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THE MANUFACTURE OF VERY ACCURATE
STRAIGHT EDGES.

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In the construction of mechanical or scientific instruments of precision, the problem frequently arises to provide means of guiding carriages through a considerable distance in as straight a line as possible.

Rectilinear motion may in general be secured by the employment of either link work or by the use of gibs or ways; but of these two methods the latter is by far the one more frequently adopted.

In the construction of the large "wave comparer," which has recently been used with such great success by Professor Michelson in his determination of the standard meter in

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terms of the wave length of light,* this same problem was presented, and the accuracy of movement demanded was considerably greater than is usually attained, even in the best comparators and measuring machines.

In this case the employment of link work was rendered impossible on account of the limited space at disposal, even granting that its employment would have secured the desired accuracy. The use of planed ways on the bed of the instrument itself, which in its general features resembled a long lathe bed, was equally inadmissible, because even if such ways could once be brought to the degree of straightness demanded, the initial strains in the casting would soon manifest themselves to such a degree as to render a resurfacing, or rather a restraughtening, of the ways necessary whenever the bed was moved. For these and other reasons it was finally determined to secure the desired accuracy of movement by the use of steel straight edges as guides for the carriages; these straight edges forming no part of the bed itself, and being secured to it in such a way as to avoid as far as possible any distortion from a twisting or warping of the support.

The manner in which this was done is shown in *Fig. 1*, which shows the bed in cross-section. The main portion of the bed *a a* is of H section, and has planed upon it two ways *r r*, on which the outer edges of the two carriages *M N* rest and slide. On the central cross bar of the H rests a long cast-iron bar *b*, against which are lightly clamped the three steel blades *c c' d*, four edges of which, viz. : the two edges of the horizontal blade and the two upper edges of the two vertical blades, form the inner ways for the carriages *M, N*, which move longitudinally along the bed, and are therefore guided in their movement by the two planed ways *r r* on the bed and the steel straight edges just referred to, the carriages being kept against the two vertical edges of *d* by the action of the inclined ways *r r*.

The actual problem was not, as might have been inferred

* *Nature*, 49, 60.

from the preliminary statement of it, to have every point in the carriage move in a straight line; this would have necessitated that the outer ways *rr* be of the same order of accuracy as the straight edges, and this even if once secured would have been extremely difficult to preserve; but it was to have the carriage so guided that the plane of section perpendicular to the line of movement should always remain parallel to a given fixed plane, or in practice so that a mirror supported on the carriage with its face in this plane should always remain as nearly as possible normal to a ray of light whose direction is parallel to the direction of movement.

Now, it is evident from this disposition and arrangement of bearing surfaces, that any irregularity or want of straightness in one of the ways *rr*, will produce simply a rotation of the carriage in a plane perpendicular to the line of intersection of the planes of the two edges, and that granting only that each of these edges be straight any surface mounted on the carriage parallel to this perpendicular plane will remain parallel to itself for all positions of the carriage. It is not necessary, either that the ways *rr* be straight or parallel to the straight edges, or that these last be parallel to one another.*

The problem, from a mechanical standpoint, then reduced itself to one of producing four straight edges, of such an order of accuracy that the extreme angular movement of the carriage in moving from one end to the other of its ways should not exceed two seconds ($2''$) of arc, which means in linear terms that the error in any part of the straight edge should not be greater than $\frac{1}{100000}$ the length of the carriage (in this case seven inches), or about $\frac{1}{16000}$ of an inch. Although this was a rather high degree of accuracy for such long straight edges ($51\frac{1}{2}$ inches), it was hoped

* A slight error is introduced on account of the finite width of the edges themselves, and the finite length of the bearing points, the bearing surfaces on the carriages being each about fifteen millimeters long, and of a width equal to the thickness of the straight edge; but the errors introduced by this cause are of the second order, as compared with the necessary irregularities in the ways themselves.

that the manufacturers would be able to make them of the required degree of accuracy by ordinary mechanical processes. After considerable correspondence, the work was intrusted to a well-known Eastern firm, whose reputation for this class of work is unexcelled in either this country or in Europe. In order, however, to check the results obtained by the manufacturers, as well as to afford a means of testing the edges after they were secured in place and at any future time during the use of the instrument, it was necessary to devise a method by which the edges might be compared with another standard edge; such, for example, as an actual metal standard, or a mercury surface.

