

that of light, the potential at any point of the line in front of the particle should tend to zero.

Again, if a sphere be described round the moving charge as centre, and the integral of normal force due to the moving charge be taken over its surface, it will be found that, with the simpler expression for the potential, the value of the integral is $\frac{ec}{2u} \log \frac{c+u}{c-u}$, whereas with the expression given above, it is e , in accordance with Gauss's Theorem.

XII. *Note on Gravitation.* By O. W. RICHARDSON, F.R.S.,
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THIS note is mainly a brief account of experiments undertaken to seek a connexion between gravitation and electricity. The line of attack is largely empirical and follows a train of thought developed prior to the current theories of gravitational relativity †. Although no positive results have been obtained it seems desirable that a short account of the experiments should be published, both on account of the importance of the subject and also because the results may set limitations on the possible scope of other gravitational theories. The central idea was that gravitation might be due to a slight modification of the law of force between the positive and negative electrons out of which matter is built up, such modification arising out of the different amounts of energy associated with the respective positive or negative electrons as evidenced by their different masses. This point of view is to a certain extent antipathetic to the relativity theory of gravitation, and its comparative failure may be considered in some degree a success for the latter.

One consequence of the point of view referred to is that the ratio between weight and mass should not be quite the same for different elementary substances. The experiments of Bessel and Eötvös show that this ratio is extraordinarily constant for a number of substances. Nevertheless experiments on this point were commenced by the writer in 1914, were abandoned during the war, and have now been recommenced by Mr. H. H. Potter. The present note does not

* Communicated by the Author.

† O. W. Richardson, *Phys. Rev.* vol. xxxi. pp. 610, 709 (1910);
'Electron Theory of Matter,' chap. xxii. (1914).

refer to those experiments which are still uncompleted. As their subject-matter forms the whole basis of gravitational relativity, their importance is obvious apart from the line of attack here considered, and they will be dealt with separately on completion.

One direction in which we might expect to find effects of the kind here contemplated is in the displacement of the spectral lines as between different isotopes. There is of course such a displacement to be expected on Bohr's theory on account of the dynamical effect of the changed mass of the nucleus. There might, however, be an additional effect due to the modification of the field of force arising from the nuclear charge owing to the change in the mass associated with it. As a matter of fact, Aronberg * and Merton † both find a small displacement for a given line as between uranium lead—ordinary lead—thorium lead. For example, Merton for $\lambda = 4058 \text{ \AA.U.}$ finds

$$\lambda (\text{Ur. lead}) - \lambda (\text{ord. lead}) = 0.0050 \text{ \AA} \pm 0.0007 \text{ \AA}$$

$$\lambda (\text{ord. lead}) - \lambda (\text{Th. lead}) = 0.0022 \text{ \AA} \pm 0.0008 \text{ \AA}.$$

Whilst these displacements are small they are nevertheless many times larger than the Bohr shift calculated from the dynamical effect of the change in the mass of the nucleus. Merton also finds a displacement as between thallium from pitchblende and ordinary thallium. On the other hand, ordinary lithium (atomic weight 6.94), which has been claimed to be a mixture of isotopes (atomic weight 7 with a small proportion of 6), shows no evidence of the expected duplicity in its spectral lines. However, there is no evidence of the Bohr shift, which should be large in this case; so that the lithium observation does not seem very helpful in this connexion.

There are, of course, other possible causes for the displacement of the spectral lines of isotopes such as, for example, a small variation in the electrostatic field of the nucleus arising from a difference in the configuration of its electronic constituents; but if, for the sake of argument, we assume it arises from a modification of the law of force of the more fundamental kind now under consideration, the connexion between such modification and the change of wave-length may be calculated as follows:—

Using the notation of 'Electron Theory of Matter,' 2nd edn. p. 616, let capital letters denote positive and small

* *Astrophys. Journ.* vol. xlvii. p. 96 (1918).

