

Calculation of the Coefficient of Self-Induction of a Circular Current of given Aperture and Cross-Section

This content has been downloaded from IOPscience. Please scroll down to see the full text.

1892 Proc. Phys. Soc. London 12 518

(<http://iopscience.iop.org/1478-7814/12/1/337>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 138.73.1.36

This content was downloaded on 03/10/2015 at 12:23

Please note that [terms and conditions apply](#).

XXXVI. *Calculation of the Coefficient of Self-Induction of a Circular Current of given Aperture and Cross-Section.* By Professor G. M. MINCHIN, M.A.*

LET ACB (fig. 1) represent a circular wire in which a current of strength i is circulating; let O be its centre and OV its axis (perpendicular to its plane); let P be any point in space and through P describe a circle, PQ, parallel to the plane of the current, its centre being V on the axis.

Fig. 1.

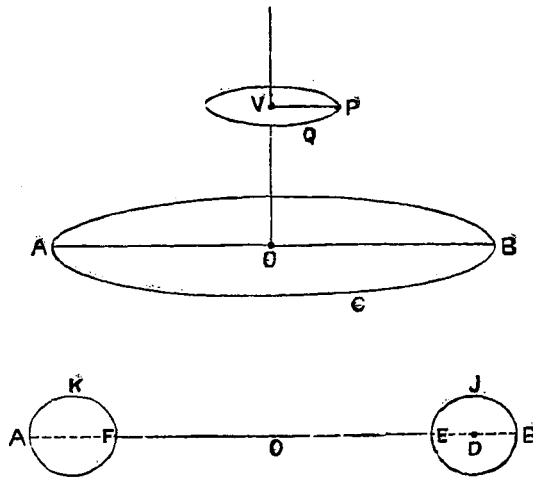


Fig. 2.

It is required to calculate the normal flux of magnetic force passing through the circle PQ. If $VP = x$, and the vector potential of the current at P is G (this latter being, of course, perpendicular to VP and parallel to the plane of the current), the component, Z , of the magnetic force at P parallel to OV is given by the expression

$$Z = \frac{dG}{dx} + \frac{G}{x} \quad \dots \dots \dots (1)$$

(See my paper on the "Magnetic Field of a Circular Current," *antea*, p. 204). This can be written

$$Z = \frac{1}{x} \frac{d(Gx)}{dx} \quad \dots \dots \dots (2)$$

* Read December 8, 1893.

The function $G.x$ is the same as Stokes's *current function* which exists for fluid motion which is symmetrical about an axis. (See Basset's 'Hydrodynamics,' vol. i. p. 12.)

Taking a circular strip of radius x and breadth dx at P, the flux of force through the strip is $2\pi Zx dx$, i. e.,

$$2\pi \frac{d(Gx)}{dx} . dx.$$

Hence, integrating this from $x=0$ to $x=VP$, we find that the total normal flux through the circle PQ is the value of

$$2\pi Gx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

at P.

Let fig. 2 represent the cross-sections of the wire at A and B in fig. 1 made by a plane through the axis OV, the radius of each being c , while the radius, OD, of the central filament of the wire is a (as in my paper on the "Magnetic Field close to the Surface of a Wire conveying an Electrical Current," *antea*, p. 379).

Then we shall calculate *the total normal flux of force through any surface which is intersected once in the positive direction by every tube of force emanating from the given current.*

This quantity, divided by the current-strength, is the coefficient of self-induction of the current. Taking the general case, viz., that in which the current-density at every point in the cross-section of the wire varies inversely as the distance of this point from the axis OV—we have found (*antea*, p. 398) that at any point, P, close to the wire

$$\begin{aligned} Gx = 4ai \left\{ \frac{L}{2} - 1 - \frac{m}{4a} \cos \phi \left[L - 1 - \frac{c^2}{4m^2} \right] \right. \\ \left. + \frac{1}{16a^2} \left[L \left(\frac{3c^2}{4} - \frac{m^2}{2} \right) + \frac{m^2}{2} - \frac{15c^2}{4} \right. \right. \\ \left. \left. - \cos 2\phi \left\{ L \frac{m^2}{2} + \frac{c^2}{4} - m^2 - \frac{c^2}{8m^2} \right\} \right] \right\} , \quad (4) \end{aligned}$$

where $m = PD$, $\phi = \angle PDA$, $L = \log_e \frac{8a}{m}$.

