

XLVI. The viscosity of water

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multiplied by r_1/r_2 . In this instance r_1/r_2 was 1.61 and $E/C=r_2/r_1 \cdot \theta_1/\theta_2=2.485$ or $E=30,800,000$ and Poisson's ratio = .243.

A tension-test on a similar piece of tube gave $E=30,700,000$.

In addition to its applications for bending and twisting the apparatus may be used for testing a variety of cases of combined stress if a pump is added to give a fluid pressure in the interior of tubes.

The accompanying diagram (Pl. XI. fig. 10) shows the results of tests to failure of bicycle-tubing when subjected to (I.) bending, (II.) twisting, and (III.) twisting combined with a uniform bending moment.

XLVI. *The Viscosity of Water.*

By RICHARD HOSKING, B.A. (Camb).*

[Plate XIII.]

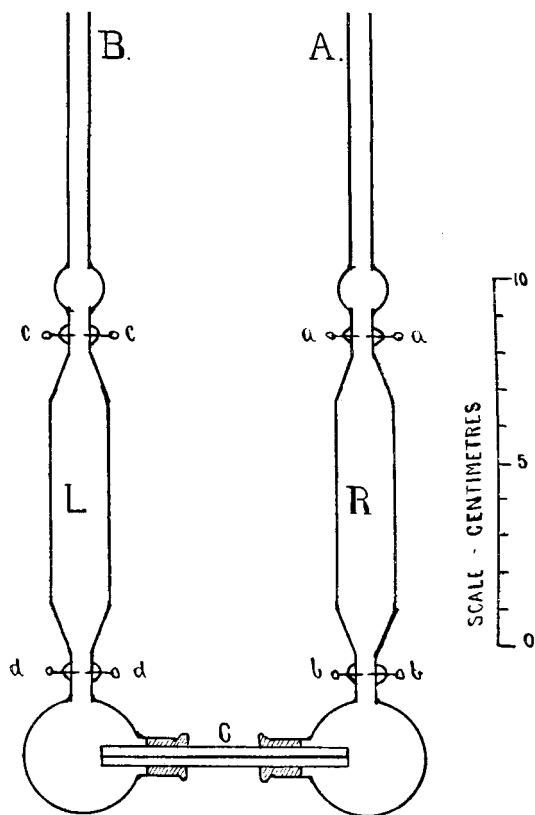
IN my previous experiments on the determination of viscosity by the efflux method†, I have always arranged to have the rate of flow of liquid in the capillary tube very small. The kinetic energy correction in the well-known reduction formula was thus always small in comparison with the first term. In the present experiments, however, I have purposely increased this rate of flow in order to test the formula in cases where the kinetic energy correction is much greater. The glischrometer used in these experiments was of the same form as those previously used by me, but the bulbs were larger. It is shown in fig. 1. At *a*, *b*, *c*, and *d*, platinum wires are inserted, which are almost touching inside the tubes. The capillary C is fitted to the limbs by rubber bands. The volume of the bulb R at 0° C. is 10.2801 ccs., and that of L at the same temperature 10.3201 ccs. Four capillaries were carefully selected for separate use in the glischrometer, and their ends were ground with fine emery, in a lathe. Their lengths, measured directly with callipers, were 5.570, 6.494, 5.408, and 6.456 cms. respectively at 0° C., and their radii approximately were .019, .019, .020, .020 cm. respectively, at 0° C. (The exact determination of the equivalent radius of each capillary was made at the end of all the experiments. It involved the cutting up of the tube and the careful measurement of the sections.) Sufficient freshly distilled water was put in to fill up the glischrometer from *b* to *c*.

* Communicated by the Author. Read before the Royal Society of N. S. Wales, June 3, 1908.

† Phil. Mag. March 1900; May 1902; May 1904.

The ends A and B were connected to the reservoir of compressed air or the outside air by means of three-way taps. Measurements of viscosity were taken first with the water flowing out of R into L, and secondly with the water flowing in the opposite way, under the pressure of air in the reservoir.

Fig. 1.



flowing in the opposite way, under the pressure of air in the reservoir. The average of the two determinations was taken as an absolute value of the viscosity. The pressure was measured by means of (1) a water manometer 200 cms. long, (2) a mercury manometer in cases where the pressure was greater than 200 cms. of water.

To facilitate the reading of the water manometer, a pair of small parallel mirrors was attached to each of the arms. These were inclined so as to make an angle of 45° with the vertical manometer scale. One was fixed near the centre of

the arm, and opposite a telescope; the other could be moved along the arm and clamped in front of the water surface. When the mirrors had been set, it was thus possible to read off the positions of the two water surfaces by means of the telescope, at the same instant; for both images were arranged to be side by side in the field. The pressures employed varied between the limits 100 cms. (water) and 42 cms. (mercury).

The time of flow was in most cases very short, the average being about one minute, but in extreme cases it was as low as 22 seconds. Special means had to be employed to register the time intervals correctly. The chronograph used was kindly supplied by the Sydney Observatory. It consisted of two electromagnets side by side. The armatures were provided with needles. Paper tape was fed through rollers immediately over the needle points at the rate of about 5 cms. per second. A special spring enabled the needles to travel forward a little on piercing the tape, and prevented the tearing of the tape. One needle was used for recording seconds by direct reference, through electrical contacts, to a standard clock. The other needle punctured the tape when a key was pressed at the transits of the meniscus in the glischrometer at the points *a* and *b* in the one case; or at *c* and *d* in the other. These transits were observed always through telescopes.

The procedure in determining the viscosity was as follows. The bath temperature was arranged to be as close to the desired temperature as possible, and the heating flame was adjusted. The pressure of air in the reservoir was raised or lowered to the proper level. Double readings of pressure, time of flow, and temperature were taken. The pressure was next altered, and more readings were taken. In most cases, the determinations were repeated. Another capillary was then placed in position in the glischrometer and the series was repeated.

