



LVII. On a porismatic property of two conics having with one another a contact of the third order

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other boulders. At Boyndie, further west, the flint boulders cover the shore; and at Delgaty, ten miles inland, they occur in great abundance, along with boulders of quartz rock, but no fossils except their own. It would therefore appear, that we owe the flint boulders and the lias boulders to different periods; and as the chalk overlies the lias, it may be that its denudation was completed, and its fossils thrown upon the high grounds of the interior, previous to the formation of the boulder clay containing the fossils of the lias. Although apparently not here, the boulder clay has in other places (as on the banks of the Thorsa in Caithness) been found to contain "fragments of chalk flints, and also a characteristic conglomerate of the oolite," as well as comminuted fragments of existing shells. (H. Miller.) These facts seem also to favour this hypothesis.

The subject altogether is one involved in considerable darkness, and it is perhaps vain to attempt any generalization upon it till the local geology has been far more accurately examined and determined.

LVII. *On a Porismatic Property of two Conics having with one another a contact of the Third Order.* By J. J. SYLVESTER, M.A., F.R.S.*

IF two conics have with one another a contact of the third order, *i. e.* if they intersect in four consecutive points, it will easily be seen that their *characteristics* referred to coordinate axes in the plane containing them must be of the relative forms $x^2 + yz$, $k(y^2 + x^2 + yz)$ respectively, y characterizing their common tangent at the point of contact †.

Hence if we take planes of reference in space, and call t the characteristic of the plane of the conics, the equations to any two conoids drawn through them respectively will be of the relative forms

$$U = x^2 + yz + tu = 0$$

$$V = y^2 + x^2 + yz + tv = 0.$$

* Communicated by the Author.

† These relative or conjugate forms are taken from a table which I shall publish in a future Number of this Magazine, exhibiting the conjugate characteristics in their simplest forms, correspondent to all the various species of contacts possible between lines and surfaces of the second degree. This table is as important to the geometer as the fundamental trigonometrical formulæ to the analyst, or the multiplication table to the arithmetician; and it is surprising that no one has hitherto thought of constructing such.

Using W to denote $V-U$, and (W) to denote what W becomes when ey is substituted for t , we see that W and (W) are of the respective forms $y^2 + tw$ and $y\theta$; showing that the former is the characteristic of a cone which will be cut by any plane $t-ey$ drawn through the line (t, y) in a pair of right lines; or, in other words, that one of the cones containing the intersection of the two variable conoids (V and U) will have its vertex in the *invariable line* which is the common tangent to the two fixed conics: this proves the theorem stated by me hypothetically in a foot note in one of my papers in the last Number of the Magazine. The steps of the geometrical proof there hinted at are as follows.

The four consecutive points in which the two conics intersect will be consecutive points in the curve of intersection of the two variable conoids. This curve lies in each of four cones of the second degree. Every double tangent plane to it passes through the vertex of one amongst these. The plane containing four, *i. e.* two (consecutive) pairs of consecutive points, is a double tangent plane, and will therefore pass through a vertex; but four consecutive points of a curve of the fourth order described upon a cone, and lying in one tangent plane thereto, can only be *conceived* generally as disposed in the form of an \int , of which the belly part will point to the vertex; or, in other words, at any point where two consecutive osculating planes coincide so that the *spherical* curvature vanishes, the linear curvature will also vanish, *i. e.* there will be a point of inflexion at which, of course, the tangent line must pass through the vertex of the cone. This is the assumption felt to be true, but stated by me hypothetically in the paper referred to, because a ready demonstration did not at the moment occur to me. The legitimacy of this inference is now vindicated by the above analytical demonstration.

The methods of general and correlative coordinates and of determinants combined possess a perfectly irresistible force (to which I can only compare that of the steam-hammer in the physical world) for bringing under the grasp of intuitive perception the most complicated and refractory forms of geometrical truth.

26 Lincoln's-Inn-Fields,
October 30, 1850.