section on reproduction which takes in the mechanism of pollination, asexual reproduction by means of bulbils, experiments on regeneration, on the behaviour of potato tubers, and on grafting. The instructions are well arranged, and they form, with accessory explanations, a fairly continuous whole. A useful appendix is added, in which the needful outfit in apparatus and reagents is given, together with hints on laboratory methods. The book is intended partly for the "cultivated layman" and partly for the students of the Gymnasium and Realschule. It will, however, prove useful to the teachers in English universities, as well as to others who have discovered the wisdom of making even advanced students perform for themselves elementary experiments.

We are inclined to think that the cultivated layman will be frightened by the first twenty pages of the book, which contain a large number of rough qualitative estimations of the chemical compounds occurring in plants. This is excellent for the laboratory, but is hardly readable by one who does not repeat the experiments—and we cannot imagine the cultivated layman working his way through them. This, however, is not the fault of the authors, and it is only fair to say that the book in general is far from being unreadable.

In a future edition the authors would be well advised to give scientific names, if only for the sake of foreign readers, who cannot be supposed to know what plants are meant by *Sommerwurz* or *Mauerpfeffer*. In some few cases the instructions want a little re-editing. Thus, in exp. 123, p. 82, the student is directed to compare the assimilation of a withered leaf with that of a fresh one, but he is not told that the absence of assimilation in the withered leaf is due to the closure of its stomata. The experiment is, in fact, incomplete; what is missing is a repetition of Stahl's proof that the leaves of certain plants the stomata of which do not close on withering are capable of assimilating in that condition. At p. 45 the treatment of the function of the stoma in gaseous interchange is not all that could be wished. The reader will have a singular view of Brown and Escombe's researches if his knowledge is confined to what he can learn in the present volume.

The experiments (p. 52) on the effect of freezing leaves would be more instructive if the ice-injection of the intercellular spaces were studied on a hardy plant such as ivy. In the second experiment, on p. 78, a Tropæolum leaf is recommended for use in experiments on the passage of air through vegetable membranes. But this is hardly allowable, since the leaf in question is well supplied with stomata on both surfaces.

In spite of a few oversights in its pages, we do not hesitate to recommend the work of the brothers Linsbauer to our readers. The methods prescribed are simple and trustworthy, and the book has a merit which is rare in text-books, namely, that it is obviously written with sincere interest in the problems set before the learner. F. D.

SOME RECENT MATHEMATICAL WORKS.

Space and Geometry. By Dr. Ernst Mach. Translated by Thos. J. McCormack. Pp. 148. (London: Kegan Paul and Co., 1906.) Price 5s. net.

Irrational Numbers and their Representation by Sequences and Series. By Dr. Henry Parker Manning. Pp. vi+123. (New York: J. Wiley and Sons; London: Chapman and Hall, Ltd., 1906.)

Auslese aus meiner Unterrichts- und Vorlesungspraxis. By Dr. Hermann Schubert. Vol. iii. Pp. 250. (Leipzig: G. J. Göschen, 1906.)

Leçons de Géométrie supérieure. By M. E. Vessiot. Pp. viii+322. (Lyons: Delaroche et Schneider; Paris: A. Hermann, n.d.) Price 12 francs.

La Géométrie analytique génerale. By H. Laurent. Pp. vii+151. (Paris: A. Hermann, 1906.) Price 6 francs.

N. H. Abel: sa Vie et son Œuvre. By Ch. Lucas de Pesloüan. Pp. xiii+169; with portrait. (Paris: Gauthier-Villars, 1906.) Price 5 francs.

Theory of the Algebraic Functions of a Complex Variable. By Dr. John Charles Fields. Pp. vii+186. (Berlin: Mayer and Müller, 1906.)

Recherches sur l'Élasticité. By P. Duhem. Pp. 218. (Paris: Gauthier-Villars, 1906.)

IF reform of mathematical teaching is to mean anything real, it is necessary that the teacher should possess a much more extended survey of his subject than is conveyed in the ordinary English text-book. There could be no more suitable book for giving the elementary or secondary teacher some intelligent ideas about geometry than Dr. Mach's series of essays. In them the subject is treated in its physiological, its psychological, and its physical aspects.

The first essay thus deals with the relation of the spatial concept to the senses. In the second we have an attempt to trace the natural development of geometry from psychological causes, while the last essay discusses the subject from the point of view of physical inquiry. Incidentally, a number of illustrations are introduced, some of which are admirably adapted for teaching purposes. There could not be a better object-lesson in the elementary properties of Euclidean space than the indefinitely extended pavement formed of equal and similar triangles discussed on p. 59. From it can be read off all the principal properties of parallels and parallelograms, the relation between the three angles of a triangle, and also the main properties of similar triangles the sides of which are commensurable.