The Reduction Formula.—In the Journal and Proceedings of the Royal Society of New South Wales are published three most important papers by G. H. Knibbs*, dealing with the history, theory, and determination of the viscosity of water by the efflux method. Knibbs has shown that the reduction formula is

$$\eta_t = \frac{\pi R^4}{8VL \left(1 + n \frac{R}{L}\right)} g \cdot \rho \cdot h \cdot T - \frac{m \cdot \delta \cdot V(1 + 2kt)}{8\pi L \left(1 + n \frac{R}{L}\right) T} \quad (1)$$

* Vols. xxix., xxx., and xxxi.

In this formula L is the length and R the radius of the capillary, and T the time taken for volume V of the liquid of density δ to flow through the capillary under a pressure gh ; nR is a small length of tube producing a loss of pressure equivalent to that arising from the friction at the ends, its value must be calculated for each series of experiments; m is the numerical factor in the kinetic energy correction, which has a theoretical value of 1.12, but which has a practical value which must be determined. The factor m has been neglected in so many recent determinations of viscosity, that it is worth the while to repeat the information given by Knibbs respecting it. In 1860 Neumann deduced the value $m=1$, and Jacobsen used it in his 'Introduction to Hæmodynamics' (1860). Hagenbach deduced the value $m=1$ in the same year. Reynolds (following Bernoulli) in 1883 used the value $m=\frac{1}{2}$. Couette in 1890 independently obtained the value $m=1$. Boussinesq (1891) obtained a more accurate value $m=1.12$. Gartenmeister stated (1890) that Finkener had in an unpublished treatise shown that Couette's value was the correct (?) one. Wilberforce (1891) pointed out the defect in Hagenbach's reasoning, and he used the value $m=1$. Knibbs has shown that theoretically Neumann's correction as deduced by Boussinesq is correct, and that experimentally its value varies considerably. Knibbs has deduced values of m from Jacobsen's results, and stated that individual results show how, even under circumstances in which uniformity might be expected, it is not realized; and that if the correction be of sensible magnitude, the deduced viscosity is, to the extent of this uncertainty, unreliable.

Determination of m and n .—Preliminary experiments were made in order to obtain correct (experimental) values for the constants m and n . The temperatures were kept as close to 50° C. as possible, and, under different pressures, the times of flow were recorded. The pressures were reduced to equivalent pressures at 50° C. Knibbs has shown that the reduction formula, for experiments carried out at a certain temperature, may be expressed in the form

$$C + c/T = \rho h T, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{where} \quad C = \eta_t (1 + nR/L) \frac{8VL}{\pi g R^4}, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{and} \quad c = \frac{m\delta}{g\pi^2} \frac{V^2}{R^4} (1 + 2kt). \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Equation (2) is that of a straight line such that if $1/T$ be taken as abscissæ, and corresponding values of $\rho h T$ as ordinates, the line passing through the points so determined

will intersect the axis of ordinates at a distance C from the origin, and make with the axis of abscissæ an angle whose tangent is c . When c has been obtained m is deduced by means of equation (4).

Equation (3) may be written in the form

$$\frac{C\pi g R^4}{8VL} = \eta_t \left(1 + n \frac{R}{L}\right).$$

The left-hand side of the equation will have different values for different capillaries in the glischrometer.

Calling the left side K we have

$$K = \eta_t + n\eta_t R/L \text{ or } K = \eta_t + lR/L, \quad . \quad . \quad (5)$$

where $l = n\eta_t$.

This is the equation of a straight line such that if values of R/L be taken as abscissæ, and corresponding values of K as ordinates, the line passing through such points will intersect the axis of ordinates at a distance η_t from the origin, and make an angle with the axis of abscissæ whose tangent is k . From the value of k obtained in this way, n is at once deduced.

Typical Observations with Tube I. in Glischrometer.

Temp. (t° C.).	Manometer Reading (h) cms.	Temp. of Manometer $^\circ$ C.	Time of Flow (T) secs.	Bulb being emptied.	ρhT (reduced to 50°) $\times 10^{-3}$.	$\frac{1}{T}$.
50.05	*108.29	22.9	71.38	R	7.713	.01401
50.03	108.27	"	71.46	L	7.718	.01400
50.00	108.31	"	71.45	R	7.707	.01400
50.00	108.31	"	71.33	L	7.697	.01402
50.10	150.28	26.0	54.34	R	8.145	.01840
50.10	150.30	"	54.37	L	8.150	.01839
50.00	196.20	22.9	43.98	R	8.598	.02274
50.00	196.14	"	43.74	L	8.546	.02286
50.00	196.10	24.2	44.03	R	8.600	.02272
50.00	196.17	"	43.70	L	8.540	.02288
50.04	+20.30	23.9	33.58	R	9.239	.02978
50.03	20.30	"	33.52	L	9.220	.02984
50.02	20.30	"	33.58	R	9.236	.02978
50.01	20.30	"	33.53	L	9.232	.02985
50.10	25.40	26.0	28.35	R	9.764	.03527
50.10	24.40	"	29.24	L	9.674	.03420
50.05	30.38	23.9	24.78	R	10.204	.04035
50.06	30.38	"	24.93	L	10.267	.04011
50.07	30.38	"	24.72	R	10.184	.04045
50.08	30.38	"	24.95	L	10.280	.04008
50.00	35.40	22.9	22.32	R	10.701	.04480
50.00	35.40	"	22.38	L	10.726	.04468
50.00	35.40	"	22.28	R	10.683	.04488
50.00	35.40	"	22.38	L	10.726	.04468
50.10	40.40	22.9	20.36	R	11.171	.04912
50.10	40.40	"	21.40	L	11.737	.04673
50.10	40.40	"	20.39	R	11.188	.04904
50.10	40.40	"	21.40	L	11.737	.04673

* Water Manometer.

