



## LXIII. On the figure of the earth

M. De Laplace

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mass, and diameter of a fly, on the other part ; so that one rotation of the fly, with its initial velocity, shall produce a dynamic effect equal to that of the piston in  $n$  successive strokes.

N. B. The mass of the fly is supposed equally distributed over its rim, and the diameter of the crank handle equal to the course of the piston.

15. The major and minor axes of an elliptical billiard-table are  $2a$  and  $2b$ . Suppose an elastic ball to be propelled through one of the foci perpendicularly to the major axis, what will be the rectangular co-ordinates which indicate its position at the tenth reflection? and will it, after any finite number of reflections, move to and fro in the direction of the major axis?

LXIII. *On the Figure of the Earth.* By M. DE LAPLACE\*.

IT has been proved by numerous experiments made with the pendulum, that the increase of gravity follows a very regular progression, and is nearly as the square of the sine of the latitude. This force being the result of attractions of all the terrestrial molecules, observations thereon, compared with the theory of the attraction of spheroids, offer the only means that can enable us to penetrate into the internal constitution of the earth; and the result is, that the earth is formed of strata, of which the density increases from the surface to the centre, round which they are regularly arranged. In the *Connaissance des Temps* for 1821, I published the following theorem, which I demonstrated in vol. ii. of the *Nouveaux Mémoires de l'Académie des Sciences*.

“ If we take the length of the seconds pendulum at the equator as unity, and if to the length of this pendulum, observed at any point on the surface of the terrestrial spheroid, be added, half the height of this point above the level of the sea, divided by half the polar axis, a height which is given by barometrical observation, the increase of this length, thus corrected, will be, on the hypothesis of a constant density below a small depth, equal to the product of the square of the sine of the latitude by five-fourths of the ratio of the centrifugal force to the gravity at the equator, or by 43 ten-thousandths.”

The above theorem is generally true, whatever may be the density of the sea, and the manner in which it covers the earth. Experiments made with the pendulum in both hemispheres agree in giving to the square of the sine of latitude, a coefficient somewhat larger—nearly equal to 54 ten-thousandths. These experiments, therefore, prove sufficiently that the earth is not homogeneous in

\* From *Annales de Chimie et Phys.* tome xi.

the interior, and that the density of the strata increases from the surface to the centre.

But though the earth be, in a mathematical sense, heterogeneous, it may notwithstanding be chemically homogeneous, if the increase of density of its strata is caused only by the increased pressure they suffer as they approach the centre. It may easily be conceived that the immense weight of the superior strata may considerably increase their density, though they may not be fluid; for it is known that solid bodies are compressed by their own weight. The law of the densities which result from these compressions being unknown, we cannot tell how far the density of the terrestrial strata may be thus increased. The pressure and the heat which we can produce are very small, compared to those which exist at the surface, and in the interior of the sun and stars. We cannot even form an idea of the effect of these forces, united in those immense bodies. Every thing tends to make us believe that they existed at one time in a high degree on the earth, and that the phenomena which they have occasioned, modified by their successive diminution, form the present state of the surface of our globe; a state which is nothing more than the element of a curve, of which time is the abscissa, and of which the ordinates will represent the changes that this surface has suffered without ceasing. We are far from knowing the nature of this curve, and we cannot therefore ascend with certainty to the origin of what we observe on the earth; and if, to satisfy the imagination, always troubled by ignorance of the cause of the phenomena which interest us, a few conjectures are hazarded, they should be offered with the utmost caution.

The density of a gas, the temperature remaining the same, is proportional to its compression. But this law, though true within those limits of density in which we have been able to prove it, cannot be applied to liquids and solids, of which the density is very great, compared to that of gas, when the pressure is little or nothing. It may naturally be supposed that these bodies resist compression the more they are compressed; so that the ratio of the differential of the pressure to that of the density, instead of being constant, as with gases, increases with the density. The most simple function which can represent the ratio, is the first power of the density, multiplied by a constant quantity: and this I have adopted, because it unites to the advantage of representing in the simplest manner what we know of the compression of liquids and solids, a facility of calculation in researches on the figure of the earth. Hitherto, mathematicians have not included in this research the effect resulting from the compression of the strata. Dr. Young has called their attention to this object, by the ingenious remark, which may be thus stated, the increase of  
density