The construction of another standard especially for this purpose involved considerable expense, which it was desirable to avoid.

Besides, in order to insure that this reference standard itself retains its form to this degree of accuracy in all positions of its use, it would have been necessary to have made it too deep and stiff to have been conveniently used in the space between the vertical edges and the sides of the bed. The two horizontal edges, c , c' , might have been compared with a mercury surface by the aid of a high-power microscope, a method used by Professor Rogers in surfacing the bed of the Rogers-Bond comparator, but it is difficult to devise any simple method for comparing a vertical edge with such a horizontal surface.

The method, proposed by Professor Michelson and used first in the process of examination and afterward, during the process of grinding, for measuring the errors of the edges and checking the grinding process, is an optical one, which has been found more convenient in practice and capable of giving more accurate results than the methods of this character which have heretofore been used. It is to the improved method of testing rather than to any improved method of working that the results which have been obtained are due, for in any method of working in which corrections are determined and applied with reference to a standard edge or standard surface, the accuracy of the result cannot, as a rule, exceed the accuracy of the standard.

In this case the reference edge, if such it may be called, which is used as a standard, is as near a geometrical straight line as it is possible to approach by any physical method, being what is termed a ray of light in the language of geometrical optics.

More explicitly stated, the method consists in projecting a parallel beam of light normally on the surface of a mirror, mounted on a carriage, which slides along the edge to be examined (the mirror being placed as nearly as possible at right angles to the edge), and measuring the angular deviation of the reflected beam as the carriage is moved along the straight edge.

The actual apparatus which was employed for the purpose is shown in *Fig. 2*. Light from the illuminated cross wires of an ordinary telescope focussed for parallel rays falls upon a plane parallel piece of glass *B*, by which a portion is reflected to the mirror *C*, mounted on the testing carriage, and forms, after nearly normal reflection from this mirror, an image of the cross wires at the focus of the observing telescope *D*, also carefully focussed for parallel rays. *D* is furnished with a micrometer eye-piece with double cross wires of the usual construction.*

a is an ordinary condensing lens, serving to concentrate the light from a lamp or gas flame on the cross wires of *A*.

Both telescopes being accurately focussed for parallel rays, and the telescope *D* being set so that its axis is in the prolongation of the straight edge under test and perpendicular to the mirror *c*, the glass plate *B* is adjusted until the illuminated field of *A* is central in the eye-piece of *D*.

The telescope *A* and the micrometer eye-piece are then rotated until the cross wires of each are parallel to the plane of the edge to be tested. The carriage is then placed at one end of the edge and the wires of *D* set so that they are just bisected by the image of the horizontal cross wire of *A*. Then if the edge be perfectly straight it is

* For the sake of avoiding confusion in the measurement it is a good plan to cut out the ordinary fixed wires and retain only the double movable ones.

evident that the relative position of the micrometer wires and this image seen by reflection from c will remain the same for all positions of the carriage, but that any elevation or depression will at once cause a deflection of the image, the amount of which is accurately measured by the micrometer. The possible delicacy of the method is limited only by the size of the telescopes employed, provided only the mirror c be of the same size as the object glasses.

An objective of quite moderate size is, however, sufficient. The dimensions and constants of the apparatus used in the present case were as follows :

Aperture of telescope,	40 mm. ($1'6''$)
Focal length of telescope,	314 mm. ($12'3''$)
Constant of micrometer,	1 div. corresponds to $\frac{1}{200}$ mm.

A deflection of the image through a distance of $\frac{1}{200}$ mm. at the focus of the observing telescope, corresponding to one division of the micrometer head, meant therefore an angular deflection of the beam of light of $3'2''$, and therefore since the deflection of the beam of light is doubled by reflection, an angular deflection of the carriage and mirror through about $1\frac{1}{2}$ seconds ($1\frac{1}{2}''$).