† Mercury Manometer.

Typical Observations with Tube II. in Glischrometer.

Temp. ($^{\circ}$ C.).	Manometer Reading (h) cms.	Temp. of Manometer $^{\circ}$ C.	Time of Flow (T) secs.	Bulb being emptied.	ρhT (reduced to 50°) $\times 10^{-3}$.	$\frac{1}{T}$
50.08	*108.41	26.7	80.41	R	8.716	.01244
50.08	108.25	"	80.27	L	8.676	.01246
50.08	108.13	"	80.81	R	8.729	.01237
50.08	108.15	"	80.48	L	8.690	.01242
50.05	151.12	26.7	60.40	R	9.113	.01656
50.06	151.02	"	60.30	L	9.084	.01658
50.07	150.96	"	60.40	R	9.118	.01656
50.07	150.96	"	60.18	L	9.065	.01661
50.02	197.10	26.7	48.58	R	9.549	.02058
50.04	197.20	"	48.15	L	9.470	.02077
50.05	197.04	"	48.45	R	9.531	.02064
50.05	197.08	"	48.15	L	9.468	.02077
50.00	†20.40	26.7	36.79	R	10.153	.02718
50.00	20.38	"	36.68	L	10.113	.02726
50.00	20.38	"	36.93	R	10.180	.02708
50.00	20.38	"	36.85	L	10.157	.02714
50.10	30.42	26.7	27.00	R	11.145	.03703
50.10	30.42	"	27.16	L	11.200	.03682
50.02	30.42	"	27.29	R	11.237	.03664
50.02	30.42	"	26.97	L	11.116	.03707
50.10	40.39	21.9	22.07	R	12.107	.04552
50.10	40.39	"	21.97	L	12.053	.04511
50.10	40.39	"	22.17	R	12.152	.04564
50.10	40.39	"	21.91	L	12.020	.04585

* Water Manometer.

† Mercury Manometer.

Typical Observations with Tube III. in Glischrometer.

Temp. ($^{\circ}$ C.).	Manometer Reading (h) cms.	Temp. of Manometer $^{\circ}$ C.	Time of Flow (T) secs.	Bulb being emptied.	ρhT (reduced to 50°) $\times 10^{-3}$.	$\frac{1}{T}$
50.07	*108.28	23.5	54.56	R	5.897	.01833
50.07	108.18	"	54.60	L	5.897	.01831
50.07	108.15	"	54.57	R	5.891	.01833
50.07	108.17	"	54.56	L	5.893	.01833
50.20	151.14	23.4	41.69	R	6.316	.02399
50.24	151.14	"	41.64	L	6.325	.02401
50.22	151.08	"	41.72	R	6.321	.02397
50.22	151.08	"	41.58	L	6.300	.02404
50.10	197.40	23.5	34.00	R	6.705	.02941
50.10	197.30	"	33.93	L	6.690	.02948
50.10	197.14	"	33.94	R	6.684	.02947
50.10	197.26	"	33.95	L	6.691	.02946
50.07	†20.30	23.2	26.55	R	7.311	.03767
50.07	20.30	"	26.47	L	7.290	.03779
50.07	20.30	"	26.52	R	7.303	.03772
50.07	20.30	"	26.44	L	7.282	.03783
50.08	30.39	23.9	19.98	R	8.237	.05006
50.08	30.39	"	19.86	L	8.187	.05036
50.08	30.39	"	19.98	R	8.237	.05006
50.08	30.39	"	19.82	L	8.171	.05046
50.10	40.42	24.2	17.51	R	9.606	.05711
50.10	40.42	"	16.43	L	9.016	.06088
50.10	40.42	"	17.54	R	9.624	.05701
50.10	40.42	"	16.22	L	9.900	.06165

* Water Manometer.

† Mercury Manometer.

Typical Observations with Tube IV. in Glischrometer.

Temp. ($^{\circ}$ C.).	Manometer Reading (<i>h</i>) cms.	Temp. of Manometer $^{\circ}$ C.	Time of Flow (T) secs.	Bulb being emptied.	ρhT (reduced to $50^{\circ}) \times 10^{-3}$.	$\frac{1}{T}$.
50.10	*108.08	26.7	61.26	R	6.614	.01632
50.10	108.10	"	60.80	L	6.561	.01645
50.10	108.09	"	61.83	R	6.675	.01617
50.10	108.10	"	61.21	L	6.607	.01634
50.04	150.97	26.7	46.49	R	6.995	.02151
50.01	151.07	"	46.30	L	6.967	.02160
50.00	150.96	"	46.50	R	6.990	.02150
50.00	150.96	"	46.30	L	7.955	.02160
50.00	197.17	26.0	37.44	R	7.348	.02671
50.00	197.07	"	37.35	L	7.326	.02671
50.00	197.04	"	37.54	R	7.359	.02664
50.00	196.98	"	37.37	L	7.326	.02677
50.10	+20.92	25.7	28.25	R	8.023	.03539
50.10	20.96	"	28.00	L	7.998	.03571
50.10	20.92	"	28.32	R	8.042	.03531
50.10	20.92	"	28.10	L	7.980	.03559
50.10	30.98	25.7	21.30	R	8.954	.04698
50.10	30.99	"	22.80	L	9.594	.04386
50.10	31.00	"	21.40	R	9.010	.04671
50.10	31.00	"	22.58	L	9.498	.04429
50.00	40.47	25.7	20.78	R	11.378	.04812
50.00	40.47	"	21.33	L	11.682	.04688
50.00	40.47	"	20.43	R	11.185	.04895
50.00	40.47	"	21.05	L	11.530	.04750

* Water Manometer.

