gas series; the nitrogen being evolved from the albuminous matter of the wort. By diminishing the pressure the amount of evolved hydrogen increased, and with this an increase in the amount of acetic acid and aldehyde. These products, though very small compared with the alcohol and carbonic acid, the chief resultants of ferment action are yet sufficient to account for the etherial odour of a fermenting tun. Pasteur had previously noticed the production of minute quantities of volatile acids. On electrolysing a weak aqueous solution of invert sugar, Brown obtained carbonic acid, hydrogen, and oxygen, and at the same time an appreciable quantity of aldehyde and acetic acid, together with a small quantity of formic acid. It may be that water is decomposed in fermentation in small quantities precisely as occurs in ordinary vegetation; though highly probable, we have, however, no definite facts in support of the assumption. Our present knowledge of the chemistry of fermentation is somewhat vague and general, and much remains to be done before we shall be enabled by purely physical means to decompose sugar so as to produce the results brought about by the yeast cell.

As we progress in knowledge, so does our power to imitate the products of life-action increase, and assuredly the time will arrive when alcohol will be produced by simple physical or chemical means. For many a year to come, however, we must continue to depend upon the wonderful organisms known as yeast. Their life history and action on liquids have been elucidated by the genius and patient toil of Pasteur, and he has enabled us to select such ferments as are required to produce the result desired, and hence we are no longer the sport of chance.

The brewer and wine-maker are not alone in the debt due to the illustrious Frenchman whose work we have briefly examined; we are all interested in the far wider field of germ action opened out by him, whereby many a disease of man hitherto as dark and unexplained as was fermentation, will be brought under the illuminating light of his teaching.

He who has thus shown us the key whereby we may open the locked-up secrets of nature may rest assured of the gratitude of his fellow-men, given with the greater earnestness and respectful sympathy from the knowledge that our guide has impaired health and sight in his labours for others.

France has given many a great name to the roll of fame, but none more noble or more worthily inscribed thereon than that of Pasteur. CHARLES GRAHAM

CAYLEY'S ELLIPTIC FUNCTIONS

Elementary Treatise on Elliptic Functions. By Arthur Cayley. (Cambridge: Deighton, Bell, and Co.).

THIS is a book thoroughly worthy of the great name of its author. It is difficult to know which to admire most, the grasp of the subject, the extreme simplicity of its exposition, or the neatness of its notation. It will, we think, at once take its proper place as the leading text-book on the subject.

In regard to notation, it seems to us to be thoroughly good throughout, not only in respect of the adoption of Gudermann's suggestion of the very short forms sn, cn, dn, for the sine, cosine, and elliptic radical of the amplitude of the function of the first kind, but throughout. In particular, we note an important typographical simplification in the suppression of the common denominator in long series of fractional formulæ, the denominator being given once for all, and its existence in each separate formula merely indicated by the sign of division (÷) This is a simplification, some equivalent of which we ourselves, and probably most of those who have worked at elliptic functions, have used in our private papers; but it is a new thing, and a very good thing, to see it introduced in a systematic form in a printed book.

Another very useful feature belonging to mechanical arrangement is, that the first chapter contains a general outline of the whole theory, so that its perusal enables the reader to see at a glance the plan and intention of the work. He is thus enabled at once to bring intelligent attention to bear upon his reading, instead of being distracted by the wonder as to what it is all driving at.

The intention of the work is, firstly, the direct discussion and comparison of the three forms of elliptic integral, and of the doubly periodic functions which are regarded as the direct quantities of which these integrals are inverse functions. Then the auxiliary functions Z, H, and H are taken up in an elementary form, and after this the transformation of the elliptic functions, by division or multiplication of the primary integral, with the corresponding change of modulus and amplitude. These are very fully and clearly discussed. In particular, the connection between the transformation of the radical in the elliptic integrals, and the formulæ of multiplication, is clearly brought out. Legendre had left this as a very puzzling, although necessary, inference, which he scarcely stopped to discuss. After this comes a discussion of the q functions, with a further discussion of the functions H and H, and then some miscellaneous developments.

The work is strictly confined to elliptic functions and their auxiliaries. The more general theories of Abel and Boole find no place in it, nor is there any general discussion of single and double periodicity such as forms the foundation of the work of Messrs. Briot and Bouquet. There are but few examples of the computation of particular values of the elliptic functions, and no account of general methods of computation, either of isolated values, or of tables, or of the arithmetic connected with them; nor are ultra-elliptic functions touched upon. The geometrical applications or illustrations of the elliptic integrals and functions are but meagre, and no mechanical applications are given.

The arithmetical work is quite rightly omitted. That will find a much better place in the hand-book or introduction which will doubtless accompany or follow the great tables of elliptic functions now being printed for the British Association. There is, however, one point which we think it an omission to notice, and that is the solution of the addition equation by means of auxiliary angles. (See Legendre, "Traité des Fonctions Ell.," vol. i., p. 22; or Verhulst, § 19, p. 40.) It is no defect, again, that the mechanical applications are omitted. These are better studied as they arise, as a part of mechanics rather than of analysis. But as regards geometry, we think there has been done either too much or too little. For instance, we have the usual theory of the representation of the arcs of the ellipse and hyperbola by those functions, and we have a long disquisition on the geometrical representation of the elliptic integral of the first kind by an algebraic curve; while there is no mention of the late John Riddle's discovery, that the arcs of the curves by which circles on the sphere are represented in Mercator's projection are directly given by, and absolutely co-extensive with, the elliptic integrals of the first kind, the amplitude being simply the longitude on the sphere. We think this quite as simple and as important as the discussion of the lemniscata. If we are to go into geometry at all, it might be as well also to make some allusion to Dr. Booth's discussion of the spherical conics, and to Mr. Roberts's integration of the Cartesians.