This constant can be and was measured directly in at least two other ways: (1) By inserting a strip of mica of measured thickness under one end of the carriage, and measuring the deflection produced, and (2) by displacing the cross wires of the first telescope through a known angle by means of the divided circle of the spectroscope table on which it was mounted, and measuring the corresponding deflection. The mean angular value (for the carriage) for one micrometer division, as found by all these methods, was for the apparatus used about $1'5''$, corresponding to a displacement of one end of the carriage through about $\frac{150000}{21000} = \frac{7}{21}$ of an inch. One division of the micrometer head corresponded then to about the limit of accuracy imposed. The resolving power of the telescope employed was such that differences of at least one-fourth this amount could be measured with

certainty;* indeed, readings were always made to tenths of a division, two successive readings rarely differing by more than 0.2 div. from one another.

A fairly typical set of readings taken during the process of grinding one of the edges is given in the following table. Here the two readings corresponding to the same position of the carriage were not taken successively, but form two of a continuous set beginning with No. 1, and ending with No. 14, according to the method of testing presently to be described. Added to the error of reading we have then the errors due to accidental particles of grit getting under the carriage between two readings, the error, if any, due to the movement of the carriage in opposite directions, and the error due to systematic changes in any part of the observing apparatus.

TABLE I.

No.	Reading.	No.	Reading.	* Mean of Two Readings for Same Position of Carriage.	Differences from Mean.	Mean Error.	Probable Error of Single Setting.
1	14.7	14	15.0	14.85	.15	From 14 observations = .2 div.	As determined from 14 observations $\epsilon = .61 \frac{\sqrt{\sum \theta^2}}{\sqrt{\pi - 1}}$ = .16 div.
2	15.3	13	15.0	15.15	.15		
3	15.0	12	15.2	15.40	.20		
4	14.1	11	13.5	13.80	.30		
5	15.0	10	15.0	15.00	.00		
6	13.7	9	14.4	14.15	.35		
7	14.0	8	14.6	14.30	.30		

When the bar was approaching completion considerably greater care was taken in cleaning the surfaces before testing, and the results were then considerably better; for com-

* Resolving power as ordinarily used is defined by the relation $\epsilon = \lambda/\Delta$ where ϵ is the smallest angle which can be "resolved," or the angle as viewed from the objective between two fine lines or points so close together as to be just distinguished apart; λ the wave length of light, and Δ the diameter of the objective. In this case $\epsilon = \frac{.00055}{40.5} = 2.5''$, a quantity consider-

ably greater than it was stated it was possible to distinguish. It must be remembered, however, that it is easier to distinguish differences between two successive positions of a single object than to determine whether two bright objects placed close together are separate or not, because in the one case there are two sets of diffraction fringes which overlap and cause confusion, and in the other case only one set whose position has only to be judged with reference to some fixed object (in this case the cross-wires) so far distant from it that no overlapping and consequent confusion results.

parison a set of readings under these more careful conditions of testing is given in Table II, which shows that we may rely on the correctness of the indicated error to at least 0.1 div., that is to about $\frac{1}{200000}$ inch.

TABLE II.
Edge No. 1, final.

No.	Reading.	No.	Reading.	Mean of Two Readings for Same Position of Carriage.	Difference from Mean.	Mean Error.	Probable Error of Single Setting.
1	37.2	16	37.6	37.40	.20	Mean error of these 16 readings 0.07 div.	Probable error of single setting $= .67 \frac{\sqrt{\epsilon \theta^2}}{\sqrt{\pi - 1}}$ $= 0.06$ div.
2	37.5	15	37.6	37.55	.05		
3	37.3	14	37.2	37.25	.05		
4	37.8	13	37.7	37.75	.05		
5	36.8	12	36.7	36.75	.05		
6	37.2	11	37.2	37.20	.00		
7	37.3	10	37.1	37.20	.10		
8	37.0	9	37.1	37.15	.05		

