THE THEORY OF PROGRESSIVE TAXATION.

I. Fixation of the Problem.

THE discussion on progressive taxation has suffered much from the fact that the actual question in point has not been stated with sufficient precision. Confusion has specially been caused by the mixing together of two different questions, namely, the question of the progression of the *income-tax*, and the question whether the *whole burden of taxes* ought to increase progressively with the income.

Moreover, neither of these questions is sufficiently determined to be the subject of a really scientific discussion. It cannot *a priori* be decided whether the income-tax ought to be calculated at a proportional or at a progressive rate, this depends first of all upon the position which is given to the income-tax in the system of taxation. If, for instance, the income-tax is to be an atonement for all wrongs caused by the indirect taxation, then, considering the present condition of most countries, a progression of a fairly strong character will be necessary.

The second problem is quite as indefinite, namely, to decide in what proportion the total burden of the taxes ought to increase. In my opinion this question cannot be decided without first considering the objects for which the taxes are levied. It can, for instance, be said that taxes on liquor, which are levied for the current expenses of the State, press on the poorer classes much more than is justifiable. But from this it does not at all follow that a high tax on liquor is always unjust. If, for instance, such a tax is used to supply a sick fund for workmen, or to combat the tubercular disease among the lower classes, then it seems to me to be justifiable, quite independently of the question whether the total burden of the taxes of the working classes is thereby increased over and above the measure which would be fixed by a strict proportion to the taxation of the higher classes. Thus no general scheme for the distribution of the total burden

of taxes can be found. In order to arrive at a really definite problem, it is necessary to leave out of consideration all taxes which are in the special interest of any given class of society; and to fix one's thoughts wholly on those expenses which, being necessary for the State, yet are not in the interest of any special group of citizens. These *general* expenses must clearly be covered according to the *principle of ability*. The income-tax is here of the utmost importance. It is not to be imagined that the income in itself is an absolute measure of individual ability; on the contrary, it is quite certain that one must take into consideration, for instance, the size of the family, the best way probably being to grant certain deductions for each child or other person whom the taxpayer is obliged to support. It is also certain that the income-tax must be supplemented in several respects, particularly by a property-tax of some kind or other. But the income-tax is in any case the key-stone in the system of taxes, which ought to cover the *general* expenses of the State.

The question is, therefore, whether an income-tax which is to fulfil this purpose ought to be made progressive. In order to grasp this problem it must be noted that this income-tax must touch *all groups of society with equal force*, because of the very position which we have given to it in the system of taxes. This is the only way available in a modern State to prevent one class of society from voting lavishly general State expenses, knowing that it will in any case be other classes who will have to pay the greater part of them. It is purely this reason of a practical political nature, characteristic of the parliamentary state, which makes us build the income-tax on the principle of *equal sacrifice.* The problem of the progressive taxation is thus reduced to the following form: " *Which is the proper scale of an income-tax which is intended to cause an equal sacrifice to all citizens?* " It is this problem we here want to discuss.

II. Determination of the tax-rate which produces equal sacrifice.

Those who have wished to deduce progressive taxation from the equal sacrifice principle, have generally tried to lay stress on the fact that £1 is a greater sacrifice for a person who has an income of £100, than £10 for one who has an income of £1,000. Fully valid reasons can be brought forward for this statement. If £1 be taken from a family which has an income of £100, this family will be deprived of its means of purchasing very important necessaries. But if an income of £1,000 is

diminished by £10, then it means as a rule only a sacrifice of very insignificant luxuries. The validity of this reasoning is most evident if one brings into the comparison an income which lies on the very margin of what is absolutely necessary for subsistence. A tax of 1 per cent. is for such an income not only very much heavier than a corresponding tax on a larger income, but the sacrifices which are necessary in such a case are, as Mill says, quite incommensurable. Like Bentham, Mill derives from this the necessity of a tax-free minimum of subsistence, and with quite logical consistency, he concludes that this minimum shall be deducted from *all* incomes, so that only those sums are to be taxed by which the different incomes exceed the minimum of subsistence. It is curious that political economists have not yet been able to agree about a thing which seems so evident from a logical point of view, and which would besides afford such great facilities from a purely technical aspect.

