

Logic and the Elements of Geometry.—The *Syllabus of Plane Geometry* (Macmillan and Co., 1875) newly issued, after much deliberation, by the Association for the Improvement of Geometrical Teaching, includes an introductory section which sets forth the logical interdependence of certain associated theorems. In particular, four typical forms of theorem are given as standing in various important relations to one another:—

If A is B, then C is D (1)

If C is not D, then A is not B (2)

If C is D, then A is B (3)

If A is not B, then C is not D (4)

(1) and (2) are said to be *contrapositive* each of the other; (3) is called the *converse*, and (4) the *obverse*, of (1). Now, says the *Syllabus*, while (2) may be always got from (1) by logical inference, it is not so with (3) or (4); each of those by itself requires a geometrical proof independent of the proof of the original theorem; but yet both do not require to be independently proved, because they are themselves in turn (logically) contrapositive one of the other. It will therefore “never be necessary to demonstrate geometrically more than two of the four theorems, care being taken that the two selected are not contrapositive each of the other.”

This view of the relations of the four propositions is not new, even in England, being found in more than one recent work. The *Syllabus*, however, makes an important advance in nomenclature. Hitherto theorem (4) has been designated by the name of *opposite*, used in such glaring inconsistency with the tradition of logical science and with common understanding—opposites plainly being propositions that cannot both be true—that it is difficult to see how the confusion could ever have been tolerated. The word *obverse*, now beginning to be employed in formal logic for what used to be called the *equipollent* proposition—a logical form that has a relation to (4) analogous to that borne by the pure logical *converse* to (3)—was suggested to the Association as a substitute for the so-called *opposite*, and, being frankly accepted, will now, it is to be hoped, for ever displace that unfortunate misnomer.

So far well, but the logician's interest in the scheme does not end with this rectification. Is it open to the geometer to appropriate the words *converse* and *obverse*, and use them in a sense which, if it is not inconsistent with, is at least different from, their original logical application? The words so aptly express the propositions which the geometer has in view, being those which in his (relatively) material science correspond to the converse and obverse of pure formal logic, that he may very fairly appropriate them. At the same time the logician may still more fairly claim that his own original use of the words shall not be put out of view, seeing it is implied (as, from the fundamental character of logical science, it cannot but be implied) in the usage of the geometer. The pure logical converse of (1) is “In at least some case where C is D, A is

B," or "If C is D, A may be B," and this is implied by the geometer in saying that *his* converse, "If C is D, A is B" (amounting to the logician's *inadmissible* simple converse of an universal affirmative proposition) needs by itself a geometrical proof. So the pure logical obverse of (1) is "If A is B, C is not other than D," and this is implied by the geometer in saying that *his* obverse, "If A is not B, C is not D," also by itself needs to be proved geometrically. Nor, if the geometer should deny that he does imply logical forms of which he may be ignorant, is the denial of any avail when he accepts (2) under the name of *contrapositive*, and thus expressly accords a place within his science to a process (contraposition) which is not only purely formal, but is, in fact, logical conversion applied in a special manner. The question of real importance, then, is the practical one, how the reference to logical principles may most effectively be made. The mode of reference adopted in the *Syllabus* cannot be pronounced in all respects satisfactory.

The scheme of the four associated theorems, though it has a certain symmetry, is open to objection in that it mixes up logical and extra-logical relations. The relation of (3) to (1), or of (4) to (1), is extra-logical, while the relation of (2) to (1) is purely logical. Would it not be simpler and better to take account only of the "converse" and "obverse" in relation to (1), and say that either of these two, by itself, needs to be demonstrated geometrically after (1), but both need not, because logic, starting with either, will give the other? Of course logic will yield a contrapositive of (1), but why particularise this as (2), when it may be assumed along with still other strictly logical transformations? In the way here suggested, a beginner would, at all events, get a distincter notion of the difference between logic and geometry; and if the plan involved the necessity of somewhat more expressly stating what is the true nature of such a logical process as contraposition, so much the better. There is some confusion in the *Syllabus* on this head.

Thus theorem (2) may unquestionably be obtained from (1) by the strict logical process of contraposition, and would now be called by most logicians its contrapositive (though, by the way, it is a negative, not a positive, proposition); but (1), although in turn it follows logically from (2), cannot be won back by contraposition, any more than a universal affirmative when converted logically into a particular affirmative can be restored, by a second conversion, to its original universal form. The process called contraposition, in all cases where it is applicable, consists of two stages—obversion and conversion. For example, the simple categorical proposition, "All S is P," becomes, when obverted, "No S is not-P," and this last, being farther converted, becomes "No not-P is S," the contrapositive, as it is called, of the original proposition. Now, obviously, this contrapositive cannot be made to yield the original "All S is P" by further contraposition (obversion and conversion), for "No not-P is S," being obverted, becomes the affirmative "All not-P is not-S," and this, being converted, gives "Some not-S is not-P," quite a different proposition from the original one. To get "All S

is P" back again we must proceed, not by obversion and conversion, which together, *in this order and only in this order*, make contraposition, but by conversion first and then obversion—an order of procedure perfectly valid in logic, but unprovided with a special name. Applying this to the case in hand, as (1) cannot be called the contrapositive of (2), so neither can (3) and (4) be called contrapositives of one another: if (4) is the contrapositive of (3), (3) cannot be the contrapositive of (4).