† Mercury Manometer.

The numbers in columns 6 and 7 were used to obtain the curves given in figs. 2, 3, 4, & 5 (Pl. XIII.). It will be noticed that for a considerable distance the lines are straight, indicating constant values for c in equation (2), and therefore for m for the particular capillary. This constancy is remarkable when we consider the enormous speed with which the water is forced through the tubes in many cases. The individual observations show also that there is no variation in any particular case, and that the value of m in the general formula can be relied upon, when determined in this way. There is, therefore, no necessity to keep the kinetic energy correction small in comparison with the first term in determining viscosities by the efflux method, provided, of course, that the time of flow and pressure can be measured with sufficient accuracy. This will be shown later, where values have been worked out. In certain curves it will also be noticed that at a particular point, there is an abrupt change in the direction of the line, indicating either a largely increased value for m , or a change in the nature of the flow. This is most marked in both the curves for Tube IV. and in one of the curves for Tube III.

The following values for C and c in equation (2) were

obtained from the curves, and the corresponding values of m were deduced by equation (4).

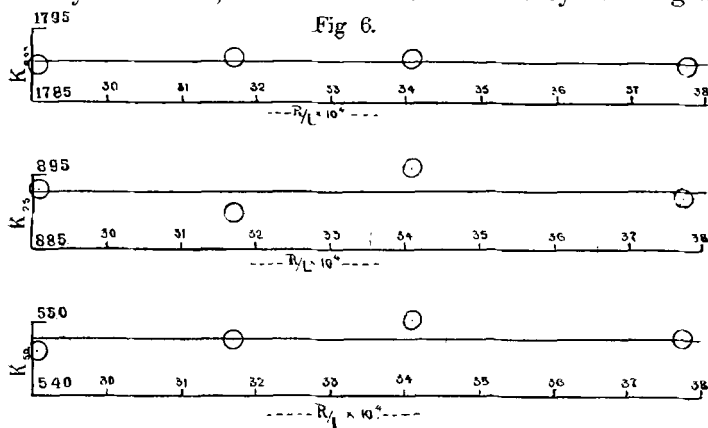
	Tube I.		Tube II.		Tube III.		Tube IV.	
	R to L.	L to R.	R to L.	L to R.	R to L.	L to R.	R to L.	L to R.
C	6350	6330	7400	7440	4570	4570	5400	5390
$10^{-4} \times c$	9.60	9.81	10.00	9.90	7.78	7.78	7.24	7.60
m	1.130	1.164	1.164	1.162	1.128	1.136	1.166	1.216

The values for m are all greater than the theoretical value. There are also two values for each capillary according as the liquid flows in at one end or the other. This fact is most marked in the case of Capillary IV.

The above values for C were used in calculating K in equation (5) for the four tubes. R/L was also calculated. These values are collected in the following table:—

	Tube I.	Tube II.	Tube III.	Tube IV.
K (mean)005564	.005463	.005480	.005478
R/L003407	.002908	.003774	.003172

It is evident that there is no linear relation between K and R/L . When the above values are plotted in the way already mentioned, it will be noticed that they lie along a



straight line parallel to the axis of abscissæ; that, therefore, K in equation (5) has zero value, *i. e.*, the value for n is zero. See fig. 6

A set of readings was taken at 25° C. and also at 0° C., and values for K_{25} and K_0 were found. The results are tabulated below, and the corresponding graphs appear in fig. 6. They bear out the conclusions arrived at in connexion with the results at 50° C.

	Tube I.	Tube II.	Tube III.	Tube IV.
$K_{(25.0)}$	·00896	·00893	·00892	·00890
$K_{(0.05)}$	·01791	·01790	·01790	·01791
R/L	·003407	·002908	·003774	·003172

In the general reduction formula, the most difficult constant to measure accurately is R, the mean radius of efflux. Capillaries are not generally right circular cylinders, nor even elliptical cylinders; and as the degree of precision with which R must be calculated is always four times as great as that required in the deduced viscosity, the examination and measurements of the capillaries must be carried on with extreme care.

Tubes I., II., III., and IV. were in the first place selected from a large number on account of their uniformity of bore—tested with a small mercury column—and their circular end sections.

The first method of measuring R was by contained volumes of mercury. The values obtained (at 0° C.) for the mean radii were ·018968, ·018926, ·020416, and ·020482 cms. respectively.

At the conclusion of all the experiments sections about $\frac{1}{2}$ cm. in thickness were cut from the tubes at regular intervals; they were ground, polished, and mounted in a brass plate. Three independent sets of readings of their dimensions were obtained by me—firstly, by direct comparison with a micrometer eyepiece in a microscope; secondly, by means of a microscope fitted to the dividing engine belonging to the Physics Laboratory, Melbourne University; and, thirdly, by means of a micrometer microscope at the Sydney University. The following average values were obtained for the radii of the sections reduced to 0° C.:—

Capillary I. (Circular).	First Method.	Second Method.	Third Method.	Mean Values.
Section 1	·01905 cm.	·01879 cm.	·01880	·018838
„ 2	·01929 „	·01919 „	·01899	·019106
„ 3	·01932 „	·01933 „	·01877	·019048
„ 4	·01926 „	·01880 „	·01881	·018882

[Mean radius (by mercury) = ·018968 cm.] Mean = ·018968 cm.

Capillary II. (Circular).	First Method.	Second Method.	Third Method.	Mean Values.
Section 1	·01899 cm.	·01889	·01880	·018862
„ 2	·01896 „	·01861	·01880	·018763
„ 3	·01912 „	·01910	·01891	·019009
„ 4	·01906 „	·01909	·01890	·018990

[Mean radius (by mercury) = ·018926 cm.] Mean = ·018906 cm.

