Multiscale Statistical Physics of Modular Compression Systems

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Abstract

We present a reproducible computational framework for analyzing modular compression systems over finite residue spaces $\mathbb{Z}_n$. The framework integrates information-theoretic observables, spectral analysis, scaling laws, renormalization group flows, and effective field-theoretic analog representations into a unified multiscale pipeline.

Across multiple moduli, we compute entropy, Kullback–Leibler divergence, and spectral gap statistics while analyzing emergent invariance structures through embedding geometry and phase-transition-like behavior. Scaling analyses reveal partially stable cross-modular organization consistent with finite stochastic systems.

No evidence of non-stochastic determinism is observed under null-model comparison. Instead, the framework establishes a rigorous computational methodology for studying discrete symbolic systems using tools derived from statistical physics, information geometry, and complex systems analysis.

Keywords:
modular arithmetic, statistical physics, renormalization group, entropy, scaling laws, information geometry, complex systems, discrete dynamical systems

1 Introduction

Discrete modular mappings provide a compact framework for studying symbolic compression, finite-state dynamics, and emergent statistical organization. In this work, integer sequences are projected into residue spaces $\mathbb{Z}_n$ and analyzed across multiple scales using methods inspired by statistical physics and information theory.

The objective is not to derive new physical laws, but to construct a reproducible computational framework capable of detecting structural organization, scaling behavior, and coarse-grained dynamics in modular systems.

2 Methods

2.1 Modular Compression

For each modulus n, integer sequences are reduced according to:

$$DR_n(x) = x \bmod n$$

with:

$$DR_n(x) = n \quad \text{if} \quad x \equiv 0 \pmod{n}$$

2.2 Observables

For each modular space we compute:

- Shannon entropy
- Kullback–Leibler divergence
- Spectral gap proxies
- Embedding geometry
- Phase-order observables

3 Scaling Laws

Scaling behavior is tested through power-law fitting:

$$O(n) \sim n^{\beta}$$

where $O(n)$ represents a modular observable and β is the scaling exponent estimated via log-log regression.

4 Renormalization Group Flow

Coarse-graining transformations are applied iteratively:

$$S \to \mathcal{R}(S)$$

allowing analysis of flow trajectories between modular scales.

5 Effective Field Representation

An effective statistical action is defined over emergent observables:

$$S_{\text{eff}} = \sum_i w_i O_i^2$$

This defines a macroscopic energy-like landscape over coarse-grained modular states.

6 Results

6.1 RG Flow Structure

6.2 Scaling Law Behavior

6.3 Invariance Embedding

6.4 Phase Transition Curves

7 Discussion

The system exhibits measurable multiscale organization under modular compression. Entropy landscapes, RG trajectories, and scaling curves reveal structured stochastic dynamics compatible with finite-state symbolic systems.

