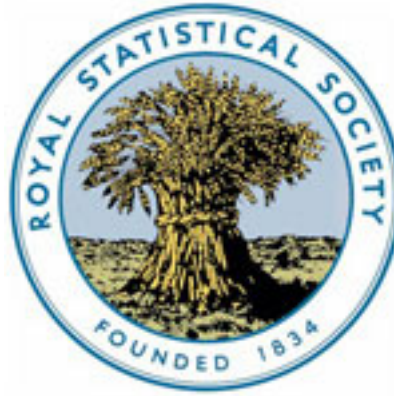


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A Practical Method of Estimating the Velocity of Circulation of Money

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II.—*A Practical Method of Estimating the Velocity of Circulation of Money.* By IRVING FISHER, *Professor of Political Economy at Yale University.*

I. *First approximation.*

HITHERTO no actual statistics for money-velocity have been attempted, while for deposit-velocity Pierre des Essars alone has given figures. In fact it has long been believed impossible to ascertain statistically the velocity of circulation of money. This opinion was well expressed by Jevons,<sup>1</sup> who wrote:—

“I have never met with any attempt to determine in any country the average rapidity of circulation, nor have I been able to think of any means whatever of approaching the investigation of the question, except in the inverse way. If we know the amount of exchanges effected, and the quantity of currency used, we might get by division the average number of times the currency is turned over; but the data, as already stated, are quite wanting.”

As we shall see, however, data do exist, capable of revealing the “amount of exchanges effected.” In fact this amount is approximately equal to the total money deposited in banks, plus the total wages paid.

This formula is as simple as it may at first seem mysterious. The chief peculiarity of the method which this formula represents, and the feature which makes it adapted to practical use, is that it utilises bank records and other ascertainable statistics as a means of discovering the total value of money transactions. The method is based on the idea that money in circulation and money in banks are not two independent reservoirs, but are constantly flowing from one into the other, and that the entrance and exit of money at banks, being a matter of record, may be made to reveal its circulation outside.

It is obvious how the bank record would be read were it true that every dollar withdrawn from banks circulated once and only once before being redeposited. Under these circumstances the annual flow of monetary circulation would exactly equal the annual withdrawal from banks prior to circulation, as well as to the annual deposits in banks subsequent to circulation. Under these circumstances every dollar used in exchange for goods would be, before its exchange, withdrawn from bank and afterwards redeposited. Since we have a record of the first and last steps of the three, viz., the withdrawals and the deposits, we would possess the means of knowing the intermediate step, the exchange of money for goods.

In actual fact, however, the situation is not so simple, for the reason that money withdrawn from banks is often circulated more than once. Yet the complications involved follow definite laws. They do not destroy the value of the bank records, but merely make it somewhat more difficult to read. We hope to show (1) that in

<sup>1</sup> *Money and the Mechanism of Exchange.* London, 1893, p. 336.

actual fact much money circulates out of bank only once, as in the hypothetical case just mentioned ; (2) that when it is paid for wages, it usually circulates twice ; and (3) that only rarely does it circulate three or more times before completing its circuit back to the banks.

We falsely picture the circulation of money when we think of it as consisting of a perpetual succession of transfers from person to person. It would then be, as Jevons said, beyond the reach of statistics. But we form a truer picture if we think of banks as the home of money, and the circulation of money as a temporary excursion from that home. If this description be true, the circulation of money is not very different from the circulation of checks. Each performs one, or at most, a few transactions outside of the bank, and then returns home to report its circuit.

For the purpose of tracing the circulation of money, and measuring it by bank records, we may classify the persons who use money in purchase for goods into three groups :—

1. Commercial depositors, *i.e.*, all engaged in business—firms, companies, and others—who have bank deposits mainly or wholly apart from personal accounts.

2. All other depositors, chiefly private persons.

3. All who, like most wage earners, are not depositors.

These three classes we shall distinguish as “Commercial depositors,” “Other depositors,” and “Non-depositors,” or *C*, *O*, and *N*. The money in the possession of “Commercial depositors” we shall call “till money,” and the rest “pocket money.”

The three groups necessarily include all in the community who circulate money. By circulating money is meant expending it in exchange, not for other circulating medium, as checks, but for goods.