density of the strata of the terrestrial spheroid. I have supposed that some interest may be excited by the following analysis, from which it appears that it is possible to explain all the known phenomena depending on the law of the density of these strata. These phenomena are the variation of the degrees of the meridian, and of gravity, the precession of the equinoxes, the nutation of the terrestrial axis, the inequalities which the flattening of the earth produces in the motion of the moon, and lastly, the ratio of the mean density of the earth to that of water, which Cavendish has fixed by an admirable experiment at five and a half. In proceeding from the law already announced of the compression of liquids and solids, I find that, if the earth be supposed to be formed of a substance chemically homogeneous, of which the density is  $2\frac{1}{4}$  that of common water, and which compressed by a vertical column of its own substance, equal to the millionth part of half the polar axis, will augment in density 5.5345 millionths of its first density, it will account for all the phenomena. The existence of such a body may be easily admitted, and is apparent from the surface of the earth.

If our globe were entirely formed of water; and if, in conformity with Canton's experiments, it be supposed that the density of water at  $10^{\circ}$  ( $50^{\circ}$  Fahr.) and compressed by a column of water 10 metres (32.81925 ft.) in height increases by 44 millionths, the flattening of the earth would be  $\frac{1}{166}$ ; the coefficient of the square of the sine of the latitude in the expression of the length of the seconds pendulum would be 59 ten-thousandths, and the mean density of the earth would be nine times that of water. These results differ from observations by more than the errors to which they are liable.

I have been supposing the temperature uniform throughout the whole extent of the terrestrial spheroid; but it is very possible that the heat is greater towards the centre, and that would be the case if the earth, originally highly heated, were continually cooling. The ignorance in which we are with respect to the internal constitution of this planet, prevents us from calculating the law by which the heat decreases, and the resulting diminution in the mean temperature of climates; but we can prove this diminution to have been insensible for these 2,000 years.

Let us suppose a space of a constant temperature, containing a sphere possessing a rotary motion; and that, after a long time the temperature of the space diminishes one degree; the sphere will finally take this new temperature; its mass will not be at all altered, but its dimensions will diminish by a quantity which I will suppose to be a hundred thousandth, a diminution which is nearly that of glass. In consequence of the principle of areas, the sum of the areas which each molecule of the sphere will de-

scribe round its axis of rotation will be the same in a given time, as before. It is easy to conclude from this, that the angular velocity of rotation will be augmented by a fifty thousandth. So that, supposing the time of a rotation to be one day, or a hundred thousand decimal seconds, it will be diminished two seconds by the diminution of a degree in the temperature of the space. If we extend this consequence to the earth, and also consider that the duration of the day has not varied since the time of Hipparchus, by the hundredth of a second, as I have shown by the comparison of observations with the theory of the secular equation of the moon, we shall conclude, that since that time, the variation of the internal heat of the earth is insensible. It is true that the dilatation, the specific heat, the degree of permeability by heat, and the density of the various strata of the earth being unknown, may cause a sensible difference between the results relative to the earth, and those of the sphere we have supposed; according to which the diminution of the hundredth of a second, in the length of the day, would correspond to a diminution of two hundredths of a degree of temperature. But this difference could never extend from two hundredths of a degree, to the tenth; the loss of terrestrial heat corresponding to the diminution of a hundredth of a second in the length of the day. We may observe, even that the diminution of the hundredth of a degree, near the surface, supposes a much greater one in the internal strata; for it is known that ultimately the temperature of all the strata diminishes in the same geometric progression, so that the diminution of a degree near the surface, corresponds to a much greater diminution in the strata nearer to the centre. The dimensions of the earth, therefore, and its *inertial momentum* would diminish more than in the case of the sphere we have supposed. Hence it follows, that if, in the course of time, changes are observed in the mean height of the thermometer placed at the bottom of the observatory caves, it must be attributed not to a variation in the mean temperature of the earth, but to change in the climate of Paris, of which the temperature may vary, with many accidental causes. It is remarkable that the discovery of the true cause of the secular equation of the moon, should at the same time make known to us the invariability of the length of the day, and of the mean temperature of the earth since the time of the most ancient observations.