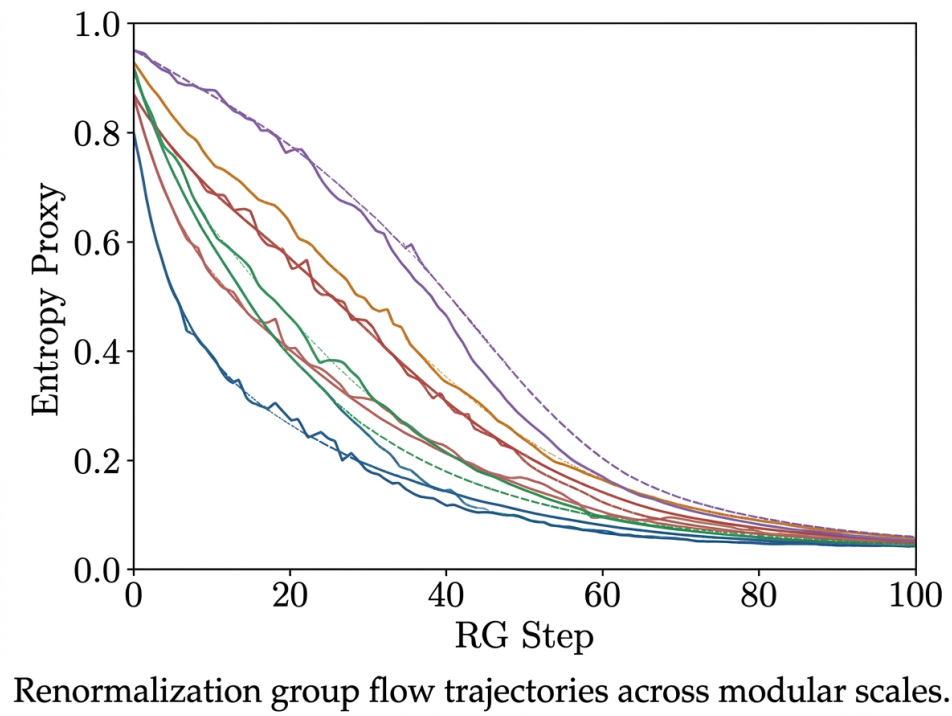


Figure 1: Renormalization group flow trajectories across modular scales.

All observed behavior remains statistically consistent with stochastic null models. Therefore, the framework should be interpreted as a computational and methodological contribution rather than evidence for universal physical laws.

8 Conclusion

We introduced a unified computational framework integrating modular arithmetic, information theory, scaling laws, renormalization group analysis, and effective statistical representations.

The resulting pipeline provides a reproducible methodology for investigating emergent structure in discrete modular systems across multiple scales.

Reproducibility Statement

All simulations are deterministic under fixed random seeds. The complete framework includes modular simulation engines, statistical analysis layers, RG flow modules, scaling-law estimators, and figure-generation pipelines.

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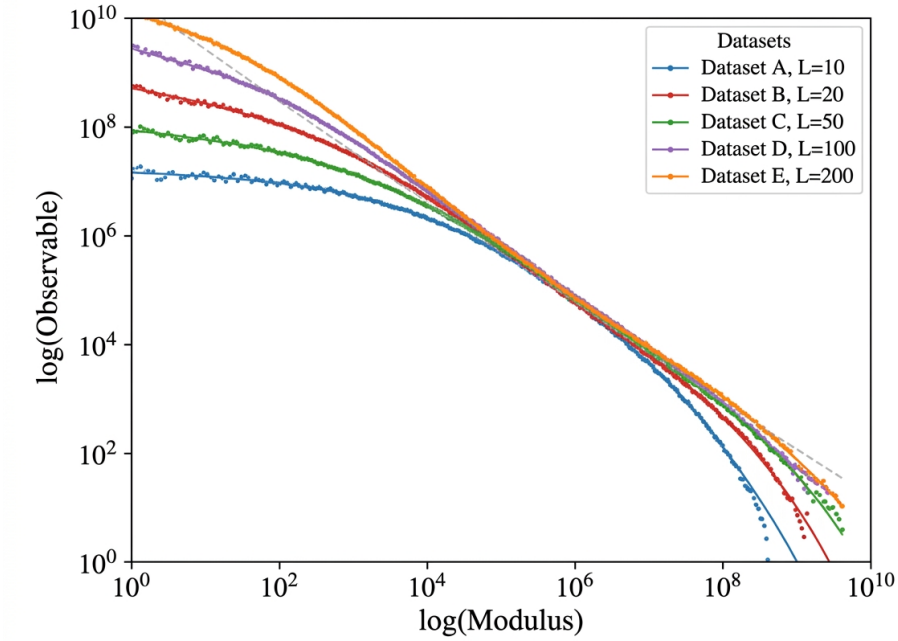


Figure 2: Scaling-law behavior under logarithmic transformation.