Then, again, we have an account of Jacobi's geometrical theorem in its original form, depending upon a family of circles having the same radical axis, while the corresponding theorem, depending upon circles having two inverse points in common, given by Chasles (see his "Géometrie Supérieure," cap. xxxi., p. 533), which much more directly represents both amplitudes and moduli, is not mentioned explicitly, although it is involved in the geometrical exposition given of Landen's theorem.

We have also been unable to find any account of Jacobi's reduction of the integral of the third kind to the form—

$$\int d\phi .\ E\phi : \Delta\phi$$

The transformations of the functions are worked out with great completeness, the results being tabulated in some rather formidable-looking, but really very convenient, schedules. This part of the work is carried almost to an extreme.

On the whole the book is one of the most important contributions to mathematical literature which has appeared for a long time. It is well done, and covers ground that was previously but ill occupied. It is clearly printed, and the fact that the proof-sheets have been revised by Mr. J. W. L. Glaisher is a guarantee for the correctness of detail. C. W. MERRIFIELD

OUR BOOK SHELF

Instruction in Photography. By Capt. Abney, R.E., F.R.S., &c., Instructor in Chemistry and Photography at the School of Military Engineering, Chatham. Third Edition. (London: Piper and Carter, Gough Square, Fleet Street, E.C., 1876.)

WE are very glad to find that Capt. Abney did not carry out the intention which he mentions in the preface of not producing another edition of his well-known "Instruction in Photography." That the little volume is widely known and appreciated is shown by the fact of its having reached a third edition, and we can only say that it well deserves its success. A photographer of the author's well-known skill and repute could not fail to be able to instruct others in his art, but when in addition he has gained large experience by continued practical teaching in such a school as that at Chatham his lessons become additionally valuable.

Capt. Abney does not enter much into theory, though he gives very good and simple accounts, illustrated by chemical equations, of the principal changes occurring during the processes described. We observe that he announces the forthcoming publication of a "Photography" among Messrs. Longmans' Text-books of Science, in which he proposes to deal more fully with the theoretical part of the subject. We shall look forward to this with considerable interest; meanwhile, for practical instruction in the art this little book distances all competitors. R. J. F.

LETTERS TO THE EDITOR

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts. No notice is taken of anonymous communications.*]

Just Intonation

THE errors and oversights—in my paper in NATURE, (vol. xv. p. 159)—with which Mr. Chappell charges me, are imaginary. To make the matter clearer, the vibration numbers of a diatonic scale started from $\frac{3}{2}$ as a tonic are—

$$\frac{3}{2},\ \frac{27}{16},\ \frac{15}{8},\ 2,\ \frac{9}{4},\ \frac{5}{2},\ \frac{45}{16},\ 3.$$

In order to keep to the same part of the keyboard, let the last five notes be depressed one octave, and we get this series:—

$$1,\ \frac{9}{8},\ \frac{5}{4},\ \frac{45}{32},\ \frac{3}{2},\ \frac{27}{16},\ \frac{15}{8},\ 2,$$

where $\frac{3}{2}$ is the tonic, and 1 or 2 the subdominant. A similar explanation applies to the scale starting from $\frac{4}{3}$ as tonic. With respect to the "comma of Pythagoras," I am not aware of any "generally adopted miscalculation." *Who* was the real discoverer of that interval is a matter of no consequence; but in a system such as the Pythagorean, which was tuned by true fifths, it would have been not only a very natural but an essential inquiry, "What definite number of fifths corresponds with another number of octaves?" This, without at all necessitating the supposition of the existence among the Greeks of instruments of an immense range in octaves, would be but an easy arithmetical calculation resulting necessarily in the conclusion that there was no exact coincidence between fifths and octaves, but that twelve fifths differed by a very small quantity from seven octaves. This small difference is therefore very aptly termed the Pythagorean comma. Now in the equal temperament system twelve fifths just coincide with seven octaves, so that the despised comma of Pythagoras is really a measure of the error of the equal temperament fifth. In fact, putting P for the comma in question—

$$P = \left(\frac{3}{2}\right)^{12} 2^{-7} \therefore\ 2^{\frac{7}{12}} = \frac{3}{2}\ P^{-\frac{1}{12}}$$

so that if 2 N be the vibration number of the lower tone of an E T fifth, that of the upper tone will be $3\,N\,P^{-\frac{1}{12}}$, which is in error by $3\,N(1 - P^{-\frac{1}{12}})$ vibrations, giving rise to a number of

$$\text{Beats} = 6\,N(1 - P^{-\frac{1}{12}})$$

per second in the E T fifth. Not, however, that it is necessary to allude to Pythagoras or seven octaves to get those beats.

The grounds upon which Mr. Chappell declines to accept 24, 27, 30, 32, 36, 40, 45, 48, as representing the vibration numbers of the diatonic scale are not very clear, certainly; and repudiating these, he can of course have no sympathy with Colin Brown's keyboard.

To revert shortly to this, the subject of my previous communication,—in his new and very interesting work on "Temperament," Mr. Bosanquet has given a description of Colin Brown's keyboard, but in so peculiar a manner that it is really difficult to recognise the instrument at all, and neither its elegance nor simplicity are brought out as I think they should be.

A. R. CLARKE

Ordnance Survey, Southampton, January 9

South Polar Depression of the Barometer

I THINK it probable that on this subject Mr. Murphy's views and my own might have appeared more in harmony if we had neither of us expressed them with so much brevity. In my letter on Ocean Currents in NATURE, vol. xv. p. 157, I was incidentally led to speak of the barometric depression round the South as greater than that round the North Pole. In speaking