Mill refuses to make any further concessions to the defenders of the progressive taxation. That a tax of the same percentage for large and small incomes (the existence-minimum having been deducted from all of them) would cause the lower incomes a greater sacrifice, seems to Mill " too disputable altogether, and even if true at all, not true to a sufficient extent, to be made the foundation of any rule of taxation." [1] And, in fact, these questions, as they are generally brought forward, are certainly too indefinite to serve as the foundation of any scientific discussion.

But it seems to me that the analysis can be carried out in a more profound and consistent manner on the line that Mill has indicated. The reason for making a deduction is that a tax which interferes with the necessaries of some classes of society cannot fulfil the conditions of " the equal sacrifice." But it is quite impossible to state any fixed sum which would always correspond to these necessaries. On the contrary it is certain that the necessaries of life of the higher classes of society are on the average considerably greater than those of the lower classes. If one once grants that a tax which is to produce an equal sacrifice must not take away any of those means which are needed to cover essential wants, then for the sake of consistency the deductions must be made greater for the higher classes of society. It is simply impossible for a professional man to live as cheaply as a common miner. He has outlays for books, paper and corre-

[1] *Principles of Political Economy*, V., II., 3.

spondence, and if he has a family he cannot live and work in only one room. He cannot, in one word, discharge his function in society, and, economically speaking, continue to exist as the same person, if he is reduced to a standard that can be considered as a fair minimum for a common labourer. Such a reduction means, therefore, to him a sacrifice of much the same nature as that of the labourer deprived of a certain part of the necessities for physical existence. Then the professional man, deprived of the possibility of carrying on his profession, will probably not be able to earn anything in a lower grade of work. For some expenses of the kind mentioned above tax-legislation can grant direct deductions; as, for instance, for an artisan's shop-rent or other business expenses. But a number of expenses remain, nevertheless, which are so nearly connected with the personal consumption of the taxpayer, that there can be no question of deducting them as expenses. There is thus no other course open than to confess the simple truth, that the necessaries of life are actually, in the present state of society, more numerous for the higher classes. To make this statement more definite we can say that the income which is necessary for a person's economical existence increases on an average with the total real income, but naturally more slowly than this. In fact there would probably be no one, of whatever class or party, who would deny this truth if he were to judge it by itself. But if that is the case one could expect that it would be recognised also in the theory of taxation, quite independently of the consequences to which it might lead.

We are, however, not yet so free from prejudice. The reason why most people are unwilling to grant tax-free deductions to the higher incomes, is probably the widely spread belief that the higher classes would necessarily be specially favoured thereby. But a closer examination proves that this supposition is quite false. If the untaxed deduction increases more slowly than the income, then clearly the remainder increases more rapidly than the income. A proportional tax levied on the remainder is then a *progressive* tax on the total income. Moreover, as we shall see presently, it is possible, by allowing such deductions and taxing the remainders at a constant rate, to obtain as strong a progression as we please.

To recapitulate : The Principle of Equal Sacrifice leads us to deduct from all incomes certain necessaries and to tax the remainders at a constant percentage. These necessaries are not the

necessaries of merely physical subsistence; but are, to use the language of Prof. Marshall,[1] " *the necessaries of efficiency.*" Then as soon as taxation would interfere with the efficiency of any special class of the community, the sacrifice of this class would be, in Mill's terms, quite incommensurable with that of the other classes, taxed only out of their surplus income above the margin of efficiency. Thus " *Equal Sacrifice*" *means deduction of the Necessaries of Efficiency and a proportional tax on the remainders.*

The question how to construct an income-tax, meant to produce an equal sacrifice, is thus reduced to the problem : What are the average necessaries of efficiency in the different grades of income ? We know already that these necessaries *increase* with the incomes, but more slowly, and, consequently, that we, by deducting them and taxing the remainders at a *uniform* rate, will arrive at a *progressive* tax.

However, before we enter upon the technical details of the construction of a progressive scale on these grounds, we must glance at the technics of progressive taxation, as far as it has been developed hitherto.

