



## XIX. On parallel straight lines

Mr. John Walsh

To cite this article: Mr. John Walsh (1824) XIX. On parallel straight lines , Philosophical Magazine Series 1, 63:310, 100-102, DOI: [10.1080/14786442408644475](https://doi.org/10.1080/14786442408644475)

To link to this article: <http://dx.doi.org/10.1080/14786442408644475>



Published online: 29 Jul 2009.



Submit your article to this journal [↗](#)



Article views: 3



View related articles [↗](#)

purpose, with the names of the metals on their respective leaves.

The whole apparatus packs up in a neat mahogany box.

This instrument differs from the galvanometers I have seen described. I use a powerful magnet for detecting and determining a slight galvanic influence; whereas in the galvanometer a feeble needle is used for that purpose. Many other differences might easily be pointed out, whether advantages or not is not for me to determine.

I am, gentlemen,

Yours, &c.

Artillery Place, Woolwich.

WM. STURGEON.

### XIX. On Parallel Straight Lines. By Mr. JOHN WALSH.

A SOLID is that which resists the touch. Surfaces are the boundaries of solids. Lines are the boundaries of surfaces. Points are boundaries of lines. When the surfaces of two solids are such, as that any one surface of the one being placed any where on any one surface of the other, there shall be no space between them,—these are called plane surfaces, and their boundaries are called straight lines. If such surface is placed any where on any other surface any how bounded, and that there is no space between them,—this other also is a plane surface. Two straight lines cannot inclose space. Two straight lines that intersect each other are in the same plane; and three straight lines that meet each other are in the same plane.

Let  $x$  and  $y$  be two straight lines intersecting each other, and let the plane of the triangle  $ABC$  be so placed on the plane of  $x$  and  $y$  as that  $AB$  shall always fall on  $x$ ; and let  $A'B'C'$ ,  $A''B''C''$  be any two positions of the triangle  $ABC$ ; then shall  $A'C'$  be parallel to  $A''C''$ . This follows from the invariability of the angles of the triangle  $ABC$ .

*Axiom.* When a straight line intersects any one of two parallel straight lines, it cannot be parallel to the other.

This is the same as Euclid's axiom. It appears to me to be more evident in this form. It is not however so logical as that of the Greek geometer, as it is intended to elucidate a property of angles.

When a straight line falls on two parallel straight lines, it makes equal angles with them both. (Preceding prop. and axiom.)

Professor Leslie, in a note to his *Elements of Geometry*, second edition, shows that M. Legendre has failed in his attempt  
to

to demonstrate, by the theory of functions, the property of plane triangles, depending on the preceding proposition, that the sums of the three angles are equal to two right angles. M. Legendre, though vanquished, continues to argue still, and is supported in his defence by a foreign geometer of eminence, M. Maurice, as well as by the celebrated British geometers Dr. Brewster and Professor Playfair. When such authorities contend, it were perhaps presumptuous to interfere. It appears to me extraordinary that M. Legendre, a geometer so deeply acquainted with the properties of numbers, should really mistake the nature of number itself. This circumstance is, I believe, common to most geometers. Number expresses the relation between two homogeneous magnitudes. It cannot represent magnitude. Numbers are therefore abstract things. To say "*abstract numbers*," is to say "*abstract abstract things*!" It is as correct language to say "straight right line."

At the origin of grammar, men, then not acquainted with the real nature of numbers, arranged them very improperly under the head of adjectives. We say, "The money is one pound sterling;" "The distance is three metres;" "The right angle is one," &c. We ought to say, "The money has the relation one to the pound sterling;" "The distance has the relation three to the metre;" "The right angle has the relation one to the right angle," &c.:—the pound sterling, the metre, the right angle, &c., being the bases of comparison.

Let  $A B C$  be the angles of any plane triangle, and  $c$  the side adjacent to the angles  $A B$ . It is required to determine the angle  $c$ . For this we have  $C = \theta(A B c)$ . Now M. Legendre says, in a note to the tenth edition of his *Elements of Geometry*, that "the side  $c$  is of a nature heterogeneous to the angles  $A B$ , and cannot coalesce with them in the equation  $C = \theta(A B c)$ . The right angle being the natural unity of angles, it is therefore a number. The angles  $A$  and  $B$  are therefore numbers. They cannot then coalesce with the side  $c$ , which is a straight line; then  $C$  is entirely determined by the angles  $A$  and  $B$  alone; therefore, when two angles of one triangle are equal to two angles of another, the third angle of the one is equal to the third angle of the other."

Now I shall demonstrate that the preceding reasoning fails in three different ways. 1st. It is said the right angle is the natural, that is to say, the necessary unity of angles. I have shown that a number cannot be put for magnitude. The right angle is not the necessary, but is made the arbitrary base of comparison in respect to angles. In respect to the sides, I shall make the metre the base of comparison; then instead of the side  $c$  I shall substitute its relation to the metre, and substituting

stituting for the angles  $A B C$  their relations to the right angle, the equation  $C = \theta(A B c)$  is an equation entirely between numbers, and consequently the number  $c$  cannot be excluded.

2dly. If the side  $c$  is excluded from the equation  $C = \theta(A B c)$ , then in this equation we have  $c = 0$ ; but when  $c = 0$ , then  $A = 0$ ,  $B = 0$ ,  $C = 0$ , then  $0 = \theta(0, 0 \cdot 0)$ . Here the reasoning fails altogether.

Finally: M. Legendre asserts that the side  $c$  is of a nature heterogeneous to the angles  $A$  and  $B$ . Now the number of degrees in the angles of any plane triangle is determined by the circumferences of the circles of which the angular points are the centres. I have demonstrated elsewhere, that  $\frac{N}{y} h = T$  is the equation of the tangent straight line to any curve;  $N$  being the normal,  $y$  the ordinate, and  $h$  any arbitrary increase or decrease of the abscissa; and that it is homogeneous to the equation which determines the length of the arc. Therefore the circumference of a circle and the side of a plane triangle are homogeneous quantities.

When we have to demonstrate a general property which necessarily involves in it notions of infinity, we must rest the demonstration as a postulate which must involve notions of infinity. The difficulty encountered in this theory of parallel straight lines, arises therefore from the constitution of the human mind, and cannot be overcome.

“Men learn the elements of science from others: and every learner hath a deference more or less to authority, especially the young learners, few of that kind caring to dwell long upon principles, but inclined rather to take them upon trust. And things early admitted by repetition become familiar. And this familiarity at length passeth for evidence. Now to me it seems there are certain points tacitly admitted by mathematicians, which are neither evident nor true. And such points or principles ever mixing with their reasonings, do lead them into paradoxes and perplexities.”—(*Berkeley, Defence of Free Thinking in Mathematics*, sec. 21.)

JOHN WALSH.

XX. *A technical Description of Chloraster, a new Genus of Narcisseæ.* By A. H. HAWORTH, Esq. F.L.S. &c.

To the Editors of the *Philosophical Magazine and Journal*.  
Gentlemen,

ALLOW me to reply to E. E. in page 7 of your Miscellany for January last, and to say that the account of the two *Narcisseæ* there mentioned was published in the Botanical Register