The nature of these three groups must now occupy our attention. In countries advanced in the art of banking, “Commercial depositors” include practically all business establishments, and little else. “Other depositors” include most persons in the professional and salaried classes and proprietors, and little else, while the class of “Non-depositors” is almost coterminous with wage earners.

It is true that these characterisations of the three classes are not quite complete. In regard to “Commercial depositors,” for instance, there are some small business establishments like street vendors, having no bank accounts. But their number is small in comparison with the number of business persons who have accounts, and, what is more to the point, the money they handle is even more negligible. In the United States at least, excepting in the rural parts of the South and a few other places, the money expenditures of which are very small, the custom of having bank accounts is practically universal among business establishments. To keep a bank account is, in fact, a necessity for an ordinary business man, if he wishes to succeed. Otherwise he practically deprives himself of three of the most essential aids in modern business, the use of circulating credit, the use of remittance by post, and the use of time credit.

Unless a dealer is obliged to pay “spot cash” or prefers to do so—and such cases are both few in number and insignificant in

the amounts of money involved—he will almost invariably find it easier to make payment by check. Moreover, the very fact that most other business men use banking facilities creates in his mind the desire to have an account himself, both because he dislikes to appear “different,” and because when others pay him by checks he finds it necessary to cash these checks, which is always more trouble than to deposit them.

Cash payments are especially inconvenient when business is done at a distance. Remitting money by post, express, or personal delivery is troublesome, risky, and expensive as compared with posting a letter containing a check. Even a post-office money order is a clumsy and expensive substitute, and its use proclaims the user an insignificant financial factor.

Again, a business man without a bank account cannot usually obtain time credit, either from dealers or from banks. In the United States a bank likes to lend only to its own depositors. A business man who asks for a bank loan usually meets with the request to open an account. If he should seek a loan from another dealer, as, for instance, his supply house, the absence of a bank account would arouse suspicions as to his business standing, and might lead to refusal.

These facts, confirmed by observation and inquiry, have led to the belief that practically all business in the United States, certainly over 99 per cent. (measured not by number, but by size), makes at least some use of bank accounts. Even in localities where there are no banks, traders usually like to have a bank account in the nearest town, in order to facilitate their dealings as purchasers. We conclude, therefore, that the category of “commercial” depositors practically coincides with the category of business establishments.

“Other depositors” include most proprietors, professional, and salaried persons. Almost no wage earners are included, and almost no business establishments or business men in a business capacity. When a single individual conducts a business he usually separates carefully his business self from his personal self. John Smith, the individual, and the John Smith Shop are distinct. The pocket money of the one and the till money of the other are not confused. Where payments are made from one to the other, the transaction is regarded as of the same nature as the payments between the shop and any other person. Originally and under primitive conditions it is, of course, true that no such distinction was observed, and even to-day there are cases where the differentiation is unmarked, *e.g.*, hucksters, pedlars, fruit stands, and small country shops. But, as we have seen, these persons are not usually depositors. Moreover, the number is small, and since by the nature of the case the money they handle is still smaller, their classification is, for practical purposes, a matter of indifference. It is true that occasional cases exist of ordinary business men who have the exclusive ownership of a business and do not take care to separate clearly their business and their personal accounts. Yet we may in such cases perform the separation in thought. Where such a person withdraws money

from his till and puts it in his pocket, we may say his business self has paid his personal self some dividends of the business. Likewise, his checks drawn are usually distinguishable as between his business or his personal expenses, even though he himself fails to keep two separate bank accounts. But such cases are rare and unimportant, because modern business of size is usually conducted by partnerships and corporations, where a strict separation of accounts is necessary to safeguard conflicting interests. So much for the line of demarkation between "other depositors" and "commercial depositors." As to the line separating "other depositors" and "non-depositors," it should be observed that, although "other depositors" include most proprietors and professional and salaried persons, some proprietors and professional men, especially in rural communities, and some salaried persons, chiefly small clerks, are "non-depositors."

Finally, "non-depositors" consist chiefly of those who are classed in statistics as wage earners. While there are some wage earners who are depositors,<sup>2</sup> they are rare; and while there are some "non-depositors" who are not wage earners, especially the agricultural proprietors (farmers) and small clerks, the amount of money circulated by them is small in comparison with the total circulation. While the line separating wages and salaries is not definitely marked in theory, it is usually recognised easily in practice.