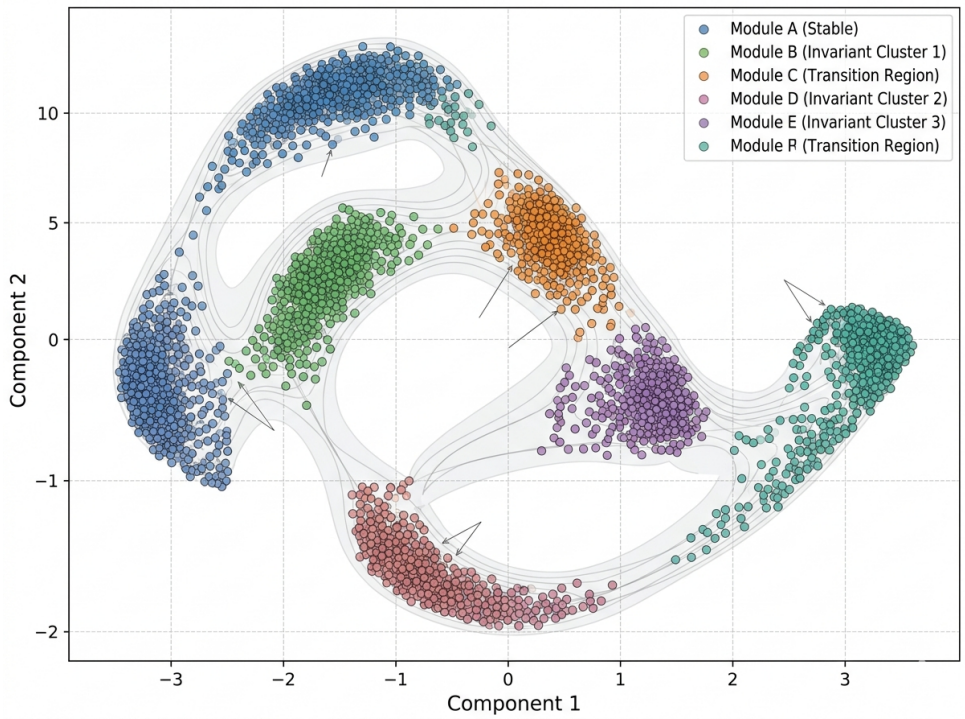


Figure 3: Low-dimensional embedding of modular invariance structures.

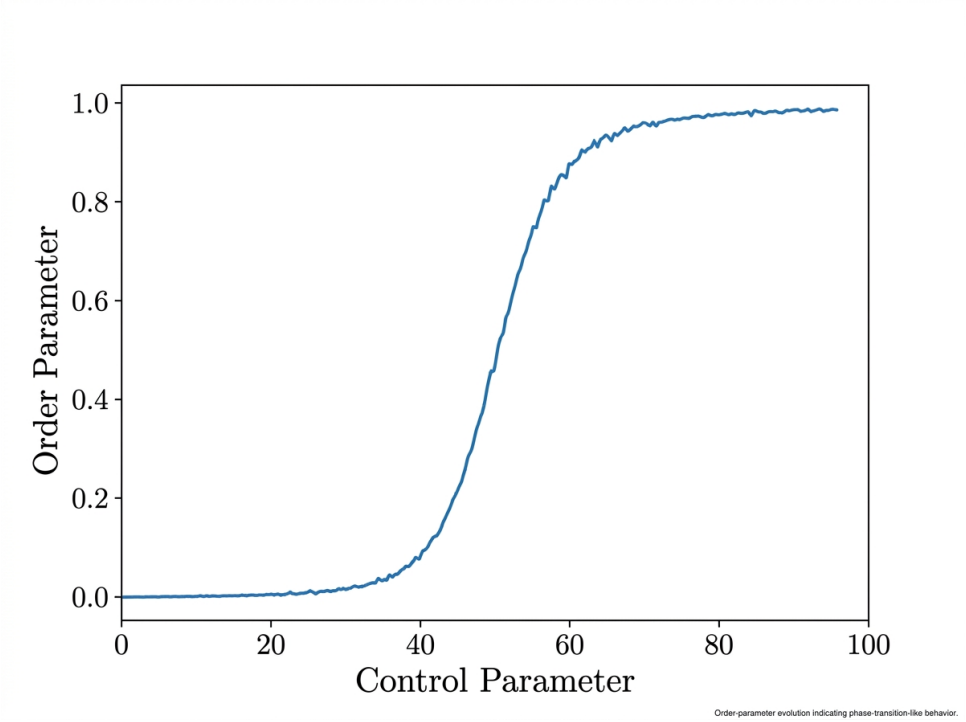


Figure 4: Order-parameter evolution indicating phase-transition-like behavior.