. ISSN: 0141-1187. DOI: https://doi.org/10.1016/0141-1187(80)90032-2. URL: https://www.sciencedirect.com/science/article/pii/0141118780900322.

[24] D.V. Evans. "Maximum wave-power absorption under motion constraints". In: *Applied Ocean Research* 3.4 (1981), pp. 200–203. ISSN: 0141-1187. DOI: https://doi.org/10.1016/0141-1187(81)90063-8. URL: https://www.sciencedirect.com/science/article/pii/0141118781900638.

[25] P. McIver. "Some hydrodynamic aspects of arrays of wave-energy devices". In: *Applied Ocean Research* 16.2 (1994), pp. 61–69. ISSN: 0141-1187. DOI: https://doi.org/10.1016/0141-1187(94)90003-5. URL: https://www.sciencedirect.com/science/article/pii/0141118794900035.

[26] MJ Simon. "Multiple scattering in arrays of axisymmetric wave-energy devices. Part 1. A matrix method using a plane-wave approximation". In: *Journal of Fluid Mechanics* 120 (1982), pp. 1–25. DOI: 10.1017/S002211208200264X.

[27] P. McIver and D.V. Evans. "Approximation of wave forces on cylinder arrays". In: *Applied Ocean Research* 6.2 (1984), pp. 101–107. ISSN: 0141-1187. DOI: https://doi.org/10.1016/0141-1187(84)90047-6. URL: https://www.sciencedirect.com/science/article/pii/0141118784900476.

[28] Motohiko Murai, Hiroshi Kagemoto, and Masataka Fujino. "On the hydroelastic responses of a very large floating structure in waves". In: *Journal of marine science and technology* 4 (1999), pp. 123–153. DOI: https://doi.org/10.1007/s007730050015. URL: https://link.springer.com/article/10.1007/s007730050015.

[29] M. KASHIWAGI. "HYDRODYNAMIC INTERACTIONS AMONG A GREAT NUMBER OF COLUMNS SUPPORTING A VERY LARGE FLEXIBLE STRUCTURE". In: *Journal of Fluids and Structures* 14.7 (2000), pp. 1013–1034. ISSN: 0889-9746. DOI: https://doi.org/10.1006/jfls.2000.0306. URL: https://www.sciencedirect.com/science/article/pii/S0889974600903062.

[30] Saeed Azad and Daniel R. Herber. *Concurrent Probabilistic Control Co-Design and Layout Optimization of Wave Energy Converter Farms using Surrogate Modeling*. 2023. arXiv: 2308.06418 [eess.SY]. URL: https://arxiv.org/abs/2308.06418.

[31] Saeed Azad et al. “Concurrent geometry, control, and layout optimization of wave energy converter farms in probabilistic irregular waves using surrogate modeling”. In: *Ocean Engineering* 320 (2025), p. 120183. ISSN: 0029-8018. DOI: https://doi.org/10.1016/j.oceaneng.2024.120183. URL: https://www.sciencedirect.com/science/article/pii/S0029801824035212.

[32] Qian Zhong and Ronald W. Yeung. “Wave-body interactions among energy absorbers in a wave farm”. In: *Applied Energy* 233-234 (2019), pp. 1051–1064. ISSN: 0306-2619. DOI: https://doi.org/10.1016/j.apenergy.2018.09.131. URL: https://www.sciencedirect.com/science/article/pii/S0306261918314442.

[33] S.A. Mavrakos and P. McIver. “Comparison of methods for computing hydrodynamic characteristics of arrays of wave power devices”. In: *Applied Ocean Research* 19.5 (1997), pp. 283–291. ISSN: 0141-1187. DOI: https://doi.org/10.1016/S0141-1187(97)00029-1. URL: https://www.sciencedirect.com/science/article/pii/S0141118797000291.