Children under, say, twelve years need not be included in any of the three categories, as they are not handlers of money; at least, not to a sufficient degree to have any appreciable influence on the total circulation.

We may now picture concretely the main currents of the monetary flow, including the circulation of money in exchange for goods. Fig. 1 illustrates the three principal types.

The corners of the triangle  $C$ ,  $O$  and  $N$  represent the three groups of "Commercial depositors," "Other depositors" and "Non-depositors," and the  $B$ 's represent banks. The arrows represent the flow of money from each of these four categories to another. Thus  $B_o$  represents the annual withdrawals from banks by "Other depositors,"  $O_c$  the spending of this withdrawn money by "Other depositors" among "Commercial depositors," and  $C_b$  the return of the money from the "Commercial depositors" to the banks. This circuit ( $B_o O_c C_b$ ) of three links is very common. A second type of circuit is represented by a chain of four arrows ( $B_o O_n N_c C_b$ ). It is illustrated by private depositors drawing money ( $B_o$ ), and paying wages ( $O_n$ ) to servants who in turn spend the money ( $N_c$ ) among tradesmen who finally deposit it ( $C_b$ ). A third type of circuit, also fourfold, is represented by the arrows  $B_c C_n N_c C_b$ . It is illustrated by commercial firms cashing their checks at Banks ( $B_c$ ) for pay rolls,

<sup>2</sup> The term "depositors," as here used, does not, of course, include savings bank depositors. A savings bank is not a true bank of deposit, providing circulating credit, but is itself to be classified as a "commercial depositor." It sells its own so-called deposits (as a kind of small investment) and deposits the money received therefore in a true bank of deposit, precisely like a shopkeeper or other commercial depositor.

paying with the cash so obtained wages ( $C_n$ ) to workmen who spend it ( $N_c$ ) among other tradesmen who re-deposit it ( $C_b$ ). These three types are not the only ones, but they are so much more important

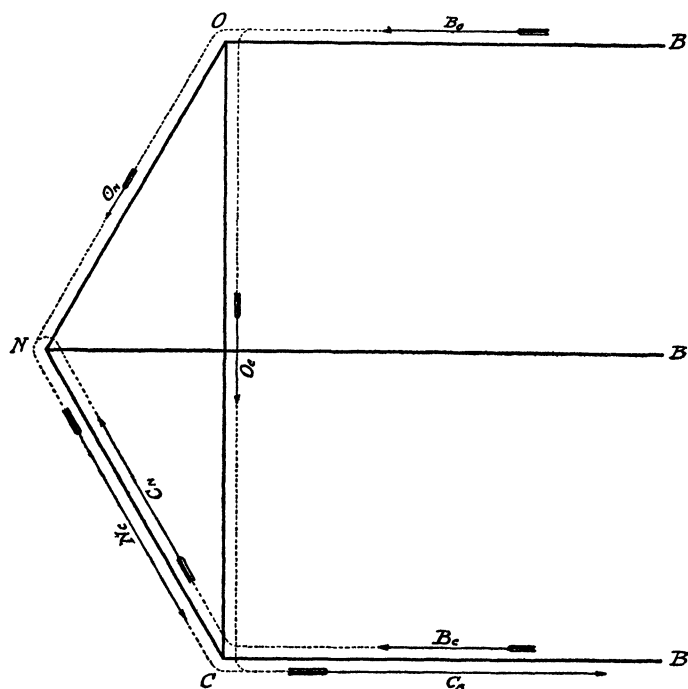


Fig. 1.

than any others that they merit our undivided attention before a completer study is undertaken. The diagram has been constructed for the purpose of exhibiting them uncomplicated by other details.

It will be noted that not all of the flows described are examples of the *circulation* of money. As already indicated, money may be said to circulate only when it passes in exchange for *goods*. Its entrance into and exit from banks is a flow but not circulation. In the diagram the horizontal arrows represent such mere banking operations, not true circulation. The arrows along the sides of the triangle, on the other hand, represent actual circulation. The diagram shows four such arrows, representing the four chief types of circulation:  $O_c$  payments of money from "Other depositors" to "Commercial depositors" in the purchase of goods,  $O_n$  payments from "Other depositors" to "Non-depositors," as when a housewife pays wages,  $C_n$  payments from "Commercial depositors" to "Non-depositors," as when a firm pays wages, and  $N_c$  payments from "Non-depositors" to "Commercial depositors," as when a wage earner buys goods of a merchant.

