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## MR. MINCHIN'S TREATISE ON STATICS

*A Treatise on Statics, with Applications to Physics.*

By G. M. Minchin, M.A. Vol. II. 3rd ed. (pp. 512 + vi.). Clarendon Press Series. (Clarendon Press, Oxon, 1886.)

THIS new edition of this work has been separated into two volumes. The first volume (351 pp.), dealing with "Equilibrium of Coplanar Forces," aims at the standard of Undergraduate Honours; it is noticeable for the frequent use of graphic methods and for a long discussion on funicular polygons (now forming so important a help in graphic applications to engineering); this was published in 1884. The second and longer volume is a masterpiece of constructive skill in the adaptation of modern methods; it is particularly noticeable for the introduction of the theory of screws and of astatic equilibrium, also for an extensive selection of excellent examples, and for the free use of hyperbolic and elliptic functions in solutions: the reading required is thus considerable; it is, in fact, intended for those who seek Honours. The work is so much improved in this edition that it merits an extended notice. The second volume is divided into only seven chapters, each of which is an essay on its special subject. The numbering of chapters and articles is continuous with Vol. I., whilst the pagination is distinct.

Chapter XIII. (the leading chapter, 64 pp.) deals with Non-Coplanar Forces, and contains the usual propositions (16 pp.) about compounding and resolving forces and couples, about resultants, equilibrium, and central axis: then follow (48 pp.) the theory of screws, cylindroids, complexes, and degrees of freedom; the constructions given for the cylindroid are neat: in one the surface is traced by the blades of a pair of scissors, which open horizontally at a uniform rate, whilst the rivet falls vertically; this gives a vivid idea of the surface.

Chapter XIV. (34 pp.) treats of Astatic Equilibrium, which is defined to be a balance amongst forces of fixed magnitudes and directions at definite points of a body subject to displacement. This is treated by quaternions, the Cartesian method being found cumbersome. It is shown that a system of forces can always be astatically balanced by a set of *three* forces in any given directions (and even by three equal rectangular forces) applied at three points lying in a plane fixed in the body, and also by two forces if these points lie in a line, or by one force if they coincide (the general proof of all this is easy by elementary methods). This subject has some practical application in electrical measurements, for which an astatic magnet-pair is much used, and in seismometry, for which it has been sought to make pendulums astatic for small displacements (see Milne's new work on "Earth-quakes," p. 26).

Chapter XV. (91 pp.) treats of Virtual Work. The term "work-coefficient" has here been with great advantage introduced to replace the lengthy and *incorrect* term "generalised component of force." Lagrange's method is treated of at length: its advantage is shown to

consist in reducing all problems to the case wherein the displacements are independent, by introducing internal forces to represent the constraints. One disadvantage is its undue length, most marked in simple cases. Another is a decided risk of error in estimating the work of the internal forces; instances of error due to this in Lagrange's researches are shown, *e.g.* the cases (1) of an inextensible surface wherein Lagrange assumes (incorrectly) that  $\delta dS = 0$  fully expresses the inextensibility; and (2) of an extensible surface wherein he assumes (incorrectly) the work of internal deformation to be simply proportional to  $dS$ ; and (3) of an elastic wire wherein Lagrange overlooks the distortion. A brief summary of Jellett's researches on inextensible surfaces is given, and it is shown that such a surface is quite determinate (and therefore not deformable) if any bounding edge of it be fixed, except it be antitlastic or developable, which latter admit of deformation when certain edges only on them are fixed. The surface-tensions of liquid-films are investigated (12 pp.), and the experimental way of producing several such forms is given, and their stability discussed.

Chapter XVI. (45 pp.). On Strings and Springs.—The properties of strings in general, also on rough and smooth surfaces, are discussed, with some cases of the extensible string; next those of plane elastic rods and plane springs; lastly, those of a twisted wire (20 pp.): this last is important in electrometers. The interesting *kinetic* analogies are shown (1) of a plane elastic rod with the simple pendulum, and (2) of a bent and twisted uniform wire with a heavy mass moving about a fixed point, *viz.* that the differential equations in the analogous problems are similar.

Chapter XVII. (123 pp.), on Attraction, is divided into four sections.

Sec. I. (29 pp.). On Attraction in General.—It is explained that the law of gravitation implies that the attracting particles must be very small compared with their distance. Notice is most usefully drawn to this limitation several times in the sequel, *e.g.* it is shown that the Cartesian expressions *seem* to give indeterminate attraction for very close points; also that for attractions more rapid than  $1/r^2$  the attraction on an internal point is really infinite.

Sec. II. (40 pp.). On Potential.—In the definition the usual idea of motion from infinite distance has been dropped, and the definition runs as the work done in bringing a tiny mass from a position of zero attraction, &c. (not from infinity): this is much better. The continuity of the gravity-potential and of its first derivatives, the discontinuity of its second derivatives, the absence of maxima or minima thereof in empty space, and the instability of equilibrium under gravitation to several masses are shown. The application of the method of inversion is given; and, amongst many examples, Thomson's solution of the attraction of a spherical shell whose density  $\propto (\text{distance})^3$ .

Sec. III. (13 pp.). On Ellipsoids.—After the usual investigation of their attraction, it is shown that the surfaces of prolate and oblate spheroids are not equipotential: various problems interesting in the figure of the earth are given.

