

PROGRESS IN THE CORRELATION OF PHYSICS AND MATHEMATICS.*

BY F. L. BISHOP,
Bradley Institute.

The progress that is being made in the correlation of physics and mathematics is so extensive as regards amount of territory covered, and the methods employed differ so greatly that it is almost impossible to rightly estimate its absolute value.

Prof. E. H. Moore of the University of Chicago writes "that distinct advances in this direction are being made in England, Germany and France, at least to the extent that first, the work in mathematics is treated as a single whole; second, it is done simultaneously with physics; third, it is done as far as possible by the same instructors. This facilitates actual and continuous correlation."

At a meeting of the Mathematical Club of the University of Chicago held during the last summer quarter reports were made showing progress on the Pacific coast, in the South and in the Middle West. A factor in this country tending toward correlation is the organization of such clubs as the Association of Teachers of Mathematics in the Middle States and Maryland. Its first meeting was held at Teachers College, New York City, on Nov. 28, 1903. Among the papers read were "The Laboratory Method of Teaching Mathematics," "Geometry in the Grammar School," and "Has Algebra Any Genuine Application?"

By such organizations as the Eastern Association of Physics Teachers this subject has received more or less consideration, the following quotation being taken from an address by Vice-President George A. Cowen, of Simmons College: "Fifteen years ago at Phillips Academy, Andover, Professor Graves performed the experiments while the boys looked and wondered. Now they do and know. With the change came the necessary demand for accurate measurement, but measurements of length and weight and force are of no use unless properly correlated. This is mathematics. Mathematics, it is affirmed, has made physics unpopular. Language without words would be about as sensible as physics without mathematics."

* Address given before the Mathematics and Physics Sections, C. A. S. and M. T., November 26, 1904.

The methods that are being employed are illustrated by the following: Professor G. W. Greenwood, of McKendree College, writes: "We are trying here to bring mathematics and physics into closer relationship by showing that algebra is a means of expressing precise relations among magnitudes which may be measured and of expressing relations deduced from given relations. I am using entirely new definitions with these ends in view, and so far as I know they differ widely from the text books. We take any verbal statement from physics or arithmetic and then state it in the form of an equation. From equations we state verbal equivalents or rules. We leave entirely in the physics department the experiments from which the laws are deduced, and so far we are making good progress."

From Professor Newhall of the Shattuck School we have the following: "The courses are conducted entirely separate, but the department of mathematics aims to teach such subjects as the metric system, ratio and proportion, variation, graph, a knowledge of the trigonometric functions, etc., before they shall be needed in the study of the sciences." The instructor in physics has written out for me a full list of the formulas, equations and geometrical proofs which occur in a year's work in physics, and I see that these identical equations and proofs are studied in the algebra and geometry. In return he emphasizes such subjects as the parallelogram of forces, direct and inverse variation, etc." Mr. Newhall also writes that they have a close correlation between the mechanical drawing and mathematics. Further he states: "I do not think I like the idea of correlating the two subjects to such an extent that either loses its identity."

Professor H. E. Cobb of Lewis Institute in outlining the interesting work he is doing says: "My experience is no doubt of value in this, that I am laboring under the difficulties that most teachers encounter when they try to get out of the beaten path. While I am free to use any method I choose in my classroom, at the close of each quarter and often during the quarter, students are transferred to or from my section. Hence I must go over the ground with the new ones, and always have my students in such shape that they can do work with other sections. In the first year algebra I use the balances and levers to illustrate the equation and the operations with positive and negative numbers. Squared paper is used in the solution of problems and graphical

work. During the last two quarters one day a week is given to concrete geometry, measurements being made in the metric and English systems, and constructions with compasses and ruler. In first year geometry great emphasis is laid on doing and on numerical computation. During the last quarter elements of trigonometry are introduced and triangles are solved, and computations of heights and distances made with the squared paper instead of tables of natural functions. In geometry various blocks are measured with both ruler and calipers, which are weighed and the volume and specific gravity computed."

Professor Donecker of the Richard T. Crane Manual Training High School of Chicago has devised a balance which he calls the "Algebraic Equation Balance," which serves the purpose of making it possible to present equations in concrete form. The primary purpose is to give a concrete idea of negative numbers. It can also be used as a basis for problems on levers. I suggest that every mathematical teacher investigate this apparatus.

