



145. A Circle and Sphere Connected with a Confocal System of Conics and a Confocal System of Conicoids Respectively

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145. [L². 10. g.] *A circle and sphere connected with a confocal system of conics and a confocal system of conicoids respectively.*

I. The well-known equation for the squares of the semi-minor axes of the two conics of a confocal system which pass through any point $x'y'$ is

$$\lambda^2 - \lambda(x'^2 + y'^2 - a^2e^2) - a^2e^2y'^2 = 0. \quad (\text{See Smith's Conics.})$$

A geometrical theorem follows at once.

For the sum of the roots is $x'^2 + y'^2 - a^2e^2$.

Hence the sum will be positive, zero, or negative, according as the point $x'y'$ is outside, on, or inside the circle $x'^2 + y'^2 - a^2e^2 = 0$.

But b_s^2 , the square of the minor axis of the hyperbola, is the negative analytical quantity, and is the negative of the square on the geometrical minor axis. Hence the circle on SS' as diameter divides those points, which are such that of the two conics through them the minor axis of the ellipse is greater than the minor axis of the hyperbola, from those points having the converse property, while for points on the circle itself the minor axes of the ellipse and hyperbola are equal.

II. Again for a system of conicoids the equation that gives the squares of the least axes of the three conicoids through any point x', y', z' is

$$\frac{x'^2}{a^2 - c^2 + \lambda'} + \frac{y'^2}{b^2 - c^2 + \lambda'} + \frac{z'^2}{\lambda'} = 1.$$

Hence the sum of the roots is

$$x'^2 + y'^2 + z'^2 - (a^2 + b^2 - 2c^2),$$

and in a similar way the points, which are such that of the three conicoids of the system through them the square on the least axis of the ellipsoid is greater than the sum of the squares on the geometrical least axes (viz. the negative of the analytical squares) of the two hyperboloids, are separated from those points for which it is less by the sphere concentric with the system and passing through the angular points of the square formed by drawing parallels through each of the foci in* each of two pairs of foci of the system that lie in the plane containing the two larger axes of all the ellipsoids of the system, lines parallel to the join of the other pair; and on the sphere itself there is equality.

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146. [R. 5. a.] *To prove that the circles on one side of the radical axis of any given non-intersecting coaxial system can be described simultaneously by a swarm of particles under the attraction of a central force.*

The above theorem was devised after the converse of Hamilton's theorem on the law of force in a conical orbit had been demonstrated to me. This converse is: The possible orbits under a central force varying directly as the distance from the centre of force and inversely as the cube of the distance from a fixed straight line are conics having the centre of force and the line for pole and polar. There is one point to be noticed, viz. orbits which are integrals of the equations of motion that are cut by the line in real points must be rejected, as by reference to the law of force it is apparent that at such points the force becomes infinite and the equations of motion used are equations of motion under assumption of finite force and preclude such a case. To prove the above theorem we see that if we take the limiting point on the same side of the radical axis as centre of force, and the line parallel to the radical axis through the other limiting point for the given line, then since by a theorem in geometry all these circles have this point and line for pole and polar, and since the line does not cut any in real points, they are possible orbits.

There remained only one point to settle.

* It will easily be seen which two of the three pairs of foci are meant.