When the finished straight edges arrived, each one was set up with its edge horizontal, being supported in position by two wooden supports, as shown in *Fig. 2*, placed about one-fourth the length of the bar from each end. One of the four-inch straight edges, No. 1 (*Fig. 1 c*), was first tested and was found to fall so far below expectations that we determined to attempt the correction of it ourselves by grinding down the edge with fine emery. Three grinding tools were therefore prepared, of the form shown in *Fig. 2 A*, of different lengths, one being about twelve inches long, another seven inches, and the third about three inches long. For the purpose of approximately determining the amount removed, a fine micrometer screw, with a large divided head, reading to $\frac{1}{1000}$ mm. = $\frac{1}{250000}$ " was mounted so as to be used as a beam caliper, but it was found that after a little practice with a given grade of emery it was possible to judge very closely of the amount removed by the time of grinding and the pressure used. Besides, the process of testing is so rapid that the whole operation of observing, reducing results, and plotting the curve requires on the average less than ten minutes, and it is, therefore, better to determine the complete curve for the edge after each grinding. The routine method of doing this was as follows:

The carriage was placed first with one end coinciding

with the end of the straight edge, and the reading of the micrometer taken; it was then moved forward its own length and another reading taken, and so on to the end. A similar set was then taken going in the reverse direction, and the mean taken of the two readings (going and coming) for the same position of the carriage, as has already been indicated in Tables I and II.

From this set of means the curve was plotted as follows: The difference between each reading and the next successive one, when multiplied by a factor depending on the constant of the micrometer and the length of the carriage, will evidently represent the difference in level between the two ends of the carriage in the second position, as referred to the line of reference, viz.: the direction of propagation of the incident beam of light. If then these differences be successively added and the numbers so obtained plotted as ordinates to a curve whose abscissæ are distances along the straight edge, the resulting curve will be that of the edge in question. For convenience in plotting, the differences are taken not between successive readings beginning with the first (although this was the plan first adopted), but between each reading and the general mean, the object being to make the ends of the bar coincide with the axis of plotting, and at the same time furnish a check on the numerical reduction of the observations. The following set of readings, Table III, will serve as an example of this method of reduction, the readings being those made on the first straight edge tested (No. 1), just as it came from the makers.

TABLE III.

No.	Position on Bar.	Mean "Going" and "Coming."	General Mean.	Difference.	Σ differences = y	$y' = .00007 \times y$ Difference of Level (Inches).
0	0	—	190.4 $\div 7 = 27.2$	—	0.00	.00000
1	7 inches	29.5	27.2	+ 2.3	+ 2.3	.00016
2	14 "	30.5	27.2	+ 3.3	+ 5.6	.00039
3	21 "	28.5	27.2	+ 1.3	+ 6.9	.00048
4	28 "	25.5	27.2	— 1.7	+ 5.2	.00036
5	35 "	25.8	27.2	— 1.4	+ 3.8	.00027
6	42 "	25.6	27.2	— 1.6	+ 2.2	.00015
7	49 "	25.0	27.2	— 2.2	+ 0.0	.00000
		190.4				

Wadsworth :

intervals on the bar determining its form, an interval taken, beginning at a distance half the length of the bar, and in return as for the first, and so on, in the same manner as the second series of points is referred by the process of abscissæ as the first, it is generally set with reference to the depression or rotation of the

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the heavy line, *Fig. 3*. It is a carriage only as long as the heavy line, and it is not for the fact that it is not necessary to test it in the process of testing at seven-and-a-half inches than that at three-and-one-half inches. It would be better to use two carriages, one, say, of six inches length, the former in the final stage of the process, the latter in the final stage of the process, too, the greater the length of the carriage, the greater the accuracy of the result.

grinding or roughness due to any minute remaining particles of emery or dust could be detected.

When properly cleaned and polished the carriage ought to "float" on the edge almost as easily as one well-made surface plate on another. When this cleaning is properly performed the readings "going" and "coming" for the same position of the carriage agree with each other and may, as already stated, be depended on to 0.1 div. of the micrometer head.

The process of working down the edge by this process, successively testing and grinding, is very rapid, provided the material of which the edge is made is uniform and homogeneous, and, what is still more important, free from initial strain due to hammering or rolling. The characteristic successive steps in working down the edge first taken are shown in *Fig. 4, a, b, c*, etc., the final stage being represented in *Fig. 4, d*, an inspection of which will show that the maximum error remaining was less than $\frac{1}{25000}$ inch, an error considerably less than the limit imposed.