Dr. Manning's book on irrational numbers contains a presentation in a simple form of another field of mathematical inquiry, such as is also eminently suited for placing in the hands of the ordinary schoolmaster. We have decided that the geometry of proportion shall be taught to schoolboys without reference to irrational quantities, but we have not yet eliminated a spirit of reckless extravagance in the quite unnecessary use of infinite series, often with total disregard for their convergency. In Dr. Manning's treatment an irrational number is defined

as forming a point of separation between rational numbers of two classes, the numbers of one class being less than those of the other. This definition appears to involve the assumption (pp. 7, 10, &c.) that the point of separation is unique, in other words, that there cannot be two irrational numbers which have not some rational number separating them. Perhaps this assumption may be regarded as a definition of equality of irrational numbers; in any case, the inquiring reader would find it necessary to examine more fully the references to Dedekind's and Cantor's writings given on p. 56. Once the assumption or definition is made, the representation of numbers by sequences readily follows. The theory of limits is discussed on p. 57, and in the following chapter the notion of a sequence is shown to give rise to that of a series. The remaining portion of the book is mainly devoted to the study of convergence, and includes the well-known multiplication theorem and applications to the still better-known binomial and exponential series.

Prof. Schubert is rightly regarded as an authority on the teaching of mathematics, but if this description leads the English reader to expect that the present selection of lecture notes will consist of a mere repetition of the "school geometry" and "graphs" which are being ridden to death in England to the exclusion of other equally important reforms, that reader will be greatly disappointed. Dr. Schubert has rather shown us what can be done by any teacher who will endeavour to make himself "a snapper up of unconsidered trifles." He finds, in the first place, that the determination of centres of gravity is not well treated in text-books either on mechanics or on the calculus; accordingly, this problem forms the subject of the first section. The discussion includes curves, areas, and figures of revolution, and we notice the three- and four-cusped hypocycloids, the lemniscate, the kissoid, and other well-known curves figuring among the worked-out examples. Next follows a chapter on Snellius's law of refraction. Some properties of the parabola deduced from the equation of the tangent are next discussed. Then follow certain stereometric problems, and in particular an extension of Simpson's rule for the volume of a frustum. Each of these sections deals with points which are not satisfactorily treated in existing text-books. The book concludes with some interesting problems in spherical trigonometry, in particular the "Heronic" triangle, in which the sines and cosines of the sides and angles are rational fractions. The book is interesting reading, and quite easy for anyone with an elementary knowledge of the subjects discussed, to follow.

"Lecons de Géométrie supérieure" consists of a collection of lecture notes on a course delivered in 1905–6, and transcribed by M. Anzemberger. The notes are *type-written*, not printed, and we can only wish that a similar method of procedure could be adopted with the mass of dry, uninteresting, superfluous, and wholly irrelevant details which so often occupy pages of printing in modern published "researches." The course can be precisely described to English readers as "solid geometry of curves, surfaces and complexes." It deals mainly with the large subject of curvature, but, in addition to considering systems of lines, the author gives some elegant discussions of systems of spheres and circles. The present reviewer has for some time past given a course of lectures on solid geometry in which the curvature of curves is treated kinematically. It is interesting to see this most useful and suggestive method adopted in the present notes, for example, in defining the osculating plane as the plane containing the tangent and the acceleration.

M. Laurent's book also deals with analytical geometry, mainly solid geometry, but treats principally those portions of the subject which are studied before curvature. It has for its object the development of geometry from a purely abstract point of view independently of any preconceived notions regarding space. It is thus based on the study of orthogonal transformations and quadratic forms, and an instance of the spirit of the book is afforded by the preliminary note, in which the periodicity of the circular functions is derived from their definition as exponentials apart from any consideration of their geometrical properties. The subject-matter includes the study of tangents and envelopes, the properties of surfaces of the second degree, their diameters and polars, the principle of duality, and a final chapter on the non-Euclidean spaces of Riemann and Bolyai. The author at the outset advises his readers to make a clean sweep of all their previously acquired geometrical notions. It is pointed out that in order to pass from the abstract to the concrete one definition is required, namely, the definition of rigid-body displacement. This definition is to be regarded as fundamental, and as superseding Euclid's axiom of parallels. Among the applications we notice Abel's theorem and an important theorem of Chasles.

The story of Abel's life has been told recently in more than one book, yet it is a story that well bears re-telling, if for no other reason because it ought to be read as widely as possible. It is natural that M. de Pesloüan should give considerable attention to the part of Abel's life which was spent in Paris, and in a concluding chapter he offers some reflections as to the causes which led to Abel's great memoir being neglected at the time it was offered to the academy. To understand these causes, M. de Pesloüan considers it is only necessary to study the trend of mathematical thought in Paris about the year 1826. At that time French mathematicians were too much engrossed with applied mathematics—such as dynamics and electricity—to give heed to a paper dealing with a property of transcendental functions, and thus nobody understood or appreciated the value of Abel's work. The author further cites the parallel case of Galois as another unappreciated mathematical genius who interested himself greatly in Abel's work. It might be easy to cite other examples, such as Grassmann. The misfortune is that there is nothing to prevent a recurrence at the present time of the circumstances which led to Abel's dying in poverty without obtain-

ing any adequate recognition of the work which in later days caused his name to be handed down to posterity.