III. Criticism of the ordinary methods of Progressive Taxation.

According to the popular scheme for progressive taxation, the incomes are divided into different groups, and each group has a fixed tax-rate. It is this method which has given to progressive taxation the character of complete arbitrariness which makes it unacceptable to most right-minded people. It is difficult to get any real hold of this problem. The division of the incomes into separate groups, as well as the tax-rate which each group ought to pay, can be varied *ad infinitum*, and particularly there is no limit to what may be required from the higher incomes.

Besides, the method—if indeed it can be called a method—is very incomplete from a technical point of view. This can best be shown by an illustration. Let us suppose that the following scale has been chosen :

Incomes from		0s. to	500s.	pay 0 per cent.
,,	,,	500s. ,,	2,500s.	,, 1 ,,
,,	,,	2,500s. ,,	8,500s.	,, 2 ,,
,,	,,	8,500s. ,,	20,500s.	,, 3 ,,
,,	above	20,500s.		,, 4 ,,

[1] *Principles of Economics*, Vol. I., book II., ch. III., § 3 ; 4th Ed., p. 137.

THE ECONOMIC JOURNAL

Then at every rung of the ladder there will be a discontinuity of the taxes. When the income exceeds the limit of 500s., the tax is suddenly increased by 5s. This is clearly unjust. At the margin of 2,500s. the tax is doubled, although the income may have been but slightly increased. In scientific and parliamentary debates, as well as in legislation bills it has often been urged that this weakness is an outcome of the very nature of progressive taxation, and cannot be avoided. But there is really nothing more simple than to arrange so as to avoid these discontinuities. To do this we have only to imagine the different shillings of which the income consists, as arranged in a certain order and set down that:

For each one of the first   500s.   0 per cent. is paid.
 ,,   ,,   next   2,000s.   1   ,,   ,,
 ,,   ,,   ,,   6,000s.   2   ,,   are paid.
 ,,   ,,   ,,   12,000s.   3   ,,   ,,
 ,,   ,,   following shillings 4   ,,   ,,

It would be an important progress if this simple reform in the technique of taxation could be agreed upon. In this manner every discontinuity is avoided, and the danger of the whole income being at the end swallowed up by an unlimited progression is eliminated. Then even if the last shillings of a very large income should happen to be taxed at an unreasonably high percentage, yet will the preceding ones thereby be left intact and be dealt with just as if they had been parts of a smaller income.

Now, it is easily seen that every continuous scale of this nature can be obtained just as well by the method of deductions from the real income. In the above supposed case we should have to allow the following deductions:

For each of the first   500s. a deduction of 100 per cent.
 ,,   ,,   following   2,000s.   ,,   75   ,,
 ,,   ,,   ,,   6,000s.   ,,   50   ,,
 ,,   ,,   ,,   12,000s.   ,,   25   ,,
For all following shillings   ,,   0   ,,

and then tax the remainders at a common rate of 4 per cent.

Or, to state it generally, if the different groups are to pay a tax of $p_1, p_2 \ldots p_n$ per cent., *the same result* can always be attained by levying the tax according to a common rate P, which

is not less than any p. But one must at the same time grant a deduction in each group of :—

$$\frac{100\ (P-p_1)}{P} \text{ per cent.}, \quad \frac{100\ (P-p_2)}{P} \text{ per cent., \&c.}^1$$

In the theory of taxation, attempts have been made to distinguish between progressive and degressive taxes. The latter expression has generally been used to denote a tax which starts with a fixed percentage; with which, however, certain deductions have been granted to the lower incomes. We see now that this difference is quite fictitious ; in fact, one can by means of the system of deductions, produce *as strong a progression* as one likes. It is, in fact, in the granting of untaxed deductions which are larger for the higher incomes and then imposing a proportionate tax that the only really rational method of constructing progressive scales of taxation is to be found.

