



Review

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young aspirants, not satisfied with the state of life into which it has pleased the powers that be to call them, and endeavouring to express in language of appropriate moderation their claims to a wider recognition of natural gifts as yet unsuspected by the community in general.

The *Yale Alumni Weekly* prints the following application for a professorship at the university :

Yale Colledge—I have a natural gift of natural science. I have had no training as a professor but with the Athorities of the Colledge I could applay a great value. I would like a good paying position like that. To test me ask me any question wich is supposed to have no answer try me by mail well enough or through the Warriors Club. found in old directories so that my trip will be quiet certain ask me anything in regards to the relations of one thing to another also of accidents or the caracter of the life of anything from the winds to the human being or the spirituel to the matearial of all supersitions of any part of the body as thought builds it I will give them to you with prove. I can regulate inclinations by diet cure decease.—Sincerely.

The following is from Georgia, and is a worthy pendent to the above :

Dear Sir as I understand the matter, that you will furnish aid or, will help a worthy project your Hon, Sir, I am going to make known to you a secreate that I have not made known to others or, to non other than my own people, or famly. The Secreat is simply this. I am a geanis, and I have underway 3 great projects, one the greatis that has ever leaped from the Brain of man, this last one menshion has many difrance action or feuturs which is difficult for me to carry out on account of not having surficient tools nither have I time to work at the project because of my financil strain, thanking you very much for your favor to reply, in favor to help me in this matter or to reject.—Yours truly.

REVIEWS.

Report on Gyroscopic Theory to the Aeronautical Committee, 1914.
By SIR GEORGE GREENHILL. Printed by the Stationery Office, and sold by Wyman & Sons, Breams Buildings, Fetter Lane, E.C. Price ten shillings.

A collection is made in this report of the scattered theory and formulas of the motion of a spinning body, such as the top, and extended to other familiar examples of the gyroscope, great and small, and introductory to the practical applications arising to-day in the steering and stability of the flying machine and airship, the gyro-compass, the single track gyroscopic car, and in the whirling effect of a revolving shaft carrying a flywheel, as in the Laval turbine. On the largest scale the theory is investigated of the Precession of the Equinox, revealed to us by astronomical observation, and illustrated too by the motion of the Moon's Node.

The mathematical treatment is carried out with analytical completeness, as far as possible in the mathematical development of to-day, to make the report serve for a collection of reference when the theoretical investigation is required without delay when a practical problem is proposed, requiring a solution within the limits of practical requirement.

The distinction arises at the outset between dynamical and statical equilibrium and stability.

In statical equilibrium the centre of gravity of a system seeks out the lowest position it can assume. But in the dynamical equilibrium, for instance of the sleeping top, the centre of gravity rises as high as it is able; and when the top is steady upright, it is said to sleep.

A man or animal sleeps lying down in statical equilibrium, with the centre of gravity at rest in the lowest position.

But in movement a man assumes the noble upright attitude of the biped, or rides upright on the back of a horse, a camel, or a bicycle, in the position of statical instability.

This is for ease of progression, and any burden, rifle or knapsack, he prefers to carry as high up as possible. Mounted still higher on stilts, or as Blondin on a tight rope, his progress is not more difficult, with the confidence of experience.

A confusion of statical and dynamical stability has led to serious mistakes and misapplication of theory. Such as ballasting a ship too low, making it bottom-heavy and uneasy among waves, according to the recommendation of Bernoulli the Swiss admiral, very good on a lake, but uninhabitable at sea; also in the medical doctor's design of a soldier's knapsack low down on the back, suitable only for a halt, on the march fatiguing and uncomfortable.

Then there was the craze for spreading the railway gauge so as to lower carriage body and boiler between the wheels.

Such theory has found itself in conflict with practical experience, realised we see to-day in the modern tramcar and motibus, as well as in the old prints (there is a good illustration in Verdant Green) of the stage-coach loaded with passengers and luggage high up on the roof, and running freer in consequence with a slower roll on the springs.