Of the remaining two books on our list a great deal might be said, but it would be difficult to give more than a bare statement of their contents in a general review of the present character. Dr. Field's development of the theory of algebraic functions by algebraic methods occupies a useful place in the literature of the subject, and is well adapted for use as an introductory treatise. In the matter of exposition, the summaries at the commencement of each chapter are valuable. The subject-matter includes a discussion of the Riemann-Roch theorem, Plücker's formulæ, and the Abelian integrals. The development of the theory, which is applicable to algebraic equations of the most general character, culminates in the complementary theorem, from which such applications as those just mentioned follow as corollaries.

Prof. Duhem's treatise has for its object the study and analytical expression of the equations of a material medium for displacements and stresses of a more general character than those considered in the ordinary analysis of stresses and small strains. It thus takes account of finite strains and of viscous in addition to elastic resistances. It includes the study of isothermal and adiabatic changes. The problem of wave propagation is discussed at considerable length, and in particular the conditions for permanence of wave motion. Hysteresis is not taken into account. The problem is a generalisation of that dealt with in 1874 by Dr. Oskar Emil Meyer. Some time back a small elementary treatise was reviewed in NATURE dealing with a somewhat cognate subject, namely, the classification of the various phenomena that can exist in a deformable medium, and the present treatise may be conveniently described as an analytical discussion of the x, y, and z equations, while the little book in question explained the A, B, C of the subject.

G. H. B.

OUR BOOK SHELF.

Arboriculture Fruitière. By Léon Bussard and Georges Duval. Pp. xii+562; illustrated. (Paris: Baillière et Fils, 1907.)

THE object of this little book, we are told, is to be useful to fruit-growers, and with that view to lay before the reader in a condensed but systematic form as complete a general view as possible of the scientific principles underlying practical methods of fruit culture.

The actual details of cultivation do not differ materially from those followed in this country, but there is a marked difference in the manner, and especially in the spirit, in which the several operations are carried out in the two countries.

Here the details of pruning, pinching, and the like are done in routine fashion, handed down from our predecessors and pursued because experience has shown the utility of the practice.

In France much more thought is given to the matter. The book before us affords an instance of this. The various shapes and positions which the buds assume and the circumstances in which they are formed are gone into with much detail, and we have descriptions of *lambourdes, dards, brindilles, cochonnets, bouquets de mai, chiffons, coursons,* and *bourses,* for many of which we have no corresponding terms in English. Nevertheless, a knowledge of these details is essential to a rational system of pruning, and apart from their practical interest they should be carefully studied by those interested in bud-variation and "mutation."

We do not think that botanists in general adequately recognise the great diversity that exists in the buds of a single tree. The study of a pear-branch or of a peach-shoot would form an excellent preliminary exercise to the investigation of bud-variation, and perhaps serve to restrain premature theoretical pronouncements. For this reason, apart from its practical utility, we can commend the work before us as well thought out and carefully written. The principal varieties are described, the illustrations are appropriate, there is a table of contents, and an index, the latter not so complete as it should have been.

Physikalische Kristallographie vom Standpunkt der Strukturtheorie. By Ernst Sommerfeldt. Pp. vi+132. (Leipzig: C. Tauchnitz, 1907.) Price 6 marks.

THE title of this book is somewhat misleading. According to the commonly accepted nomenclature of crystallography the book would be described as a geometrical account of the structure-theory with a few physical applications. The ground covered is hardly wide enough to warrant the name "physical crystallography."

The author's style and method are obviously modelled on those of Sohncke. His account of the 230 possible types of crystal-structure is descriptive rather than logical, and will appeal far more to a practical crystallographer who wishes to have some slight acquaintance with modern developments of the structure-theory than to a mathematician who regards the subject as an application of the group-theory. The latter will probably feel a little irritated at the absence of exactness in definition and completeness in proof. For instance, the "space-partitions" on which the argument is based are nowhere clearly defined, and the reason given (p. 65) for assuming fifteen of these partitions as fundamental is quite unconvincing. Surely the partitions should either be limited to the fourteen possible space-lattices or be extended to include such figures as Kelvin's fourteen-walled cell. Sohncke's systems are illustrated by photographs of excellent models, but such diagrams probably convey very little to a reader unless they are arranged for stereoscopic use. The author gives, however, figures showing the projections of these models on a plane, which will doubtless be an assistance to the student, though they might with advantage be clearer.

The last forty pages of the book are devoted to a discussion of some physical applications of the structure-theory. Here the author appears at his best, and has some very interesting things to say on the subject of etched figures and rotatory polarisation. His suggestions on etching of low symmetry seem to be new; those on rotatory structure, twinning, &c., are to be found in other books, but the author has brought the argument well up to date. All this part of the treatise is well worth reading, except that in the chapter on crystals with a trigonal axis the real point at issue is a little obscured.

H. H.