This was so encouraging that the remaining edges were corrected in the same manner.

Edge No. 3, the next one taken in hand, is represented in its initial state in *Fig. 5*. As will be seen, it was considerably better than edge No. 1, but that the errors were considerably larger than could be allowed. In the process of working down this edge an unexpected difficulty presented itself—a difficulty due probably to a want of proper annealing of the steel bar of which the straight edge had been made. During the grinding the bar changed its form repeatedly, the changes observed between two successive grindings being far greater than could have been caused by the simple removal of too great an amount of material at local places. Another curious thing was that the local grinding of a high spot seemed in many cases to increase rather than diminish the error at the point of grinding; in other words, the metal seemed to swell up under the action of the grinding tool. Then, after a series of successive attempts to reduce the "hump" with very little success, the material seemed suddenly to give way all at once, and a

deep hollow was the result. The grinding down of the extreme ends was a matter of special difficulty, as the springing there was especially bad. It seemed almost impossible to attain as high a degree of accuracy in this case as in the preceding, and after spending a good deal of time on the bar it was finally left in the state shown in *Fig. 6*, with the intention of making the final corrections after it had been clamped in place, hoping that the clamping might prevent the springing which proved so troublesome.*

The correction of edges Nos. 2 and 4, the opposite edges of the one bar *c*, proved the most difficult task of any. The warping, presumably on account of bad annealing, was specially bad, and working on either edge almost invariably affected the form of the other, so that it was necessary to reach the final result by a series of successive approximations, working first on one edge until it was nearly right, then on the other, then on the first again, and so on until the amount to be taken off from each side was so small that its removal did not affect its neighbor.

It was necessary also to make a specially light bronze testing carriage for this bar, in order to avoid errors due to the deflection of the bar by the weight of the carriage, errors for which it is true allowance might have been made, but which it seemed better to avoid if possible.

The deflection of the bar due to its own weight here becomes of importance.†

It was reduced as far as possible by placing the supports about one-fourth the length of the bar from each end, under which conditions the deflection at the centre is only about one-fortieth that experienced when the bar is supported at

* It must be remembered that the movements observed are, after all, extremely small, the springing rarely amounting to more than '00005 inch, and usually much less.

† The deflection of a uniform bar supported at its ends is

$$\Delta = \frac{5}{32} \frac{s l^4}{E d^3}$$

where *s* is the weight of the bar per cubic inch, *l* the length (in inches) and *d* the depth. In this case assume *E* = 25,000,000. Δ is found to be '0027 inch, a quantity corresponding to about fifty div. of the micrometer.

Wadsworth :

slight outstanding error in section neglected, as it was done *in situ*, thus eliminating the error. The initial curves in Fig. 7, the final curves in Fig. 8, are mainly for the purpose of illustrating the capabilities of the method and the accuracy with which it was practically possible to obtain an expenditure of time and effort. At the first point, a glance at the extreme error for edge 1

The clamping screws having been screwed firmly down, a second test was taken, and showed that the form of the edges had not been materially changed by the clamping action. The testing carriages used were now the actual carriages to be used in the final work, a mirror being mounted on each one in such a way that any one of the four edges could be tested without moving the optical part of the testing apparatus. An innovation was introduced in the way of grinding tools, which was in many respects a step backward.

Instead of the iron grinding tools already described, a number of rectangular blocks of emery, of different lengths and of varying degrees of fineness, were prepared. To protect one edge from the action of the grinding block while the adjacent edge was being worked upon, one side of the block was covered with paper.

The reduction of the large irregularities was performed by the mechanician of the laboratory, but in the final stages of the work it was found necessary to return to the iron grinding tools previously used.

After the edges had been brought to about the condition shown in *Fig. 5*, we took the grinding in hand ourselves, and in a few days, with the experience already acquired, brought the edges to their final condition, shown in *Figs. 9, 10, 11, 12*.