Consequently we need not discuss, in the theory of progressive taxation, any other question than what deductions shall be granted to the different incomes. Within the limits thus given to the discussion, all materially different scales of progression can be advocated. Now I hold that these deductions must be computed in accordance with the Principle of Equal Sacrifice, interpreted in the way we have done above ; and therefore a material and, indeed, very necessary restriction is laid upon the infinite arbitrariness of the general conception of progressive taxation. I think that no valid objection can be raised against this proposition ; but, of course, the views will diverge considerably when it comes to the arithmetical determination of the necessaries of efficiency of the different classes of society. If we allow very high margins for the fullest efficiency of the manual labourers and for the lower middle-class, we will get a very strong progression. On the other side, if we, as many conservative people do, think that the lower classes need nothing more than they have, indeed, cannot make any good use of more, we should perhaps limit their deductions to the barest minimum of physical subsistence and thus come to a nearly proportional scale. We see thus, that the two principles which struggle with one another in the battle on progressive taxation, the democratic and the plutocratic principle, are still acting with full force on the limited field we have allowed for the

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¹ It is easily seen, in the same way, that generally *every* progressive scale—even a discontinuous one—can be obtained through allowing suitable deductions to the different incomes, and taxing the remainders at a constant rate.

discussion. But they are deprived of their usual arbitrariness and vagueness, and the problem of progressive taxation is brought into immediate connexion with the general views of the end and the means of the progress of the human race taken up by the different parties.

The English income-tax is generally considered as the type of a degressive tax. Opponents to progressive taxation regard England as the country from which to get the model of a true income-tax. On the other hand, in Germany, where progression is generally carried out, people pride themselves on being far in advance of England in development, since England has not yet ventured to leave the primitive stage of degression. We can easily realise the absurdity of such talk, by drawing the curve which denotes the English income-tax, as it has been during the last years, and on the same paper the curves of some of the income-taxes, which have been suggested, or are in use on the Continent. It will then be seen that the English curve begins below the others, and ends in exceeding many of them. And if we look at the English income-tax since the last increase to 14*d.* in the pound, we must admit that England now occupies the leading position in Europe as regards a strongly progres-sive taxation,—and this in spite of her so-called " degressive " system.

IV. Technique of the Progressive Taxation.

It is of course a crude method to let the untaxed deductions increase in the way we have indicated in our illustration. This method is also unsuitable from a practical point of view ; because as yet it gives room to too much arbitrariness, and it affords but little guidance to the fixing of the amount to be deducted from the different groups of incomes. In a progression founded on a really scientific basis, the deduction must increase as a continual function of the income. This function ought, moreover, to be chosen as simple as is consistent with the freedom necessary to its adjustment to varying circumstances and to the needs of different countries. A linear fractional function seems the best for the purpose. If we suppose the income to be x, and the untaxed deduction to be y, then we have :

$$y = \frac{ax + \beta}{\gamma x + \delta}.$$

This function contains three independent constants. The con-stant tax percentage, P, must also be taken into consideration, so

that we have really four constants at our disposal, in order to adjust the rate of taxation to existing circumstances. The tax s must be calculated according to the formula :

$$s = \frac{P}{100}(x - y).$$

In order to obtain a continual tax-rate which corresponds as much as possible to that given in our illustration, we must make

$$s = \frac{4}{100}\left[x - \frac{14000\,x + 1000000}{x + 15500} \right].$$

The taxes calculated by means of this formula correspond to those which were obtained by means of the former method for the incomes 500s., 2,500s., 8,500s., and 20,500s., amounting in percentage of the whole incomes respectively to 0 per cent., 0·8 per cent., 1·65 per cent., and 2·44 per cent. The rate of percentage increases for the higher incomes, yet without ever reaching 4 per cent.

In order to make the general formula practically useful, it is necessary to write it in such a form that the meaning of the constants becomes quite clear. For this purpose it is best to introduce the untaxed minimum of existence, e, as a unit, and accordingly write :

$$x = eu \; ; \; y = ev,$$

where u and v denote the income and the deduction respectively, expressed in the unit e. We further introduce a new term, the *maximum of subsistence*, by which we understand the upper limit of the untaxed deduction which is granted to any income.[1] This maximum of subsistence we denote by em. The untaxed deduction, which lies midway between the maximum and the minimum, we might call *medium of subsistence*, and denote by $e\frac{m + 1}{2}$.