Capillary III. (Elliptical).	First Method.	Second Method.	Third Method.	Mean Values.
Section 1	·02023	·02066	·02069	·020603
„ 2	·02016	·01969	·02020	·020023
„ 3	·02083	·02064	·02066	·020682
„ 4	·02016	·02009	·01993	·020022
„ 1	·02124	·02119	·02069	·020948
„ 2	·02043	·01983	·01996	·020000
„ 3	·02097	·02032	·02075	·020643
„ 4	·02043	·02010	·01998	·020095

Mean radius by mercury = ·020416.

Mean $\begin{cases} (a) \cdot 020719 \\ (b) \cdot 020035 \end{cases}$

Capillary IV. (Elliptical).	First Method.	Second Method.	Third Method.	Mean Values.
Section 1	·02138	·02125	·02088	·021087
„ 2	·02023	·02070	·01976	·020152
„ 3	·02130	·02118	·02086	·021040
„ 4	·02043	·02007	·01984	·020015
„ 1	·02144	·02143	·02068	·021057
„ 2	·02023	·01954	·01976	·019765
„ 3	·02183	·02125	·02079	·021042
„ 4	·02023	·01997	·01994	·019723

Mean radius by mercury = ·020482.

Mean $\begin{cases} (a) \cdot 021056 \\ (b) \cdot 019914 \end{cases}$

In determining the mean value for each section, the values obtained by the three methods were weighted in the following manner: Method 1, weight 1; Method 2, weight 2; Method 3, weight 3; thus for Capillary 1, Section 1, by adding

together $\cdot 01905$ cm., twice $\cdot 01879$ cm., and three times $\cdot 01880$ cm., and dividing the sum by 6, we obtain the value $\cdot 018838$ cm. The mean obtained in this way is, I consider, the best value the individual results will produce, taking in account the experimental difficulties in measuring such small bores in the three cases.

The mean values by measurement were then combined with the values obtained by mercury column, and in this way the final values were obtained.

Capillary I., circular cylinder, radius .018968 cm. at 0° C.

„ II., „ „ „ „ 018916 „

III., elliptical cylinder, semi-axes (a) $\cdot 020762$

(b) 020076

IV., " " " (a) .021061 "

(b) 019919

III., mean radius of efflux R (where $R^4 = \frac{2a^3b^3}{a^2+b^2}$)

$$= \cdot 020413 \text{ cm.}$$

IV., mean radius of efflux $R = 0.020474$ cm.

An accuracy of 1 in 1000 was aimed at throughout the experiments. The values for the constants in the reduction formula having been obtained, the viscosity was determined from the various observations which had been made from time to time, including those already mentioned.

A special set of experiments at 0° C. was made at a fixed pressure in order to obtain accurate values of the viscosity at 0° C. The thermometer registered 0°·05 C. throughout the series. With Tube I. in the glischrometer, under a pressure of 197 cms. of water, the average time of flow was 109·7 secs. The first term of the reduction formula was found to be ·018690, and the second term ·000769. The value for the viscosity at 0°·05 was ·01792; which reduced to 0° C., becomes ·01795. A second determination gave the same values practically. The observations and reductions are given on p. 513, also a similar set at 25° C. p. 514.

Most of the experiments, however, were carried out at 50° C. with the water flowing under different pressures. In the following tables (pp. 515-517) are collected the various results. The pressures are given in centimetres of mercury, the times of flow in seconds, the values of viscosity (double observations) in absolute measure, and the kinetic energy correction—second term—also in absolute measure. The pressures and times are approximate, and the viscosity values are reduced to the even temperature 50° C.

Temperature 0° C.

Tube in Glsbro- meter.	Temp. (°).	Pressure (h.).	Time, T.	Direction of Flow.	First Term.	Second Term.	η .	η reduced to 0° C.
I.	0.05	197.00	109.90	R to L	-0.18990	-0.00769	-0.1792	-0.1794
	0.05	197.00	109.55	L to R	-0.18930	-0.00769	-0.1792	
	0.05	197.00	109.84	R	-0.18990	-0.00770	-0.1789	
	0.05	197.00	109.55	L	-0.18990	-0.00769	-0.1792	
II.	0.05	196.90	127.74	R	-0.18410	-0.0576	-0.1783	-0.1789
	0.05	197.00	127.30	L	-0.18425	-0.0576	-0.1735	
	0.05	196.90	127.87	R	-0.18430	-0.0577	-0.1783	
	0.05	196.90	127.63	L	-0.18465	-0.0574	-0.1789	
III.	0.05	196.95	80.60	R	-0.18960	-0.01066	-0.1789	-0.1795
	0.05	196.95	80.42	L	-0.18990	-0.01065	-0.1795	
	0.05	196.95	80.72	R	-0.18990	-0.01066	-0.1792	
	0.05	196.95	80.36	L	-0.18980	-0.01065	-0.1792	
IV.	0.05	197.20	93.69	R	-0.18670	-0.0809	-0.1786	-0.1793
	0.05	197.10	93.88	L	-0.18770	-0.0805	-0.1796	
	0.05	197.10	93.81	R	-0.18683	-0.0808	-0.1787	
	0.05	197.10	93.65	L	-0.18723	-0.0806	-0.1792	

These results give mean value for viscosity of water at 0° C. = -0.1793.

Temperature 25° C.