† *Roy. Soc. Proc. A*, vol. xevi. p. 388 (1920).

letters negative charges. For an electron charge e' mass m' revolving round a nucleus containing positive electrons of total charge E and mass M and negative electrons of total charge e and mass m , the force at distance r apart is

$$F = \frac{1}{4\pi r^2} \left\{ (E+e)e' + a[(E+e)m' + e'(M+m)] \right. \\ \left. + c \left[(E+e) \frac{m'^2}{e'} + e' \left(\frac{M^2}{E} + \frac{m^2}{e} \right) \right] + g(M+m)m' \right\}.$$

For isotopes $E+e$, e' , and m' have the same value but M , m , E , and e are different. The difference in F for two isotopes is thus

$$\delta F = \frac{1}{4\pi r^2} \left\{ ae'\delta(M+m) + ce'\delta \left(\frac{M^2}{E} + \frac{m^2}{e} \right) + gm'\delta(M+m) \right\}.$$

To a first approximation M/E and m/e are respectively constant, being the oxygen nucleus and negative electron values, also m/M is small, so that

$$\delta F = \frac{1}{4\pi r^2} A \delta W,$$

$$\text{where } A = ae' + ce' \frac{M}{E} + gm'$$

$$\text{and } W = M + m.$$

The forces will thus be the same as if the nuclear charge were increased by an amount

$$\left(a + \frac{M}{E}c + g \frac{m'}{e'} \right) \delta W \equiv B \delta W.$$

According to Bohr's theory the frequency of any particular spectral line is

$$\nu = 2\pi^2 \frac{M_1 m_1 E_1^2 e_1^2}{(M_1 + m_1) h^3} \left\{ \frac{1}{\tau_r^2} - \frac{1}{\tau_s^2} \right\},$$

e_1 and m_1 being the charge and mass of the circulating electron and E_1 and M_1 the charge and mass of the nucleus and electron system about which it circulates. τ_r and τ_s are positive integers and h is Planck's constant. The corresponding frequency for the isotope is

$$\nu' = 2\pi^2 \frac{(M_1 + \delta W) m_1 (E_1 + B \delta W)^2 e_1^2}{(M_1 + m_1 + \delta W) h^3} \left\{ \frac{1}{\tau_r^2} - \frac{1}{\tau_s^2} \right\}.$$

Thus

$$\frac{\lambda'}{\lambda} = \frac{\nu'}{\nu} = 1 + \delta W \left[\frac{m_1}{M_1(M_1 + m_1)} + 2 \frac{B}{E_1} \right] \\ + \delta W^2 \left[\frac{m_1}{M_1(M_1 + m_1)^2} + 2 \frac{B}{E_1} \frac{m_1}{M_1(M_1 + m_1)} + \frac{B^2}{E_1^2} \right] + \dots,$$

and the relative shift

$$-\frac{\delta\lambda}{\lambda} = \delta W \left[\frac{m_1}{M_1(M_1 + m_1)} + 2 \frac{B}{E_1} \right] \\ + \delta W^2 \left[\frac{m_1}{M_1(M_1 + m_1)^2} + 2 \frac{B}{E_1} \frac{m_1}{M_1(M_1 + m_1)} + \frac{B^2}{E_1^2} \right] + \dots$$

The atomic weights of the different leads tested by Merton do not appear to have been measured, but we need not go beyond the order of magnitude of the different terms, and for this accuracy it will be sufficient to put

$$-\delta\lambda/\lambda = 1.2 \times 10^{-6}, \quad \delta W = 0.5, \quad M_1 = 207, \quad \frac{1}{m_1} = 1860.$$

The first term on the right represents the Bohr shift. It amounts to about 6×10^{-9} and therefore forms only a small fraction of the observed $\delta\lambda/\lambda$. It appears that all the other terms are small except the second which has to account for practically the whole of $\delta\lambda/\lambda$. On the type of view here considered then it appears that B/E_1 must be of the order 10^{-6} .

If B/E_1 were as large as this it would have several notable consequences. In terms of the coefficients the value of B/E_1 is $\frac{1}{E_1} \left(a + \frac{M}{E} c + g \frac{m'}{e'} \right)$. The last term in this expression represents the ordinary Newtonian gravitational force due to the mass m' of the electron. On substituting the numerical data the value of $\frac{gm'}{E_1 e'}$ is found to be 2.5×10^{-16} .

It is thus a very minute fraction of the whole of B/E_1 . This would mean that in a gravitational field positive and negative electrons would be acted on by opposite forces of nearly equal magnitude but much larger than the Newtonian forces for neutral particles of equal masses. In an insulator at rest this would give rise to an electric polarization proportional to the gravitational intensity. In a conductor in equilibrium we should expect a separation of the charges giving rise to an equilibrating electric field. In the earth's gravitational

field, if $B/E_1 = 10^{-6}$, this field should be equal to 2.13×10^{-3} volt/cm.