These four types of circulation of money occur in the three

circuits already described, being sandwiched between the flows from and to the banks. The first,  $O_c$ , is contained within the circuit  $B_oO_cC_b$ , and, since no "Non-depositors" intervene, represents money circulating *once* between its withdrawal from bank and its re-deposit there. The remaining types ( $O_n$ ,  $C_n$  and  $N_c$ ) are contained within the two other circuits ( $B_oO_nN_cC_b$  and  $B_cC_nN_cC_b$ ), and, owing to the fact that "Non-depositors" intervene, represent money circulating *twice* between withdrawal and re-deposit.

In short, one of the three circuits ( $B_oO_cC_b$ ) shows money circulating *once* out of bank. Both the others pass through  $N$ , and show money circulating *twice* out of bank. The diagram, then, represents all circulating money as springing from and returning to the banks; all of it as circulating *at least once* in the interim; and that portion handled by "Non-depositors" as circulating *once in addition*. Therefore the total circulation exceeds the total flow from and to banks by the amount flowing through "Non-depositors." In other words, the total circulation in the diagram is simply the sum of the annual money flowing from and to banks and the money handled by "Non-depositors." The quotient of this sum divided by the amount of money in circulation will give approximately the velocity of circulation of money.

We have, however, still to consider the correction to be made for the less important forms of monetary circulation excluded from the diagram.

## II. The complete formula.

In order to estimate the degree of accuracy of the first approximation for the circulation of money, we need to compare this approximation with a complete formula framed to include all possible transfers of money against goods.<sup>3</sup> There are nine possible kinds of transfers, three being respectively *within* each one of the three groups  $C$ ,  $O$ , and  $N$ , and six being *between* each pair of these three, in either direction.

The exchanges possible within a class are (1) those between one "Commercial depositor" and another "Commercial depositor"; (2) those between one "Other depositor" and another; and (3) those between one "Non-depositor" and another. The transfers possible between classes are (4 and 5) those between "Commercial depositors" and "Other depositors" in either direction; (6 and 7) those between "Other depositors" and "Non-depositors" in either direction; and (8 and 9) those between "Non-depositors" and "Commercial depositors" in either direction. The three intraclass kinds and the six interclass kinds make the nine possible kinds of transfers of money against goods.

<sup>3</sup> That is, all transfers *within* the community considered. If it is desired to include as part of a community's circulation the sums exported or imported in foreign trade, these may most conveniently be added at the end. But even if they be included they will be of trifling significance, partly because foreign trade is usually very small compared with domestic, and partly because money is so little used in foreign trade, especially if we exclude bullion from the category of money.



Fig. 2 gives a complete picture of all these nine flows of money in exchange for goods, that is, the entire "circulation of money." The nine flows are represented in the diagram by the nine arrows

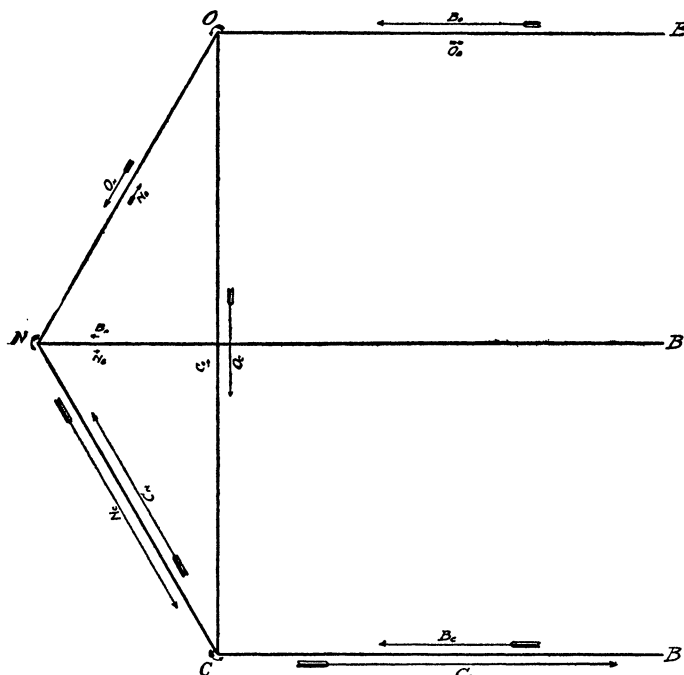


Fig. 2.

