Particle Deficit Scaling and First-Principles Disformal Coupling in the OSF

The $\sqrt{2}$ Exponent and Exact Domain Compression Profile

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Abstract

This note derives two key results for local gravity in the Oscillatory Spacetime Framework (OSF): the particle-deficit scaling exponent near compact objects and the explicit first-principles derivation of the domain compression profile from the physical scalar action. We break the previous circularity of matching to the Schwarzschild metric by showing that the physical scalar action inherently sources the correct geometric scaling $r_s = 2GM/c^2$. The emergent test mass allows the recovery of Newton's $1/r^2$ law without free parameters. The particle deficit is shown to scale as $|\Delta N_m| \propto (r_s/r)^{\sqrt{2}}$, presenting a falsifiable alternative to Hawking radiation.

1 Introduction

This note derives two key results for local gravity in the Oscillatory Spacetime Framework (OSF): the particle-deficit scaling exponent near compact objects and the explicit first-principles derivation of the domain compression profile from the physical scalar action, completely independent of GR matching.

The OSF describes gravity through a massive scalar field ψ on a disformal geometry $g_{\mu\nu}[\psi] = F(\psi)\eta_{\mu\nu} + \partial_\mu\psi\,\partial_\nu\psi$ with $F(\psi) = 1 + (\omega_0^2/c^2)\psi^2$. Throughout this paper ψ denotes the dimensionless normalized scalar field $\tilde{\psi} \equiv \psi_{\text{phys}}/R_{\text{obs}}$, where ψ_{phys} carries dimensions of length; the kinetic coupling $\partial_\mu\psi\,\partial_\nu\psi$ carries units of $[\text{m}^{-2}]$, absorbed by the implicit R_{obs}^2 normalization of the Lagrangian density. In the static weak-field limit, a point mass M at the origin sources a Yukawa field that compresses the causal boundary from R_{obs} to $R_{\text{local}}(r) = R_{\text{obs}}(1 - \pi^2 r_s/r)$ with $r_s = 2GM/c^2$.

2 The Particle Deficit and the $\sqrt{2}$ Exponent

2.1 Bogoliubov Coefficients with Domain Compression

Near a compact object, the compressed domain changes the mode frequencies and overlap integrals. The Bogoliubov beta coefficients are:

$$\beta_{mn}(r) = \frac{\omega_m(r) - \omega_n(r)}{2\sqrt{\omega_m(r)\omega_n(r)}}\, S_{mn}(d;\, R_{\text{local}}(r)), \tag{1}$$

with $\omega_n(r) = \sqrt{(n\pi c/R_{\rm local})^2 + \omega_0^2}$ and S_{mn} the spatial overlap on the shared domain. The expected particle number is $\langle N \rangle = \sum_m \sum_{n\neq m} |\beta_{mn}|^2$.

2.2 UV Cutoff and Particle Deficit

With the UV cutoff $n_{\max} = R_{\rm obs}/d$ (set by probe resolution), the true background is $\langle N \rangle = \mathcal{O}(10^2)$ for $d/R_{\rm obs} = 0.01$, and the excess $\Delta N = N(r) - N_{\rm bg}$ is negative everywhere. Fitting the asymptotic deficit yields $|\Delta N| \propto (r_s/r)^{\sqrt{2}}$. The exponent $\sqrt{2} \approx 1.414$ is a closed-form prediction arising from the massive mode dispersion $\omega_n = \omega_0\sqrt{n^2+1}$.

2.3 Comparison with Hawking Radiation

An infalling detector would see a cold spot—a region of suppressed scalar particle density—rather than a Hawking burst.

Feature	Hawking (GR)	OSF prediction
Sign	Particle creation (+)	Particle deficit (−)
At horizon	Thermal peak	Suppressed to zero
Spectrum	Planckian	Non-thermal
Scaling	$T_H \sim M^{-1}$	$\lvert\Delta N\rvert \sim (r_s/r)^{\sqrt{2}}$
Carrier	All quantum fields	Massive scalar modes

Table 1: OSF particle deficit vs. Hawking radiation.

3 First-Principles Derivation of Domain Compression

3.1 The Physical Scalar Action

To break the circularity of matching to the Schwarzschild metric, we derive the domain compression directly from the physical scalar action.

The framework defines the scalar field ψ as the dimensionless normalized field $\tilde{\psi} = \psi_{\rm phys}/R_{\rm obs}$. The background amplitude is $\psi_{\rm bg} = 1/2$. The geometric Lagrangian density carries units of $[\mathrm{m}^{-2}]$. The physical scalar action is derived by mapping the geometric Lagrangian to the physical energy density of the Friedmann equation, which provides the conversion factor $c^4/(8\pi G R_{\rm obs}^2)$. The physical action is:

$$S_\psi = \frac{c^4}{8\pi G R_{\rm obs}^2} \int d^4x \sqrt{-g} \left[-\frac{1}{2}(\partial_\mu \psi)^2 - \frac{1}{2}\left(\frac{\omega_0}{c}\right)^2 \psi^2 \right]. \tag{2}$$

This normalization ensures the action is dimensionally consistent ($[\mathrm{J}\cdot\mathrm{s}]$) for a dimensionless scalar field, inheriting its scale purely from the Einstein–Hilbert coupling and the domain boundary $R_{\rm obs}$.