Professor Risley of the Mathematical Department of Armour Institute of Technology outlines their work as follows: "We have gotten outside problems from the work in Physics and used them in our class work. We have not discarded a text, as some have advocated. In those subjects requiring mathematical statements and calculations our instructors agree that the difficulty is fundamentally one of arithmetic. The mistakes are made in adding, etc., and in getting the decimal point in the right place. Handling ordinary common fractions and in placing the work in a clear logical order. In our plane geometry practically all the work is original. This is somewhat slow at first and usually a little discouraging to the student for about six weeks. After that if the instructor has been sufficiently strenuous in exacting the niceties of geometrical logic, and clearness, there is a most pleasant outlook ahead. In our geometrical notebook we have a great deal of construction work illustrated by the following problems: Draw 5 lines of different lengths, measure them in centimetres and inches, find the ratio of the length of an inch to that of a centimeter in each case; also the ratio of the length of a centimeter to that of an inch in each case. Obtain the mean ratios. Why do your values differ from the true ratios. As another example: Draw 5 different angles, acute, obtuse and reflex. Measure each three times and obtain the mean. Describe the

process of measurement carefully. The student will not study construction as such from his text this term. The idea being to have him become thoroughly familiar with his inch and centimeter rules, his compasses and dividers, protractor, etc. Later he will consider the parallelogram of forces from data obtained in actual experiments. One primary object in our work is the development of the initiative, and we hold that analysis bears an important place in this development.

Dean Raymond of the department of Physics writes as follows: "We have found at Armour Institute of Technology that in attempting to do specified physics experiments with our mathematics classes, we sacrificed the formal drill in the manipulation of algebra expressions. This is too important a part of the training of an engineer to be studied in any but a rigid manner. In place of the "booky" problems that are found in almost every text, we have supplied a long list of problems from physics, especially mechanics, etc. The law is stated and the student has problems to solve that arise from this law. The interest of the student is assured at the outset, knowing that he will later meet with the principle in his engineering work. Besides the interest of the student, he is being drilled to manipulate those forms which will make the study of the subject of mechanics or physics very much easier when taken up. Complaints were frequent from those teaching the applied mathematics that the men could not handle the mathematics of the subject after having made the application. The list of problems were written after scanning the books used in the engineering classes, in hopes that the men might be better trained to handle the work. From results attained thus far, we feel that the work is proving very beneficial."

It would be much easier for Prof. Comstock or Prof. Plant, who have so successfully and untiringly pushed the work of correlation at Bradley Institute, to outline their work for you and explain exactly how and why it was begun. From the point of view of the physicist it commenced some six years ago when the mathematics department discarded the algebras then in use and made out in outline one which used extensively the graph and introduced a large number of physical problems. These problems were selected in what appears to me now an almost ideal manner. The physics department furnished a list of all the typical equations used in elementary physics and later a series of problems

which covered every type of equation. The mathematical department then selected from these and added many others which appeared especially well adapted to students in elementary algebra. From physical problems to simple apparatus, such as balance, lever, thermometer, etc., was only a step, and the logical outcome of the introduction of such work. This work has been developed along this line until it seems to me as I come in contact with the students taking the course, to be very successful.

In geometry the students were first given a few plane figures to find their dimensions in both the English and metric systems. Various geometric problems that had a more or less direct bearing on the physics were introduced.

Two years ago the mathematical laboratory for work in concrete geometry was established. This simply meant the better systematizing of the experiments already given and the addition of many more. Some of these experiments were taken directly from the physics, while others were original and not ordinarily given at the present time in the elementary physics. Great care was taken in the selection of these experiments to include only in general those that have a geometrical proof, thus enabling the student to obtain a very clear comprehension of the practical applications of his geometry. The second object that was aimed at was the selection of experiments which required only the simplest form of apparatus. Those which do not fulfill this latter condition are, in my opinion, absolutely worthless for elementary mathematics. As soon as the apparatus becomes sufficiently complicated to need explanation by the instructor, the student has his mind turned from the essentials of the experiment to the apparatus. It is not the aim of this laboratory course to teach the student manipulation of complicated apparatus.

It was early recognized that it would be impossible to have a close correlation between the physics and mathematics unless the instructors were familiar with the work of both departments. For this reason one mathematical instructor taught three-fourths of his time in mathematics and one-fourth in physics, while one of the physics instructors gave three-fourths of his time to physics and one-fourth to mathematics, i. e., the physics instructor had one class in geometry or algebra and the rest of his time in physics. This also furnished a bond of interest between the two departments, in that each knew the aims and objects of the other.