Tube in Gliscro- meter.	Temp. (<i>t</i>).	Pressure (<i>h</i>).	Time (<i>T</i>).	Direction of Flow.	First Term.	Second Term.	ηt .	η reduced to 25° C.
I.	25.38	196.97	60.35	R	.010245	.001401	.00884	.00894
	25.38	196.93	60.17	L	.010268	.001400	.00887	
	25.38	196.93	60.36	R	.010242	.001400	.00884	
	25.38	196.95	60.25	L	.010282	.001400	.00888	
	25.00	197.00	60.22	R	.010347	.001385	.00896	.00897
	25.00	197.05	60.71	L	.010356	.001385	.00897	
	25.00	197.10	60.98	R	.010366	.001384	.00898	
	25.00	197.10	60.74	L	.010364	.001384	.00898	
II.	25.24	196.78	68.84	R	.009906	.001067	.00884	.00889
	25.24	196.69	68.65	L	.009912	.001066	.00885	
	25.24	196.85	68.79	R	.009902	.001068	.00883	
	25.24	196.69	68.72	L	.009922	.001066	.00886	
	24.98	197.04	69.19	R	.009998	.001058	.00893	.00894
	24.98	197.00	69.08	L	.009995	.001055	.00894	
	24.98	197.10	69.25	R	.010005	.001057	.00895	
	24.98	197.00	69.08	L	.009995	.001055	.00894	
III.	24.96	197.10	45.85	R	.010792	.001871	.00892	.00894
	24.96	197.00	45.84	L	.010831	.001864	.00897	
	24.96	197.00	45.82	R	.010778	.001872	.00891	
	24.96	197.00	45.84	L	.010831	.001864	.00897	
IV.	25.08	196.15	52.04	R	.010315	.001456	.00886	.00887
	25.08	195.95	52.01	L	.010340	.001451	.00888	
	25.08	196.05	52.03	R	.010309	.001456	.00885	
	25.08	196.05	52.02	L	.010345	.001451	.00889	

Mean value for viscosity of water at 25° C. = .00893.

Results at 50° C. with Tube I. in Glischrometer.

Pressure (<i>h</i>).	Time of Flow (T).	Viscosity values $\eta \times 10^5$.	Viscosity (mean) $\eta \times 10^5$.	Second Term $\times 10^5$.
7 cm.	71 sec.	$\begin{Bmatrix} 552 \\ 551 \\ 554 \\ 554 \end{Bmatrix}$	553	118
11 "	54 "	$\begin{Bmatrix} 552 \\ 554 \\ 553 \end{Bmatrix}$	553	156
15 "	44 "	$\begin{Bmatrix} 552 \\ 552 \\ 551 \\ 550 \end{Bmatrix}$	551	193
20 "	33 "	$\begin{Bmatrix} 550 \\ 550 \end{Bmatrix}$	550	250
25 "	28.8 "	551	551	292
30 "	24.9 "	550	550	338
31 "	24.4 "	547	547	348
33 "	23.5 "	549	549	357
35 "	22.4 "	$\begin{Bmatrix} 552 \\ 554 \\ 556 \end{Bmatrix}$	554	376
40 "	20.9 "	$\begin{Bmatrix} 590 \\ 592 \end{Bmatrix}$	591*	400

Results at 50° C. with Tube II. in Glischrometer.

Pressure (<i>h</i>) (Approx.).	Time of Flow (T) (Approx.).	Second Term $\times 10^5$.	Viscosity $\times 10^5$.	Mean Viscosity $\times 10^5$.
7.3 cm.	80.5 sec.	91	$\begin{Bmatrix} 549 \\ 550 \\ 549 \end{Bmatrix}$	550
11 "	60.3 "	121	$\begin{Bmatrix} 549 \\ 549 \\ 549 \end{Bmatrix}$	549
15 "	48.3 "	151	$\begin{Bmatrix} 549 \\ 548 \end{Bmatrix}$	548
20 "	36.8 "	199	$\begin{Bmatrix} 549 \\ 548 \end{Bmatrix}$	549
30 "	27 "	269	$\begin{Bmatrix} 553 \\ 553 \end{Bmatrix}$	553
40 "	23 "	316	$\begin{Bmatrix} 557 \\ 561 \end{Bmatrix}$	559 *
42 "	21.5 "	340	565	565 *

Mean .00550

With Tube I. in the glischrometer, the formula gives constant values up to a pressure of 35.4 cms. of mercury, when the time of flow is about 22 seconds. It will be noticed that at this point the kinetic energy correction is more than 60 per cent. of the viscosity, and the average velocity in the tube is 400 cms. per sec.

In the case of Tube II. the formula breaks down suddenly at a pressure between 35.7 cms. and 36.9 cms., the time of flow being about 24 secs., and the correction at pressure 35.7 cms. more than 55 per cent. of the viscosity. The velocity in this case is 370 cms. per sec.

Results at 50° C. with Tube III. in Glischrometer.

Pressure (<i>h</i>) (Approx.).	Time of Flow (T) (Approx.)	Second Term $\times 10^5$.	Viscosity (double readings) $\eta \times 10^5$.	Viscosity (mean) $\eta \times 10^5$.
7.3 cm.	54.6 sec.	156	$\left\{ \begin{array}{c} 552 \\ 551 \\ 550 \\ 549 \end{array} \right\}$	550.5
11 "	41.6 "	205	$\left\{ \begin{array}{c} 553 \\ 551 \end{array} \right\}$	552.5
15 "	33.9 "	252	$\left\{ \begin{array}{c} 552 \\ 551 \end{array} \right\}$	551.5
20 "	26.5 "	323	$\left\{ \begin{array}{c} 554 \\ 554 \end{array} \right\}$	554
30 "	19.9 "	430	$\left\{ \begin{array}{c} 557 \\ 555 \\ 558 \\ 556 \\ 559 \end{array} \right\}$	557 *
35 "	17.9 "	478	$\left\{ \begin{array}{c} 558 \\ 562 \\ 565 \end{array} \right\}$	562 *
40 "	17.0 "	500	$\left\{ \begin{array}{c} 615 \\ 607 \\ 617 \\ 613 \\ 604 \\ 609 \\ 611 \end{array} \right\}$	612 *

Mean .00552

For Tube III. the figures are, pressure between 20.4 cms. and 30.4 cms., time of flow less than 26 secs., second term

60 per cent. of the viscosity; and for Tube IV. when the pressure is 27 cms. and the second term is nearly 60 per cent. of the viscosity, the deduced value is satisfactory, but an increase of pressure of 1 cm. brings about some decided change. The highest velocity reached in both cases, before

Results at 50° C. with Tube IV. in Glischrometer.