Sec. IV. (42 pp.). Spherical Harmonics.—Green's equa-

tion is deduced and its consequences investigated, especially in helping to find potential. Spherical Harmonics occupy the next thirty-two pages. The very convenient and appropriate name "Laplacian" is here assigned to the important "Laplace's coefficients": by analogy the name "Legendrian" might well be applied to Legendre's coefficients; short terms of this kind are useful, and commemorate the inventors. The usual developments are given; the applications to symmetric bodies are interesting, *e.g.* a potential function (*i.e.* one such that  $\nabla^2 v = 0$ ), which is the potential of a symmetric body for all points on its axis, is *the* potential of the body.

Chapter XVIII. (103 pp.), on Small Strains and Stresses, is divided into three sections.

Sec. I. (32 pp.). Small Strains.—This treats of the *small* strains (changes of shape or size) of a body without reference to their causes. It is shown that straight lines, planes, and parallels remain such, whilst spheres become ellipsoids, &c., and there is always one line of no rotation at every point. It is also shown that every strain may be resolved into a pure strain and a rotation, and that the strain proper may be produced by three elongations, or by one elongation and a contraction all round an axis (this is called traction). Torsion is shown to be equivalent to shear, and shear to be equivalent to an extension and contraction, &c.

Sec. II. (22 pp.). Stress.—This treats of internal stress apart from concomitant strain. The usual composition and resolution are investigated, the work of an actual strain and the virtual work of virtual strain are found, and the latter is shown to be an exact differential.

Sec. III. (49 pp.). Stress and Strain.—The relations between the moduli of compression ( $k$ ) and distortion ( $\mu$ ), the contraction-coefficient ( $\eta$ ), and Young's modulus ( $E$ ) are first traced for isotropic bodies, and the strain- and stress-potentials found for the same, and it is shown that every force-system produces definite strain. The work in pure compression and pure twisting is investigated, and it is shown that twisting couples applied at ends of a cylinder produce pure torsion only in a circular cylinder, so that in other cases the plane sections are deformed. The theory of the slightly bent plane beam is investigated as far as the theorem of three moments. In heterotropic bodies it is shown the conservation of energy reduces the number of independent elasticity-coefficients from thirty-six to twenty-one. St.-Venant's reduction to fifteen for cases where the mutual action of two particles is independent of other particles is discussed, and is shown to lead to the value  $\eta = \frac{1}{4}$  for the lateral contraction-coefficient of an isotropic body. Maxwell's researches on the propagation of gravitation are reproduced, and are described as showing that gravitation could be produced by a certain stress over a closed surface propagated through an all-pervading medium (ether) transferring strain like a solid, but further research shows that this ether is not quasi-solid.

Chapter XIX. (45 pp.). Electrostatics.—After the usual elementary propositions it is shown that a "line of force" meeting an electrified conductor obliquely is refracted, and that the charge-distribution over an isolated body is determinate: this leads to interesting problems in soap-bubbles. It is shown from Green's equation that a

hollow conductor screens its contents from outer electric disturbance; this has a practical application in protection of delicate instruments inside a metallic cage. Lastly, the theory of electric images is discussed, and examples given.

From the full analysis given it will be seen that the work is a most important one: it is, in fact, one of the best treatises of the day.

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### THE CRUISE OF THE "BACCHANTE"

*The Cruise of Her Majesty's Ship "Bacchante," 1879* 82.

Compiled from the Private Journals, Letters, and Note-books of Prince Albert Victor and Prince George of Wales, with Additions by John N. Dalton. Two Vols. (London: Macmillan and Co., 1886.)

TO us the chief interest of these two bulky volumes lies in the fact that they are the record of what we may call the technical education of our future King and his brother. It was a right and proper thing for the Prince of Wales to do to see that his sons should become personally acquainted with the leading sections of that great Empire with the conduct of which they will in the future have so much to do. Indeed, in these times, when our colonies are coming so conspicuously to the front, when their affairs are regarded as of Imperial importance, it might be a good thing to insist that our Colonial Secretaries should follow the princes' example, and that no one should be considered qualified for the post of Minister for the Colonies who had not studied their affairs on the spot. Technical education is considered essential nowadays to any one occupying a responsible position in even the humblest of callings; but we fear that statesmanship is still beyond the pale of science.

In the volumes before us Canon Dalton has the lion's share. The princes' contributions have been edited by him from their diaries, note-books, and letters; while he himself contributes long sections in which he brings together much useful information, and discussions on the affairs of the various colonies visited. Of course the writings attributed to the princes are no doubt much indebted to the superintendence of their tutor; at the same time the boyish hands can be traced throughout. The whole work is creditable both to the princes and to Canon Dalton. They certainly worked hard both at their books and at their duties as middies; for in all respects when on board ship they were treated precisely as their mates. They evidently took a genuine interest in their duties on board; took a pride in mastering all the details of navigation and the working of a war-ship like the *Bacchante*; were as eager to pass their examinations as if their future careers depended on the result. Much of their share of the work consists of details as to the day's cruise, their own work as officers, the exercises proper to such a ship, and the incidents of the gun-room. Mixed up with this are the results of their own observations in the countries visited, information gathered during their visits or from books, their experiences when sojourning in the colonies, in Japan and other countries, with occasional reflections suggested by all this. Canon Dalton's contributions are more solid and serious. He enters into