And to-day the modern locomotive has the boiler pitched as high as possible to pass under a bridge.

It is the limitation of the loading gauge of bridge and tunnel of the timid early railway engineer, brought up in the traditions of canal work, which has hampered railway development, and not so much the restriction of the gauge of the rails, the original Roman gauge of 5 feet outside, adopted all over the world; and 7-foot gauge is all pulled up; while the megalomania of the Irish gauge, 5 feet 6 inches, has not spread out of that island; any tendency discernible so far is towards a reduction of gauge, especially in the delightful toy railways, to open out picturesque scenery.

Practical experience has not borne out the early theory of Brunel and Crampton, statical and misapplied, that a low centre of gravity is conducive to steady easy running of a locomotive or railway carriage on springs; and their high single driving wheels, symptom of the small boiler scant of breath, have disappeared to-day with the increase in boiler power to cope with the heavier trains.

A lecturer on gyroscopic theory with experiments will do well to avoid the use of string to spin a top. A glance of the audience at the length of his string will show if the lecturer is going to fizzle his spin.

The length should not exceed half the stretch of the arms, as this is well known to the cheapjack as the place where the relative velocity of the hands is a maximum, a fundamental principle too in boxing.

Better use some apparatus more visible to the audience than a top; and a bicycle wheel, ready made to hand, is excellent for the purpose, with the axle prolonged into a stalk; this can be spun by hand in a smooth cup on the table; and it can be taken up and brandished to feel the muscular sense of the gyroscopic reaction, and so imitate the feelings of the flying machine, when making a turn or circle, due to the revolving screw.

Hold the stalk in a fixed position, with the axle horizontal or inclined, and the wheel makes an excellent pendulum if put out of balance by a bar between the spokes; the angle of oscillation may be made as large as desired, or the wheel may make complete revolutions to illustrate pendulum theory. Or else the bicycle complete may be stood on its back on the table, and the front wheel used as a pendulum put

out of balance ; while the back wheel gives the effect of rotation on an axle not a principal axis.

To show off complete gyroscopic motion the bicycle wheel must be suspended to hang freely by the stalk in a vertical position. Alt-azimuth suspension can be given from a bicycle hub on an iron bracket bolted to the under side of a beam or sleeper ; or a mere swivel hook will serve with the stalk hooked on to it to swing and revolve freely in altitude and azimuth. Spin enough can then be given to the wheel by hand.

As the motion is in three dimensions some such apparatus is essential to visualise it, and preserve the distinction of right and left, up and down, with the clock and against it, deasil and widdershins, distinctions impossible to keep clear in the Flatland of a blackboard drawing, or on a sheet of paper.

The first chapter of the report is restricted to steady gyroscopic motion, and an exact geometrical treatment is given with as few symbols as possible ; and the same geometry can be extended to a number of allied problems, as of the steady motion of plate, dish, egg, or wine-glass on the table, or of a stud or coin rolling on the floor ; also of a wheel, bicycle, or motor-car on a road or banked track.

Kelvin's rule—Hurry the precession and the top rises—can then be extended to the bicycle to explain why it must be steered into a smaller circle to remove the upright position, and how the stability is maintained by incessant attention to over-correction.

This over-correction is made automatically in the top or gyroscope, so that a slight tremor of nutation is always present, even if invisible in the motion apparently steady. The slight nutation is evidence of the stability of the motion, and is capable of elementary treatment. But when the nutation becomes visible and considerable, the treatment requires the Elliptic Function in all its complexity, and Chapter III gives it without passing over any difficulty. The history of the general unsteady motion of the axis of the top makes an interesting chapter in showing the influence of dynamical requirements on analytical development in creating the theory of the Elliptic Function.

This function had not been invented 150 years ago when Euler first undertook the problem of gyroscopic motion. Euler identified the motion of the axis in altitude or nutation with the beat of a pendulum swinging through a large angle, like the bicycle wheel. This pendulum beats the Elliptic Functions of the first kind, and Euler's idea is still useful for giving a concrete illustration of these functions. But the motion in azimuth brings in the Third Elliptic Integral, and theory had to wait another sixty years for the treatment by Legendre, and then only of the complete integral required for the apsidal angle.