Pressure (<i>h</i>) (Approx.).	Time of Flow (T) (Approx.).	Second Term $\times 10^5$.	Viscosity (double readings) $\eta \times 10^5$.	Viscosity (mean) $\eta \times 10^5$.
7.3 cm.	61.5 sec.	123	{ 546 } 552	549
11 "	46.3 "	162	{ 547 } 547	547
15 "	37.5 "	201	{ 544 } 545 543 547	545
20 "	28.1 "	268	{ 546 } 546 546 546	546
24 "	25.3 "	297	...	554
25 "	24.4 "	308	{ 546 } 546	546
26 "	23.8 "	316	...	551
27 "	23.2 "	325	{ 550 } 548	549
28 "	22.7 "	332	...	556 *
29 "	22.6 "	333	...	588 *
30 "	22.3 "	339	...	593 *
31 "	21.9 "	343	...	598 *
32 "	21.9 "	344	...	634 *
34 "	21.5 "	350	...	668 *
36 "	21.2 "	355	...	709 *
38 "	21.0 "	359	...	751 *
40 "	21.0 "	359	{ 822 } 806	814 *
44 "	20.2 "	373	{ 835 } 832	837 *

Mean .00548

the change, was about 340 cms. per sec. The curves in fig. 5 (Pl. XIII.) indicate that when the change takes place, there is a large increase in the value of *m*, if the formula still holds; but the individual results do not agree sufficiently well to enable one to draw definite conclusions from them.

Viscosity Values ($\times 10^5$) at 50° C. collected.

Pressure.	Tube I.	Tube II.	Tube III.	Tube IV.	Average.
7.3 cm.	553	550	551	549	551
11.0 "	553	549	552	547	550
14.7 "	551	548	552	545	549
20 "	550	549	554	546	550
24 "	554	554
25 "	551	546	549
26 "	551	551
27 "	550	549	550
29 "	551	(588)	551
30 "	550	553	(557)	(593)	551
31 "	547	(598)	547
33 "	549	549
34 "	551	(668)	551
35 "	554	...	(562)	...	554
36 "	(574)	(709)	...
37 "	(558)	...	(571)
38 "	(578)	(751)	...
39 "	(573)	...	(599)
40 "	(591)	(559)	(612)	(814)	...
42 "	...	(565)
43 "	(837)	...
Average =	·00551	·00550	·00552	·00548	

Mean Value = ·00550

The values for the viscosity of water at 50° C., obtained with the various capillary tubes in the glischrometer, are collected in the foregoing table. The mean value is ·00550.

Results.

(1) The constants in the reduction formula were all determined with the greatest possible degree of accuracy, including R , n , and m .

(2) For each capillary in the glischrometer—four were used separately—two values for m were found, one for each of the ends. These values were in every case greater than the theoretical value 1.12.

(3) For the series of capillary tubes used, experiments at temperatures 0° C., 25° C., and 50° C. gave in each case zero values for n .

(4) Absolute values for the viscosity of water at 0° C., 26° C., and 50° C. were obtained, namely, ·01793, ·00893, and ·00550; which are probably correct to 0.1 per cent.

(5) The values obtained for m were constant over a big range of pressure; and at a very high pressure there was an indication of an abrupt change in the value of m , or in the nature of the flow. The velocities at this pressure were much below the critical velocities for the various tubes, but were all above the lower limit of critical velocity.

(6) Consistent values for the viscosity of water at 50° C. were obtained in cases where the kinetic energy correction was as high as 60 per cent. of the viscosity.

I have much pleasure in acknowledging my indebtedness to Professor Sir J. J. Thomson, Cavendish Laboratory, Cambridge; Professor Lyle, Melbourne University; Professor Pollock, Sydney University; and Mr. G. H. Knibbs, F.R.A.S., Federal Statistician, formerly Director of Technical Education, N.S.W., and Lecturer in Surveying, University of Sydney, for valuable assistance during the progress of this research, which was commenced at the Cavendish Laboratory, Cambridge, and completed at the Sydney University.

Note on the Viscosity of Solutions.—The viscosity of certain lithium chloride solutions was determined with the glischrometer described in the previous paper. The only novel feature of the measurements was the automatic recording of the time of flow. The inner platinum wires at b and c (fig. 1, p. 503) were connected by insulated wires, also the inner wires at a and d . Wires were fastened to the outer wires at a , b , c , and d , and were connected to four plugs on a double reversing key. The two remaining plugs were joined by wires through a battery and one of the electromagnets already described. With the key in one position, there was electrical communication between the battery and electromagnet and the outer c on the one side, and the outer b on the other side. The circuit was complete only when the solution filled the spaces at both c and b . With the key reversed, the battery was connected to the outer d and the outer a , and the circuit was complete when the solution filled the spaces at a and d . By regulating the amount of solution in the glischrometer the signals could be made as short as necessary, at the beginning and end of the flow from R to L, or in the opposite direction, and the time of flow could be read off accurately on the tape.

The following set of readings will be sufficient to illustrate

the accuracy with which determinations of the viscosity of solutions can be made with this arrangement:—

Lithium Chloride Solution, Temperature 20°·75 C.