The difficulties encountered in this work were greater even than had been experienced in the course of the work on the free bars. In the first place, the initial strain in the bars was not allowed to manifest itself freely as before, on account of the clamping action of the screws, which was not, however, great enough to entirely prevent what seemed to be a gradual or often sudden "creeping," which proved very troublesome. Moreover, the work on any one edge was almost certain to affect the trueness of the adjacent edge, so that, in this case, all the edges had to be reduced by the process of approximation, which has already been referred to as necessary in the previous work, in case of edges Nos. 2 and 4, some of the individual steps of which were more satisfactory as regards a single edge than the final result for that edge. It is especially worth noting,

however, that all, or nearly all, of the difficulties encountered were due to the bad material, or rather the bad state of the material, of which the bars were made. Had the steel been uniform in texture, and either thoroughly hardened and seasoned or thoroughly annealed, the time required to complete the work would have been reduced one-half, and probably more. Even in the state of strain, however, there was no difficulty in reducing the errors of the edge to an order of $\frac{1}{50000}$ inch. An hour or two hours' work usually sufficed to bring about this order of accuracy when the initial error was from five to ten times this amount. The time taken to bring all four edges to their final state, as shown in *Figs. 9-12*, was between 40 and 50 hours, an average of from 10 to 12 hours' work on each edge. As will be seen by inspection, the accuracy is rather greater, on the whole, than was attained in the first work, the greatest absolute error for two of the edges, at least, being not greater than $\frac{1}{50000}$ inch, with a mean error of less than $\frac{1}{100000}$ inch in the whole length of 52 inches. To make a straight edge of this length accurate to this degree, by the ordinary method, would require an expenditure of time which would render it impracticable as a commercial operation. The results of the preceding work, however, show conclusively that, with suitable material, such edges may be prepared from the planed bar at an expense (for labor) not exceeding \$3 per foot, of length for edges, four feet long, accurate to within $\frac{1}{50000}$ inch for their whole length.

For longer edges the cost would probably be somewhat less, for shorter ones somewhat more per foot of length. *There is no limit to the length which may be made straight to this order of accuracy quite without reference to any material standard*, an important consideration when it becomes a question of making standards six to ten feet in length.

Errors due to flexure are corrected in the process itself, it being only necessary that the edge be tested when supported in the same manner and the same position in which it is to be used. This allows the use of comparatively light bars instead of the deep girder forms necessary when the ordinary

method of correction by working three edges to one another is used, a process which requires that at least two of the bars be stiff enough to avoid a perceptible change of form when turned upside-down, a condition very difficult to secure when the bars become of unusual length.

It is difficult to see what obstacles remain to be overcome in the successful application of this method to commercial work, except the single one of obtaining homogeneous and well-tempered bars of metal upon which to operate. As has been seen, it is possible to produce straight edges of the order of accuracy herein stated rapidly and cheaply, even when the material is not properly prepared, but these edges are not sufficiently permanent under handling to warrant their being brought to such a degree of accuracy unless they are to be permanently clamped in place.*

Given a bar of proper material, it may be confidently stated that straight edges of from five to ten feet in length, or even longer, if desired, may be produced with an error not exceeding $\frac{1}{100000}$ inch in their whole length. The same method may be applied also to the correction of the ways of measuring machines, dividing engines, comparators or any apparatus in which a very accurate rectilinear motion is to be secured. To these last purposes it is especially applicable, as it allows the testing to be made and the corrections to be applied in any desired position of the edges or ways upon which the work is being done, which renders it possible and practicable to always correct the ways when the bed which supports them rests in the position which it is destined finally to occupy, compensating then at once for all the effects of flexure due to want of rigidity, and twist or other distortion due to the manner of support.

To sum up briefly, the principal advantages which the method seems to possess are:

- (1) Possibility of producing one edge, and one edge only, of any required length and of any required order of accu-

* There has been no perceptible change in the edges since they were first finished, which, at the date of writing, was over six months ago, during which time the instrument has been in use almost continuously.

racy, without reference to any previously existing edge or material standard.

(2) The possibility of measuring definitely the errors in this edge and mapping them for future reference, so that it will not be necessary to carry the work on any edge further than the desired order of accuracy.