Some years later the Theta Function was invented by Jacobi for the expression of the Third Elliptic Integral, incomplete or complete. But the mixture of real and imaginary argument in the application to the top renders the expression useless by the Theta Function, although analytically complete.

As the Gyroscopic Report is intended to serve for reference when a practical problem arises, no details have been passed over in this Chapter III required in a complete treatment.

The dynamical constants are three in number in a state of top motion, making the choice of an illustrative numerical example an embarrassing one in its variety.

But some representative diagrams must be selected carefully to illustrate an actual case of motion ; here Abel's theory of the pseudo-elliptic integral comes in useful, in which the theta function quotient is made algebraical by the selection of the elliptic parameter a simple

aliquot part of a period, and so makes an oasis in the desert of the general theory.

By the choice of another constant the motion may be made to close in on itself and be expressible algebraically. But in an experiment this state must not be expected to persist, as a very slight amount of friction is enough to mask the features in changing the apsidal angle, as in the double pendulum tracings.

The simplest cases of such algebraical motion are discussed in Chapters V and VI, and illustrated by the appropriate diagram.

Otherwise Darboux's theory in Chapter IV of the associated deformable hyperboloid can be used to replace the analytical constants by geometrical lengths; calculation is thus replaced by measurement on a drawing.

Chapter VII discusses the Spherical Pendulum motion, realised when the gyroscopic apparatus is projected without any rotation of the bicycle wheel.

Chapter VIII shows the application of moving axes to the motion of a rolling sphere, and of a body alive inside with flywheels, or vessels of liquid, like an airship or flying machine.

The report concludes in Chapter IX with problems of rolling bodies, intractable at present in the general state of unsteady motion; but susceptible of treatment when steady, with a slight tremor superposed of nutation.

In this way the stability may be discussed of a hoop rolling along upright in a straight line, as well as of the more general classical problems, due chiefly to Poiseux. The treatment finds a place here of the gyro-compass, and of the rotating shaft carrying a flywheel, in its stability against whip, as for instance with a screw propeller pitching out of water, and flying round unsupported at the end of the shaft.

Hitherto the last word on gyroscopic theory has been found in Routh's *Advanced Rigid Dynamics* and the *Kreisel-Theorie* of Klein-Sommerfeld-Noether; and this report will serve to coordinate and extend the treatment and applications. G.

Theory and Applications of Finite Groups. By G. A. MILLER, H. F. BLICKFELDT, and L. E. DICKSON. Pp. xvii + 390. 17s. net. 1916. (New York, John Wiley & Co.)

It is rare to find a treatise introductory to a branch of higher mathematics which is divided into parts, each written by an expert who has interested himself in the special subject of which he writes. This is the idea of the present work. The advantages are obvious enough. We might have anticipated corresponding drawbacks; some lack of unity perhaps, or a tendency on the part of each writer to revel in a mass of detail taken from his own researches, which is doubtless interesting enough to the author, but hardly suited to the needs of the student. However, the temptations seem in the present work to have been resisted. The close contact between the authors and their mutual criticism of each other's work has secured a reasonable balance and connexion between the parts; and the emphasis laid on each author's own work is only sufficient to give a personal interest, and has not involved sacrifice of important matter.

The question of nomenclature is always a difficult one in group-theory. The authors take "substitution" as meaning "permutation," and "transformation" as meaning "linear substitution"; which seems satisfactory enough. We may also note "co-set" as equivalent to "Nebengruppe" or "partition," "cross-cut" for "greatest common subgroup" or "Durchschnitt," and so on. The definition of "product" of two transformations is not that usually adopted in this country: the point is discussed on p. 197. The same applies to the "canonical form" of a linear transformation, though the two definitions agree for transformations of finite order, which are alone discussed in the present work.