Pressure (<i>h</i>).	Time (<i>T</i>).	Correction. × 10 ³ .	η .	η (mean)
{ 198·0 cm.	57·2 sec.	150	·01201	·01200
{ 198·2 "	56·4 "	151	·01198	
{ 179·3 "	62·8 "	136	·01204	·01200
{ 179·1 "	62·8 "	138	·01197	
{ 151·0 "	73·1 "	118	·01200	·01200
{ 151·2 "	73·0 "	118	·01200	
{ 127·1 "	85·6 "	101	·01197	·01198
{ 127·0 "	85·1 "	101	·01198	
{ 100·5 "	106·6 "	81	·01200	·01201
{ 100·0 "	106·8 "	81	·01203	

Average ·01200

XLVII. *The Doppler Effect in Positive Rays.*

By JOHN TROWBRIDGE*.

[Plate XIV.]

THE discovery of canal rays by Goldstein, and that of the Doppler effect in these rays, marks an epoch in the study of the discharge of electricity through gases; for before these discoveries the multitude of confusing effects which arise in the space between the anode and the cathode made it difficult to observe any translation movements. The space, however, behind the cathode is comparatively free for the passage of the positive ions.

We now recognize, in addition to the positive rays behind the cathode—the canal rays—retrograde positive rays which are directed to the anode, or rather away from the cathode in the direction of the anode †.

This later discovery leads one to expect that the Doppler effect should be found also between the anode and the cathode. The result of my study shows that the effect does exist in this region and indicates a movement away from the cathode and toward the anode.

* Communicated by the Author.

† Wehnelt, Wied. *Ann.* A. 1899, p. 421; Runge & Paschen, Wied. *Ann.* lxi. 1897, p. 644; Paschen, Wied. *Ann.* xxiii. 1907, p. 247; Villard, *Comptes Rendus*, cxliii. 1906, p. 673; Goldstein, *Phil. Mag.* March 1908, p. 372; Jacob Kunz, *Phil. Mag.* July 1908, p. 161; J. J. Thomson, *Phil. Mag.* Oct. 1908, p. 657.

FIG. 2.

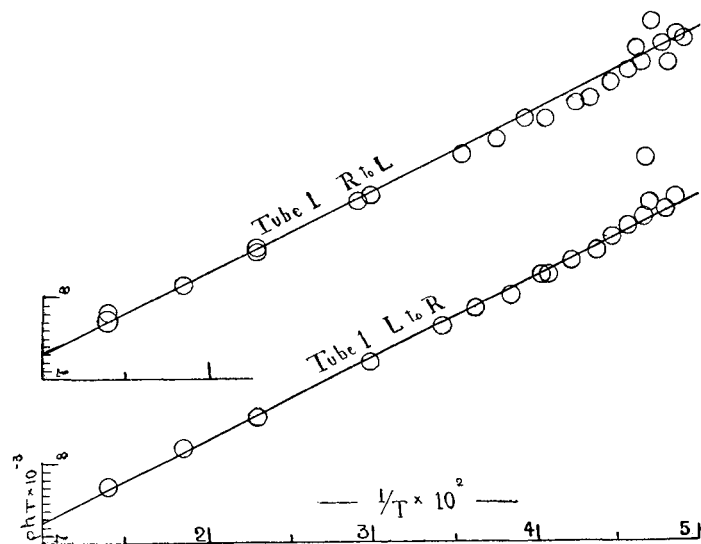


FIG. 3.

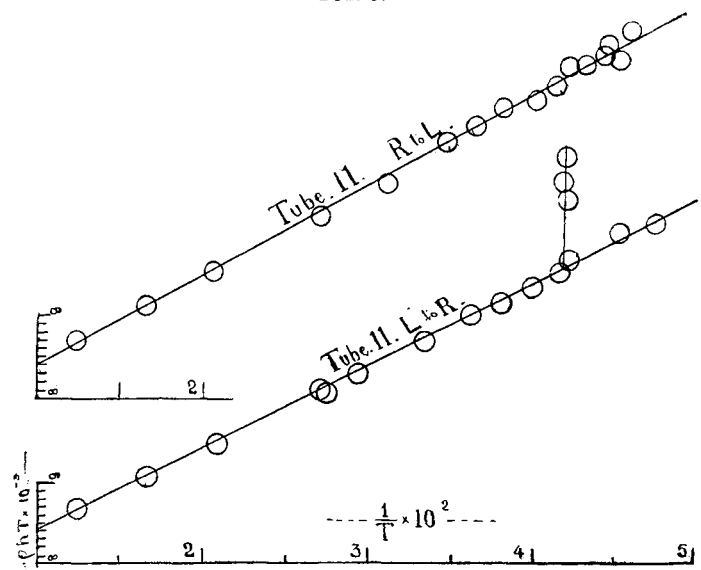


FIG. 4.

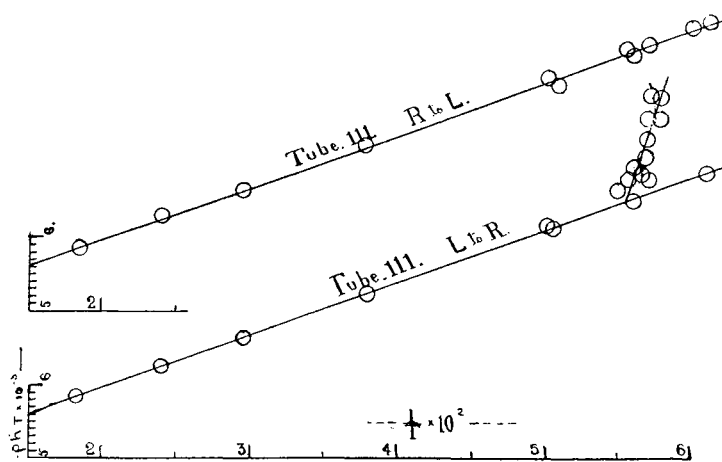


FIG. 5.