The results obtained from this arrangement cannot be overestimated. It seems to me doubtful if as much could have been accomplished in any other way, at least it would have required a much longer time. Another feature which contributed materially to the success of this correlation was the introduction three years ago of a course in physiography, which is taken by all students during the first quarter of the first year. This course given under the direction of the physics department is made an introductory course in science, so that the student is more or less familiar with the words and phrases that he will be required to use in his problems in algebra and geometry.*

Thus we have outlined briefly the work of correlation as it is being carried on in some schools. There are many others where the work is probably as far advanced as in the cases noted, but I was unable to obtain accurate and specific information concerning them. A pertinent question would be: What assistance has this been to physics? The student is familiar with many of the words like velocity, acceleration, force, centigrade, etc., the metric system in detail, and the graph. He can solve all algebraic equations occurring in physics with numerical problems under each. The working of examples in the composition and resolution of forces with the trigonometric functions—sine, cosine and tangent, is but a continuation of his work in geometry. The laws of the lever, reflection and refraction of light, of the inclined plane, and the relation between the Centigrade and Fahrenheit thermometers come as easy as the simplest equation in algebra. From his laboratory work he is familiar with the method of doing accurate laboratory work. He knows the *degree of accuracy* which he may expect to obtain, i. e., a clear relation between the theory and practice. He is also familiar with the sources of error, form of laboratory report and he knows the best methods of computation.

There are of course many others, but beside all these which we can state more or less definitely he possesses the power to use his mathematics to a degree that is almost unknown to students who have not had this work.

Another question which is often asked and which is certainly much to the point is: Do these students know their pure mathe-

*Note.—These experiments were published in full in a report of the committee on the correlation of mathematics and physics in secondary schools made to this association in 1903.

matics as well as students who have had only the abstract mathematics? This is, of course, a very difficult question to answer. I have made an attempt to find an answer in this way. I have in my classes not only students who have taken this work, but also students who have come to us from first-class high schools where I know that the preparation in pure mathematics is very good. I have sometimes asked for the proof of some geometrical theorem that we have been using, as for instance the Pythagorean theorem. I have never yet found a student with only the abstract preparation who would attempt to demonstrate one of these off hand. While I have found that a large number of the students who have had the concrete geometry were able to give the demonstration.

While this cannot be considered in any sense a proof it certainly indicates to me that the student has lost none of his reasoning powers by taking up this applied work. That I am not the only teacher who believes that the concrete work has added materially to the student's power to deal with mathematics is easily seen from the following: Professor Cobb of the mathematics department of Lewis Institute, who has made decided progress in the correlation of physics and mathematics, says: "I am thoroughly convinced that my students are getting hold of mathematics in a way that is not possible under the old formal method of teaching." Again, to quote from Professor P. B. Woodworth: "I take great pleasure in reporting progress in the correlation work in mathematics and physics at Lewis Institute. I have been more than pleased with the results as they develop this year. The students who had the mathematical work based upon actual measurements are much better prepared than those who have had the same amount of abstract work in mathematics. The work seems in some way to have developed a thinking mathematical method which largely prevents those mathematical blunders which have been so exasperating to physics teachers. I also think the attempt at precision measurement has increased the students' reverence for mathematics. The only ill effect I have observed is that those who have not had the course given by Professor Cobb are having a hard time to get in line." Professor R. A. Milliken of the University of Chicago quotes Mr. Lynde, instructor in physics at the School of Education, as saying that the students who took the concrete mathematics and have now

entered physics take to the graph like ducks to water; beyond this he is not prepared to make a report at present, as this is only the second year that the physics courses at the School of Education have been in operation.

From the statements of various persons quoted in this paper, and from many others which I have obtained, it seems that correlation does not mean that either physics or algebra or geometry is to be eliminated, but, as Dr. Milliken very aptly expresses it, "you can teach all the physics you want in algebra and geometry and then there will be plenty left for us."

It appears that the correlation is an accomplished fact in so far that problems from physics are made the basis of the original work in algebra and that wherever this work is carried on we find both the mathematics and physics teachers enthusiastic concerning the progress of the student in his power not only to use his mathematics, but he seems to possess a clearer and a more logical and a more correct idea of the abstract mathematics. It would seem that to attain the highest degree of efficiency in this work, not only must the teacher of mathematics be a student of physics, but if possible he should teach physics for a time in order that he may better comprehend the needs of the physics teacher and at the same time study carefully in all ways the effect that the correlation is having on the student's work.

RECENT ADVANCES IN METEOROLOGY.*

BY HENRY J. COX, A. M.,

*Professor of Meteorology, United States Weather Bureau,
Chicago, Ill.*

(Continued from the February number.)

GENERAL AND SECONDARY MOTIONS OF THE ATMOSPHERE.

The kite and balloon observations, and the international cloud observations which were carried on all over the world in 1896 and 1897, seem to throw new light upon the motions of the atmosphere, and have led meteorologists to give up Ferrel's canal theory and the German vortex theory, which was at first advanced by Overbeck. The motion of the air in a cyclone, coming more and

* An address delivered before the Earth Science Section of Central Association of Science and Mathematics Teachers, Nov. 26, 1904.