Let it not be said that the point here insisted on is a trivial one—that it is a mere question of naming. If it is important for learners to distinguish between a geometrical process and one purely logical, as the placing of this "Logical Introduction" at the head of the *Syllabus* implies that it is, there can be no controversy as to the necessity of exactly determining the character of the logical process. To call (1) and (2), or (3) and (4), contrapositives of one another, tells the geometrical learner little more than that there is a process called contraposition, which, if applied, will often save him much trouble. As long as he works with simple typical instances of theorems like (1) and (2), it is easy for him to see that the logical equivalence, by whatever name it is called, must hold in both directions, if it is asserted in one; but, when he comes to deal with actual geometrical propositions, even not very complex ones, he will find it difficult to assign the correct contrapositive, unless he is told definitely by what fixed line of logical transformation it may always be reached. In default of special instruction, he will hardly be able to draw from the examples of contraposition signalled throughout the *Syllabus* a consistent notion of the process. At the best, these examples need a good deal of transformation, verbal, if not logical, before they could be seen by a young student to correspond with the typical theorems which are all he has to guide him. One example, on p. 16, illustrates the graver confusion, or rather the positive error of reckoning as contraposition the passage from (2) to (1). It is there said that Theorem 24, "Straight lines that are parallel to the same straight line are parallel to one another" is the contrapositive of Axiom 5 (p. 15)—"Two straight lines that intersect one another cannot both be parallel to the same straight line." In truth the theorem follows almost directly from the axiom, which is a universal negative proposition, by the process of simple (logical) conversion: there is farther necessary a change in the expression amounting to (formal) obversion, but the first was the really critical step. Here, then, it is not logical contraposition, but logical conversion, which it concerns the geometrical student to understand, not to say again that contraposition always involves formal conversion. In short, it is impossible to frame any notion of the process of contraposition which shall apply, as is required in the *Syllabus*, equally to affirmative and negative propositions, unless it is taken to mean simply the establishment of logical equivalence; and even then it would still be necessary, before making any use of the process, to determine in what different ways equivalence may be secured. We are thus

inevitably brought back to the assumption of more than one process, however called.

The conclusion, then, to which I venture to come is that, unless logical principles are set forth more explicitly than in the *Syllabus* and other recent geometrical books, the reference to them is little likely to be of practical service to beginners. One thing is certain that, if logical principles were familiar to the geometrical beginner, he would both learn geometry better and at the same time, in the process, singularly strengthen his grasp of logical principles. The notion will be scouted that a boy should be expected to have learned logic before beginning geometry, and I by no means argue that he should; but I would yet maintain that nothing could be easier than to give boys along with instruction in grammar all the knowledge of logical principles that is necessary as a preparation for their instruction in geometry. For this, doubtless, it would be necessary that teachers of grammar should have learned logic, but that is not a very extravagant requirement.

EDITOR.

XII.—NEW BOOKS.*

Fragments on Ethical Subjects, by the late GEORGE GROTE, Murray.

FROM the large accumulation of manuscripts left by Mr. Grote, it has been possible to rescue some interesting fragments, partly didactic and partly historical, bearing upon Ethics. These are now collected into a volume, and arranged into six separate Essays.

Four of the Essays are occupied with the more usual questions discussed in modern times in connection with Ethics—the nature of Conscience and the Standard of Morals. To the first of the two—the nature and mental origin of the Moral Sentiment or Conscience—the greatest part of these four Essays is devoted.

Mr. Grote's positions are much the same as those taken by Utilitarians generally. He disputes the instinctive origin of the moral sentiment, endeavouring to show how it can be otherwise accounted for. He disputes the personal or individual nature of conscience, alleging that it has neither meaning nor existence except with reference to society. On the same ground he lays great stress on the correlation of Obligation and Right; the ethical sentiment, he says, is a sentiment of *regulated social reciprocity* as between the agent and the society wherein he lives.

"With regard to the way in which ethical sentiment was first generated, on the original coalescence of rude men into a permanent social communion, we have no direct observation to consult, and must therefore content ourselves with assigning some unexceptionable theory. But with regard to the way in which ethical sentiment is sustained and transmitted, in a society once established, we have ample experience and opportunity for observing before our eyes. We know perfectly that children are not born with any ethical sentiment: they acquire it in the course of early education, and we

* See p. 6 above.—Ed.