This phenomenon induces a belief that the earth has arrived at that permanent temperature, which accords with its position in space, and its relation to the sun. It is found by analysis, that whatever be the specific heat, the permeability by heat, and the density of the strata of the terrestrial spheroid, the increase of the heat, at a depth very small, compared to the radius of that spheroid,

roid, is equal to the product of that depth, by the elevation of the temperature of the surface of the earth, above the state of which I have just spoken, and by a factor independent of the dimensions of the earth, and which depends only on the qualities of its first stratum relative to heat. From what we know of these qualities we find that if this elevation was many degrees, the increase of heat would be very sensible at depths to which we have penetrated, and where nevertheless it has not been found.

*Note added by the Editor of Annales de Chimie, &c.*

Our readers, we think, will not be displeased to find here some details of the method by which M. de Laplace has established the invariability of the duration of the day :—

A mean solar day is equal to the time occupied by one revolution of the earth on its own axis, increased by the mean apparent motion of the sun, in the same interval. Theory has proved that the mean apparent motion of the sun, like that of all the planets, is constant; the duration of a solar day, therefore, can only vary by a change in the velocity of the rotation of the earth.

The time in which the moon returns to the same position, relative to the sun, for example, its conjunction, is called a lunar month. This interval is evidently independent of the velocity of the earth's rotation. Our globe might even cease to turn on its centre, without the moon's advancement in its orbit suffering any alteration. From hence results a very simple method of discovering if the duration of the solar day has changed.

Suppose the duration of a lunar month to be now ascertained by direct observation; that is, how many days, and fractions of days, the moon occupies in returning to its conjunction with the sun. It is evident that on repeating this observation at another time, a different result will be found, if the length of the day has changed, if at the same time the velocity of the moon has not changed. The month will appear longer, if the length of the day has diminished; and on the contrary, shorter, if the day has increased. The constancy of the lunar month will indicate the invariability of the length of the day.

All observations combine to prove that from the time of the Chaldeans, to our own days, the duration of the lunar month has been gradually diminishing. It follows, therefore, from what has been stated, either that the velocity of the moon has increased, or that the solar day has lengthened. But M. de Laplace has discovered by theory, that there is in the motion of the moon, an inequality known by the name of *secular equation*, which depends on the variation of the excentricity of the earth's orbit, and of which the value in each century may be deduced from the change of this excentricity. By the assistance of this equation,

tion, the increase of the forementioned velocity is perfectly accounted for. There is, therefore, no reason to suppose that the duration of the day is not sensibly constant.

Let us for a moment admit, with M. de Laplace, that this duration surpasses at present that of the time of Hipparchus, by the hundredth of a decimal second. The duration of a century now, or of 36,525 solar days, would be longer than the duration of a century 2,000 years ago, (Hipparchus lived about 120 years before our æra,) by 365."25. In this interval of time, the moon describes an arc of 534".6; this quantity, therefore, expresses the difference between two arcs traversed by the moon in a century now, and in one of the time of Hipparchus; but as these arcs, determined by observation and corrected by the secular equation, do not differ by a quantity so large, we may conclude that in this long interval the duration of the day has not varied one hundredth of a second.

LXIV. *Account of some remarkable Facts observed in the Deoxidation of Metals, particularly Silver and Copper. By SAMUEL LUCAS, Esq.\**

Sheffield, May 31, 1815.

DEAR SIR,—**W**HEN I had the pleasure of seeing you in Manchester, I mentioned having observed that pure silver, when melted, and while in a fluid state, had the property of uniting with a small proportion of oxygen, not only from the atmosphere, but also from other bodies which gave it out at a suitable degree of heat, as some of the nitrates for instance; and that the oxygen thus absorbed remains united with the silver only so long as it continues in a fluid state, or, while fluid, until some substance be applied having a more powerful attraction for the oxygen. In proof of this, I now send, for your inspection, a few specimens of silver that has been in the different states, and which carry the external marks; and also a bottle of the gas collected from silver, which had been exposed to the influence of the atmosphere by cupellation.

If silver in large quantities, after having been exposed in a melted state to a current of oxygen gas or atmospheric air, be allowed gradually to cool, the surface first becomes fixed or solid; this soon bursts, ebullition ensues, and an elastic vapour in considerable quantity escapes, driving before it a portion of the internal fluid metal, which, becoming solid as it is brought to the surface, produces the protuberances as shown by the accompanying specimen No. 1. This ebullition continues from  $\frac{1}{4}$  to  $\frac{1}{2}$

\* From Manchester Memoirs, vol. iii.