The surface through which we shall take the flux of force is that which is represented in section by BJE OFKA, i. e., a surface consisting of the upper half portion of the anchor-

ring formed by the wire and of its central aperture (which latter is a circle whose diameter is FE). Obviously this surface is intersected by all the tubes of force. Any surface starting from B and going round to A, *i. e.*, any surface having the circle of diameter AB for bounding edge, would do equally well, so far as the above condition is concerned ; but the calculation is simpler for the first.

The flux through the aperture FE is, then, the value of $2\pi G \cdot x$ at E, which is obtained by putting $m=c$, $\phi=0$ in the above.

Thus the flux through the aperture is

$$8\pi ai \left\{ \frac{L}{2} - 1 - \frac{c}{4a} \left(L - \frac{5}{4} \right) - \frac{c^2}{128a^2} (2L + 19) \right\}. \quad (5)$$

To calculate the normal flux through the upper half of the anchor-ring, we must take the value of the magnetic potential, Ω , at any point close to the ring. This is the resultant conical angle subtended by the circuit at the point, multiplied by i ; and it is therefore (*antea*, p. 392) given by the equation

$$\Omega = i \left\{ 2(\pi - \phi) - \frac{\sin \phi}{a} \left(mL + \frac{c^2}{4m} \right) + \frac{1}{16a^2} \left[8c^2(\pi - \phi) - \left\{ (6L - 5)m^2 + c^2 + \frac{c^4}{2m^2} \right\} \sin 2\phi \right] \right\}. \quad (6)$$

Now the normal force at any point on the anchor-ring is the value of $-\frac{d\Omega}{dm}$ with $m=c$; it is therefore

$$i \left\{ \left(L - \frac{5}{4} \right) \frac{\sin \phi}{a} + \frac{c}{16a^2} (12L - 17) \sin 2\phi \right\}. \quad (7)$$

If P is any point on the anchor-ring, the distance of P from OV is $a - c \cos \phi$, and the area of a narrow circular strip of the ring parallel to the plane of the aperture of the ring is $2\pi(a - c \cos \phi) \cdot c d\phi$; and this multiplied by (7) is the normal flux of force through the strip. Integrating the product from $\phi=0$ to $\phi=\pi$, we get the flux through the whole of the upper half of the ring. The result is simply

$$\frac{2\pi ic}{a} \left(L - \frac{5}{4} \right) \int_0^\pi (a - c \cos \phi) \sin \phi d\phi,$$

the second term in (7) being neglected because it gives a term of the third order. Thus this part of the flux is

$$4\pi ci \left(L - \frac{5}{4} \right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Adding this to (5), we have the whole flux sought, divided by i , equal to

$$\pi \left\{ 4a(L-2) + 2c \left(L - \frac{5}{4} \right) - \frac{c^2}{16a} (2L+19) \right\}, \quad (9)$$

where $L = \log_e \frac{8a}{c}$. This, then, is the Coefficient of Self-Induction. If in this expression a and c are taken in centimetres, the result is the coefficient measured in absolute units; and if this is divided by 10^9 , we have the coefficient of self-induction in secohms.

Thus, for example, a circular current running in a wire the diameter of whose cross-section is 2 millim., while the diameter of its central filament is 2 centim., has a coefficient of self-induction of about 59·207 absolute units, or $\frac{59\cdot207}{10^9}$ secohms; and if the dimensions of the cross-section were neglected, this number would be 58·866.

Clerk Maxwell (Elec. and Mag. vol. ii. art. 704) gives the coefficient of self-induction as $4\pi a(L-2)$, which agrees with (9) in the principal term.

In the same way we may find the coefficient for a superficial current in the wire. For (*antea*, p. 221) if q is the total quantity of the superficial current, we have the value of Gx at E equal to

$$2q \left[a(L-2) - \frac{c}{4}(2L-1) + \frac{c^2}{16a} \left(L + \frac{11}{4} \right) \right],$$

while from the value of the potential we find the normal flux of force through the upper half of the anchor-ring equal to

$$2\pi qc(2L+1).$$

Hence the Coefficient of Self-Induction is

$$\pi \left\{ 4a(L-2) + 2c(L + \frac{3}{2}) + \frac{c^2}{16a}(4L+11) \right\}, \quad . \quad (10)$$

which is somewhat greater than the value (9) for a steady current.