Now, according to this agreement as to the signification of our letters, v ought to be a linear function of the income u. We write this function thus :

$$v = m - (m - 1)\frac{n - 1}{u + n - 2}.$$

We see that when $u = 1$, then $v = 1$; when u is infinite $v = m$; when $u = n$, then $v = \frac{m + 1}{2}$; or in other terms, when $x = e$,

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[1] Thus the terms "minimum and maximum of subsistence" are to be understood as "minimum and maximum of necessaries of efficiency."

then $y = e$; when x is infinite $y = em.$; when $x = en$, then $y = e\dfrac{m + 1}{2}$. That is to say that when the income is equal to the minimum of subsistence, then the deduction is equally large, so that the taxable income is *nil*, as it ought to be. Further, when the income increases to a certain point en, then the deduction is equal to he medium of subsistence. Finally, the maximum of subsistence is the upper limit, towards which deductions grow, when the income grows *ad infinitum*.

In order to make the tax-rate quite definite, it is thus necessary to fix the following *four* elements; (1) the minimum of subsistence; (2) the maximum of subsistence; (3) the income which is to enjoy a deduction equal to the medium of subsistence; (4) the constant tax-percentage.

It seems, nevertheless, unnecessary to have so many elements at our disposal. The number can be further reduced by *one*, by deciding once for all that the income at which the deduction is equal to the medium of subsistence must be equal to twice this medium. That is to say, that when the deduction has reached the medium of subsistence, it amounts to half the income. This decision touches really but the form of the tax-rate, not the limits between which it moves. And it is always possible to obtain a good and even progression by disposing of the three remaining constants in a suitable way. Thus the arbitrariness which attends a progressive taxation is reduced to a minimum; at the same time a freedom of adjusting details fully sufficient for all conditions, is guaranteed.

By applying this simplification we can put $n = m + 1$, and thus we obtain:

$$v = m - \frac{m(m - 1)}{u + m - 1}.$$

We must add to this formula:

$$x = eu, \; y = ev,$$

and

$$s = \frac{P}{100} (x - y).$$

For instance, if we suppose the minimum of subsistence in Great Britain to be £100, the maximum to be £600, and the tax-rate to be 8 per cent., then the tax must be calculated according to the formula:

$$s = 8 (u - v)$$
$$= 8u \frac{u - 1}{u + 5},$$

where u means the income, reckoned in hundreds of pounds. We see that the tax-rate, starting from *nil* at the income of £100, rises continually to 4 per cent. at the income of £700, to 6·4 per cent. at £2,500, to 7·4 per cent. at £7,500, and so on without ever reaching the rate of 8 per cent. But such a tax would probably yield less than the actual English income-tax.

Thus to construct a progressive scale of taxation we have first to come to an agreement as to the necessaries of efficiency in the different grades of incomes. This point once settled, we must deduct these necessaries from the real incomes and see how much remains at the disposal of the Chancellor of the Exchequer. To calculate this aggregate taxable sum we must have complete statistics of the actual distribution of incomes in Great Britain. Finally, we must divide the so calculated tax-capital into the sum which the income-tax is supposed to yield in the current year : the quotient is the constant tax-percentage P to be paid out of all the taxable incomes.

We see then that for the complete determination of the pro   tax it is only required to fix three elements, namely,
          inimum of subsistence, e.
2. *The maximum of subsistence, e m.*
3. *The tax-percentage, P.*

The first and last of these three elements must be determined, whatever tax-rate is chosen. Only one new determining element, the maximum of subsistence is added in the case of a progressive rate, such as it has here been suggested. But this element has quite a definite meaning, and it is comparatively easy, with some knowledge of existing conditions in the country, to find a fairly true standard of this maximum of subsistence. It must be equivalent to the sum which constitutes for the higher classes a real minimum of subsistence, below which they cannot possibly go without failing in the fulfilment of their essential duties.

The theory of progressive taxation seems herewith reduced to its simplest possible form ; and at the same time it may be affirmed that no country is in need of a tax-rate which does not allow itself to be included in the form given above.

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