We might also expect currents to be generated in a circuit of which part was falling under gravity relative to the rest. I have made experiments to test this point. About 200 lb. of mercury were allowed to fall in a continuous stream from a container through a short tube 9 mm. in diameter for a distance of about a metre to a container at a lower level. The containers were electrically connected by mercury-filled glass tubes (to avoid thermoelectric disturbances) with a galvanometer having a resistance of 600 ohms and a sensitivity of 1 division for 2.6×10^{-10} ampere. The resistance of the rest of the circuit was negligible in comparison. No deflexion amounting to a scale-division could be observed when the mercury was running steadily. At the start and finish rather irregular deflexions were observed, but there are a number of ordinary explanations which might account for them. However, I do not consider that these experiments rule out the possibility of such effects at the start and finish. In order to settle this point definitely more elaborate apparatus would be required than I happened to have available.

On the view we are considering the force on an electron in a gravitational field would be quite different from the ordinary Newtonian attraction on its mass. The acceleration of an electron in the earth's gravitational field, for example, would be about 10^{12} cm. sec.⁻². A correlated effect is that the apparent weight of equal electric charges should, to a very close approximation, change sign with the sign of the charge. (The approximation neglects the gravitational term in B , which is about 10^{-10} of the whole.) This can be tested by placing a very light earth-connected plate in a median position in a flat insulated metal box and finding if the apparent weight of the plate varies with the sign of the charge on the box. Very careful experiments on this principle have been made in the Wheatstone Laboratory by Dr. L. Simons, and as the result is important they are described in the appendix. Dr. Simons finds no displacement, although he concludes that he could have detected a deflexion equal to about one-third of that to be expected if $B = 10^{-6} E_1$.

The charges which would develop on a conducting sphere of gravitating matter are apparently much too small to account for the earth's magnetic field. I find that the equations are solved by an electric distribution of uniform volume density throughout the sphere, and if the sphere is uncharged an

equal and opposite total charge of uniform surface density over the surface. If this rotates uniformly in a periodic time τ the magnetic field at external points is the same as that of a small magnet along the axis of rotation having a magnetic moment

$$\mu = \frac{4\pi}{15} \frac{BM}{c\tau} R^2,$$

where M is the mass and R the radius of the sphere and c is the velocity of light. Putting $B=10^{-6} E_1$ and the values of M and R for the earth, this gives the magnetic intensity at the pole as 5.9×10^{-12} .

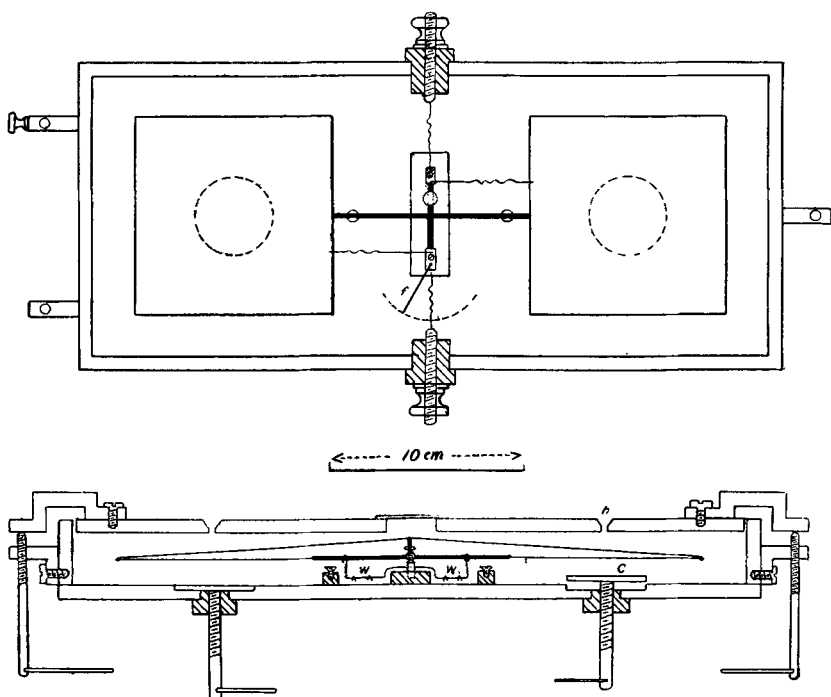
One positive conclusion which may be drawn from these notes is that the displacement of spectral lines as between isotopes cannot be regarded as furnishing evidence of the existence of cross terms in the law of force between electric charges and mass elements. Its explanation is probably to be sought in an electrostatic effect of some variation in the geometrical structure of the elements of the nucleus.

