



XLII. On the solid of greatest attraction

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glass stool; they imagine that something formidable is about to happen to them, when perhaps they have only to have the electric stream drawn from the affected parts with a wooden point. When the multiplicity of apparatus required in the usual way to go through a series of experiments, and when the expensive and brittle nature of the principal materials, as well as their being often used in the dark, are considered, I flatter myself that the simplicity and advantage of the construction here recommended, will be apparent to every practical electrician; at any rate a machine so constructed will be found a very convenient "working tool."

I am, Gentlemen, yours, &c.

Dereham, Norfolk, July 15, 1830.

G. DAKIN.

XLII. *On the Solid of greatest Attraction.* By SAMUEL SHARPE, Esq.*

TO determine the form of the solid which attracts a body on the surface with the greatest force.

The attraction of each particle of the solid must be resolved into two parts, and that part rejected which is not towards the centre of gravity of the solid; and the solid is such that every point of its surface attracts the body equally towards the centre; otherwise the sum of the attractions might be increased by removing that portion which attracts least, and placing it beside that which attracts most.

Let A (fig. 1.) the body attracted, be a point on the surface of the solid.

A B ($= a$) the diameter through A and the centre of gravity.

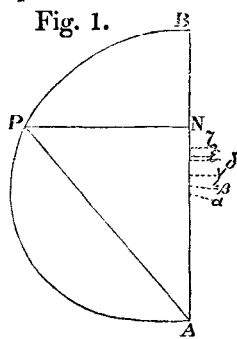
A P ($= c$) the chord joining A and any attracting particle P.

Then letting fall PN ($= y$) perpendicular on A B, A N ($= x$) will be the corresponding abscissa.

The attraction of the particle at B $= \frac{1}{a^2}$ and of P $= \frac{1}{c^2}$, but in the direction of the diameter (this latter) $= \frac{x}{c^2}$, and

therefore, $\frac{c^2}{x} = a^2$, and the equation of the curve $(x^2 + y^2)^{\frac{1}{2}} = x a^2$ Q.—E. I.

The curve may be easily drawn thus: make $z = \frac{c^2}{x}$; then



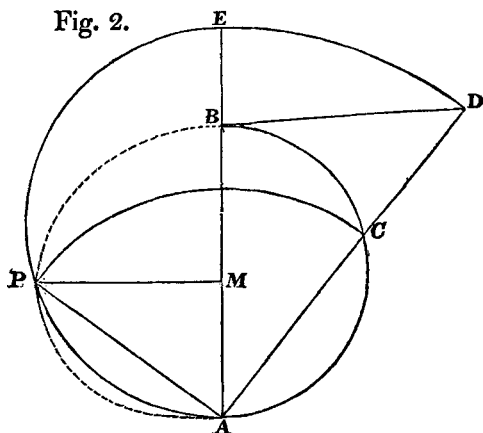
* Communicated by the Author.

$$a^2 =$$

$a^2 = cz$, and $c : a :: a : z$ and $z : c :: c : x$; from the first proportion z may be drawn, and from the second x .

On the diameter AB (fig. 2.) draw a semicircle ACB , and from A draw the chord AC produced till it meet a perpendicular from B at D ; then if AC be $= c$, $AD = z$. On the diameter produced make $AE = AD$; on AE describe a semicircle APE , and with the centre A and radius AC describe a circle cutting APE in P ; P will be a point in the curve required: for letting fall PM on AB ,

Fig. 2.



$$AP \text{ or } AC : AB :: AB : AD, \text{ and} \\ AD \text{ or } AE : AP :: AP : AM;$$

and if several other points P be thus laid down, the curve will be drawn. Q. E. D.

2. The solid is obviously made by the revolution of the area APB on the axis AB : now to measure that area,

$y x$ the fluxion of that area $= a^{\frac{4}{3}} - x^{\frac{4}{3}} \cdot \frac{1}{2} x^{\frac{1}{3}} x$ whose
fluent corrected $= \frac{a^{\frac{4}{3}} - x^{\frac{4}{3}} \cdot \frac{3}{2} - a^2}{2}$ which when $x = a$ becomes $\frac{a^2}{2} =$ the area $APBM$.

3. To determine the solid contents; let $p = 3.14159$ &c. the area of a circle whose radius is unity; then py^2x is the fluxion of the solid whose fluent is $\frac{5}{2} p a^{\frac{4}{3}} x^{\frac{5}{3}} - \frac{1}{3} p x^3$ which when x becomes a is $= \frac{4 p a^2}{15} =$ the whole solid contents.

4. Now since the contents of a sphere with radius b is $= \frac{p b^3}{6}$, by making $\frac{p b^3}{6} = \frac{4 p a^2}{15}$ we obtain $a = b \frac{855}{1000}$ or $b = a \frac{1170}{1000}$, being the proportion between the axis of this solid and a sphere of equal contents.

5. The attraction of the solid to the point A is the fluent of $2p \sqrt{x - \frac{xx}{c}} = 2px - \frac{6pq - \frac{4}{5}x^{\frac{5}{2}}}{5}$ which when x becomes a , is $= \frac{4pa}{5}$.

6. And since the attraction of a sphere to a point on its surface $= \frac{2pb}{3}$, which with equal contents $= \frac{2p}{3} \times \frac{1000a}{855}$ the attraction of this solid to that of a sphere is as 1026 : 1000.

7. Again, since the attraction of this solid $= \frac{4pa}{5}$, and of a sphere $\frac{2pb}{3}$ if we make their attraction equal $\frac{b}{2} = a \frac{6}{10}$.

8. Let d be the distance from A of a point on the axis, into which the whole solid might be concentrated without altering its attraction on A, then the attraction which $= \frac{4pa}{5}$ also $= \frac{4pa^3}{15} \times \frac{1}{d^2}$, and $d = a \frac{577}{1000}$, which in a sphere $=$ radius $= a \frac{585}{1000}$.

9. The distance of its centre of gravity from A is $=$
$$\frac{\text{fluent of } \frac{1}{2}py^2xx}{\text{fluent of } \frac{1}{2}py^2x} = \frac{\frac{3}{8}a^{\frac{4}{3}}x^{\frac{8}{3}} - \frac{1}{4}x^4}{\frac{2}{3}a^{\frac{4}{3}}x^{\frac{5}{3}} - \frac{1}{4}x^3} = (\text{when } x \text{ is } = a) a \frac{468}{1000}.$$

10. The ordinate is a maximum when the fluxion of $y^3 = 0 = \frac{2a^{\frac{4}{3}}x}{3x^{\frac{1}{3}}} - 2xx$: hence $x = a \frac{439}{1000}$ when y is a maximum.

11. Hence we obtain the following very curious points on the axis of the solid (fig. 1.), with their distances from A (a being 1000).

α . 439. The place at which its ordinate is a maximum.

β . 468. The centre of gravity.

γ . 500. The centre of the axis.

δ . 577. The point into which the whole contents might be concentrated without altering the attraction on A.

ϵ . 585. The centre of a sphere of equal solid contents (ϵ A being its radius).

ζ . 600. The centre of a sphere with equal attraction on A, a point on its circumference.

Canonbury, May 11, 1830.

SAMUEL SHARPE.