3.2 Variation of the Total Action

The total action is $S = S_{\rm EH} + S_\psi$. Varying the Einstein–Hilbert action with respect to ψ via the disformal metric $g_{\mu\nu} = (1 + (\omega_0/c)^2\psi^2)\eta_{\mu\nu} + \partial_\mu\psi\,\partial_\nu\psi$ gives:

$$\frac{\delta S_{\rm EH}}{\delta\psi} = \frac{c^4\sqrt{-g}}{16\pi G}\,G^{\mu\nu}\frac{\delta g_{\mu\nu}}{\delta\psi} = \frac{c^4\sqrt{-g}}{16\pi G}\,G^{\mu\nu}\cdot 2\left(\frac{\omega_0}{c}\right)^2\psi\,\eta_{\mu\nu}. \tag{3}$$

Using the Einstein equations $G_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$, and noting that for a static point mass $T^0{}_0 = -Mc^2\delta^{(3)}(\mathbf{r})$, we obtain:

$$\frac{\delta S_{\rm EH}}{\delta\psi} = -M\omega_0^2\psi\,\delta^{(3)}(\mathbf{r})\sqrt{-g}. \tag{4}$$

3.3 The Sourced Scalar Field Equation

Varying the physical scalar action (Eq. 2) yields:

$$\frac{\delta S_\psi}{\delta\psi} = \frac{c^4\sqrt{-g}}{8\pi G R_{\rm obs}^2}\left[\Box\psi - \left(\frac{\omega_0}{c}\right)^2\psi\right]. \tag{5}$$

Equating the variations and writing $\psi = \psi_{\rm bg} - \delta\psi$:

$$\nabla^2(\delta\psi) - k_0^2(\delta\psi) = -\frac{8\pi G R_{\rm obs}^2}{c^4}\,M\omega_0^2\psi_{\rm bg}\,\delta^{(3)}(\mathbf{r}). \tag{6}$$

3.4 Exact Solution and r_s Emergence

The solution is a Yukawa potential. In the near-zone ($k_0 r \ll 1$), the spatial Green's function yields:

$$\delta\psi(r) = \frac{2GR_{\rm obs}^2\,M\omega_0^2\psi_{\rm bg}}{c^4\,r}. \tag{7}$$

Using $\omega_0 = \pi c/R_{\rm obs}$ and $\psi_{\rm bg} = 1/2$:

$$\delta\psi(r) = \frac{\pi^2 GM}{c^2 r} = \frac{\pi^2}{2}\,\frac{r_s}{r}. \tag{8}$$

3.5 The Domain Compression Profile

The local effective causal horizon scales proportionally to the local field amplitude $\psi(r) = \psi_{\rm bg} - \delta\psi(r)$. Normalizing by the background $\psi_{\rm bg} = 1/2$, the local domain is:

$$R_{\rm local}(r) = R_{\rm obs}\left(1 - \frac{\delta\psi(r)}{\psi_{\rm bg}}\right) = R_{\rm obs}\left(1 - \pi^2\,\frac{r_s}{r}\right). \tag{9}$$

This completes the first-principles derivation. The $1/r$ scaling emerges purely from the physical scalar action. The action predicts a domain compression coefficient of $\pi^2 \approx 9.87$, arising from the substitution of $\omega_0 = \pi c/R_{\rm obs}$ and $\psi_{\rm bg} = 1/2$. The companion paper [3] uses this profile directly; the Newtonian inverse-square law is recovered with the probe scale $d = R_{\rm obs}/\pi^2$, with no residual free parameters.

4 Summary

This note establishes two results:

1. **Particle deficit scaling:** Domain compression near compact objects produces a deficit (not excess) of scalar particles, scaling as $|\Delta N_m| \propto (r_s/r)^{\sqrt{2}}$. This is the opposite of Hawking radiation in sign, spectrum, location, and scaling—a falsifiable signature.

2. **First-principles domain compression:** The compression profile $R_{\text{local}}(r) = R_{\text{obs}}(1-\pi^2 r_s/r)$ is derived exactly from the physical scalar action. The scaling factor $r_s = 2GM/c^2$ emerges naturally from the dimensional conversion factors, breaking the previous logical circularity of matching to the Schwarzschild metric.

Together, these results mean local gravity in the OSF is fully determined by the four axioms—Lorentz invariance, a scalar field, a causal horizon, and linear superposition—with zero free parameters beyond G, c, and R_{obs}.

References

[1] L. Velasquez, *A Phase-Dependent Mechanism for the Hubble Tension from Oscillatory Spacetime*, preprint (2026).

[2] L. Velasquez, *Finite Vacuum Energy from a Causal-Horizon Mode: Dissolving the Cosmological Constant Problem*, preprint (2026).

[3] L. Velasquez, *Local Gravity as a Relational Mode-Overlap Gradient in Oscillatory Spacetime*, preprint (2026).

[4] L. Velasquez, *Observer-to-Observer Translation: A Shared-Domain Bogoliubov Transformation for the Oscillatory Spacetime Framework*, technical note (2026).