APPENDIX by Dr. L. Simons.

The apparatus consisted of a heavy brass box of internal dimensions $34 \times 14 \times 2.5$ cm., the lid, resting on three screws, being adjustable. A silica framework consisting of two squares of 10 cm. side were mounted coplanar with their centres at a distance of 20 cm. apart, and a silica cross-piece arranged to carry brass ferrules at its ends through which there passed pointed steel screws which rested on steel planes. Aluminium foil 0.002 cm. thick was fixed over the two squares, and the whole balanced symmetrically within the box which was lined with the same aluminium in order to avoid irregularities arising from contact e.m.f. The total mass of the moving parts of the balance was approximately 4 gm. The sensibility could be very finely adjusted either by means of the weights (w) or by means of the screw pivots. The motion of the balance could be observed by means of a mirror attached to the cross-bar and a telescope and millimetre scale at a distance of about 1 metre, the adjustment for zero being made with the flag (f), which was a short strand of silica movable by a lever from the outside of the wooden box containing the apparatus. The vanes could be charged from the outside *via* thin copper wires joining them to the brass ferrules, thence across the steel pivots to the steel planes on which they rested, which in turn were connected to terminals outside the box.

The sensibility was found directly. A number of small weights were cut from the aluminium foil mentioned above, the mass of each being 2.18×10^{-5} gm. These could be picked up on a small brush and pushed through the holes (*h*) directly over the centre of each vane. Independent determinations gave the sensibility to be 1.42×10^{-6} gm. per millimetre deflexion on the scale, and as this was considerably magnified by the telescope a motion of one-tenth of a division could easily be observed.

Fig. 1.



The method of experimenting was as follows. The lid of the box was first adjusted so that it was approximately parallel to the bottom and at a distance of about 2.6 cm. from it. One vane and the box were kept earthed and the other vane charged to, say, 100 volts; generally there was a motion of the balance, but by means of the flag the vane to be charged could be adjusted so that it lay practically midway between the top and bottom of the box. On re-charging, it was now almost indifferent as to which way it was attracted, up or down. The final adjustment was made

by screwing the lid up or down by a small amount. In practice it was found to be quite impossible so to arrange matters that there was equilibrium on charging the vane, but no matter how finely the adjustment was made it was found that the behaviour of the vane at this sensibility was quite independent of the sign of the charge upon it up to a potential of 1000 volts.

It should be mentioned that had any difference in behaviour been observable it was intended to isolate a circular portion (c) of the bottom of the box, adjust the opposite vane, as above, for temporary equilibrium when it was charged, say, positively. On switching over to the negative let us suppose that this vane experienced a small downward force, this force could now be counterbalanced by putting a small potential simultaneously on the isolated portion of the bottom of the box. However, as no difference in behaviour between the vane being charged positively or negatively was observed, this portion of the apparatus was not used but was kept screwed down flush with the bottom.

Having regard to the dimensions of the vane and its distance from the top and bottom of the box, the quantity of electricity upon it when charged to 1000 volts was approximately 41 e.s.u. Now a deflexion for a mass of 1.5×10^{-7} gm. on the centre of a vane could have been observed, and therefore there was no observable difference in the force on ± 41 e.s.u. of charge in the gravitational field greater than $1.5 \times 10^{-7} \times 980$ dyne, or in other words there was no observable resultant gravitational electrostatic field greater than approximately 1.8×10^{-6} dyne/e.s.u.

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XIII. *Characteristic X-Rays from Boron and Carbon.* By
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THE purpose of this investigation was to see if some progress could be made towards filling up the gap between the shortest ultra-violet waves hitherto produced and the longest X-ray waves known. The $K\alpha$ emission-line of sodium, λ 11.8, and the $L\alpha$ emission-line of zinc, λ 12.3, are the longest wave-lengths which have been identified in the X-ray region. There is no reason to believe that X-rays of still longer wave-length cannot be produced, but no crystal has been found, or is likely to be found, with

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