(3) Ability to test and correct an edge in any position, and to readily verify its correctness at any future time, recorrecting it if necessary without removing it from position.

(4) Rapidity and cheapness in securing a desired result.

This last is an important practical advantage which leads me to hope that the article may not be without interest to the intelligent mechanics and instrument-makers of this country, who are always ready to adopt any practical methods which will lead to greater accuracy in the production of certain classes of high-grade work.

A number of modifications of the general method which may prove serviceable in certain cases may be proposed. Those familiar with the optical arrangements will at once see that instead of two telescopes, arranged as shown with the glass *B*, all that is absolutely necessary is a single one, occupying the position of *C*, and provided with a pair of illuminated cross-wires, the image of which will be seen in the field of the eye-piece when the telescope is focussed for parallel rays and the mirror is nearly perpendicular to the axis of the telescope. The deflection of the image from the cross-wire itself might be measured as before, or perhaps even more simply by the aid of a fine scale in the eye-piece. As it is desirable for the sake of accuracy to use a double movable wire to fix the position of the deflected image, and as it is also desirable to be able to move the set of wires whose image is observed without moving those in the micrometer eye-piece, a better arrangement would perhaps be that shown in *Fig. 13*. Here the illuminated cross-wires are placed at the end of a short tube *b*, placed at right angles to the axis of the telescope. At *c*, the intersection of the axis of *b* with the axis of the telescope, is placed either a plane parallel glass at 45° covering the whole field,

or a small right-angled prism covering one-half the field. *D*, as before, is simply a condensing lens which serves to concentrate the light on the cross-wires in *b*. With this device the whole arrangement is rendered very compact and convenient to use, as both sets of wires are close to the observer. Means might easily be provided by which the observer could himself move the carriage along the bar to any desired position without leaving his seat; but as the one who is doing the grinding is always at liberty during the operation of testing, the necessity for such a device is obviated.*

For horizontal edges and ways, a somewhat simpler method may sometimes be used. Instead of measuring the deflections of the testing carriage optically, they are measured by means of a delicate level which is secured to it. Level bubbles can now be obtained so sensitive that one division on their scale corresponds to one second, and as one-half a division is easily read with certainty, it follows that for a carriage of the same length deflections nearly as small as those measured optically may be determined more directly in this manner.

By mounting the level as shown in *Fig. 15*, it would be possible to examine edges inclined at any angle in a vertical plane; but as the method is a gravitational one, it fails for all edges out of this vertical plane.

* It is usually best to have two persons in doing the work, one of whom attends to the grinding, the other to the testing. One observer, however, can easily test as rapidly as three or four can grind.

The usual method of optical testing is to place a telescope on the carriage which moves on the straight edge, and direct this telescope either at a very distant object, or better, at the cross-wires of a second collimating telescope placed in line with the straight edge as shown in *Fig. 14*.

We can then observe in either telescope (their axes being now in the same straight line) the movement of the image of the cross-wires of the other as the carriage is moved along.

The primary disadvantage of this method is that it makes the testing carriage much heavier than the mirror does, a very serious objection when great accuracy is demanded. Then, for a given size of telescope it is only one-half as delicate as the mirror method, as the angle through which the image is deflected is, in this case, the same as that of the carriage, instead of double that angle as in the case of the mirror method.

SUPPLEMENT.

Aside from its applications to the mechanical operations already indicated, the optical arrangement described in this paper would be admirably adapted to the experimental investigation of certain problems connected with the theory of elasticity.

Formulae for the elastic curves of beams and columns under different conditions of loading have been worked out for a great many cases, but, as far as I am aware, no complete experimental verification of any of these formulæ has been made.

Such* a verification, aside from its interest on purely theoretical grounds, affords data for the determination of E , the coefficient of elasticity. This coefficient is often obtained by measuring the bending of a bar supported at the ends by a considerable weight supported at the center, the deflection due to the weight being measured by the aid of a micrometer microscope. This method involves the use of quite as much apparatus as the one which is here proposed, and which gives us, in addition to the absolute deflection at the center, knowledge of what is going on at all parts of the length.

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