about the triangle, six being along the three sides of the triangle and representing interclass circulation, and three ( $c$ ,  $o$ , and  $n$ ) at the corners to represent intraclass circulation. (The remaining six arrows on the horizontal lines represent, of course, mere banking operations.) The total circulation or monetary flow ( $F$ ) in exchange for goods is therefore the sum of the magnitudes represented by these nine arrows, viz.:—

$$F = O_c + C_o + N_c + C_n + O_n + N_o + c + o + n. \quad (1)$$

This is an exact formula for the circulation of money. We shall now compare it with the inexact first approximation, namely, "money deposited plus expenditures of 'non-depositors.'" This comparison will express the error of the first approximation, and will suggest a method of transforming the exact formula (1) into a shape more suitable for statistical application. First, we need to express algebraically the first approximation. This may easily be done by inspecting the diagram, Figure 2. The total money deposited is  $C_b + O_b + N_b$ , while the total expenditure of



"Non-depositors" is  $N_c + N_o$ . The sum of these two expressions we shall call  $F'$  and is:

$$F' = C_b + O_b + N_b + N_c + N_o, \quad (2)$$

which is therefore the algebraic expression for the first approximation.

To obtain the difference,  $F - F'$ , between the exact and the approximate formula, we subtract (2) from (1), cancelling  $N_c$  and  $N_o$  and placing the negative terms first. We thus obtain for a remainder ( $r$ ) the following:—

$$r = F - F' = -C_b - O_b - N_b + O_c + C_o + C_n + O_n + c + o + n. \quad (3)$$

That the value of  $F - F'$  is small may be seen clearly by transforming (3). We shall transform it by using the principle that the net outflow (*i.e.*, outflow minus inflow) from the contents of any reservoir must equal the net decrease in its contents during the same time; or (algebraically expressed) that the net outflow (positive or negative) plus the net increase (negative or positive) must be zero. We may apply this principle to any reservoir or store of money, but shall here find it most helpful to apply it to the reservoir of money contained among the "*Commercial depositors*" and "*Non-depositors*" taken together as one group. Let us designate the combination of these two as the "*CN group*." The total outflow indicated in the diagram from this "*CN group*" is evidently  $C_b + C_o + N_b + N_o$ , and the total inflow  $B_c + O_c + B_n + O_n$ . Hence the net outflow, so far as the diagram shows us, is:

$$C_b + C_o + N_b + N_o - B_c - O_c - B_n - O_n.$$

This, plus the net outflow *not* shown in the diagram, is the true net outflow. Since the diagram was constructed to show only flows against goods (monetary circulation), and flows to or from banks, we have still to take account of money flowing in the community in exchange for something else than goods, and that flowing without any exchange at all, as well as any net outflow *outside* of the community.

We have thus to take account of three undiagrammed flows. The first is the net outflow of money from the "*CN group*" to the *C* group, which, though in exchange, is not in exchange for goods. This means simply cashed checks, for, according to the classification we are here using, "goods" are taken to include anything exchangeable, not either money or checks. Our first correction is therefore the net outflow of money from the "*CN group*" for cashing checks, *i.e.*, the difference between the checks cashed by the "*CN group*" for the "*O group*" and those cashed in the opposite direction.

It will be understood that we have nothing to do here with the cashing of checks at banks, for this is included in the diagram ( $B_o$ ,  $B_n$ , and  $B_c$ ). Moreover, we have nothing to do here with cashing of checks *within* the "*CN group*," as when a storekeeper cashes a check presented by a "Non-depositor." We only have to do with the net outflow for cashed checks from *CN* to *O*. This net outflow (which may be positive, negative, or zero) we shall designate by the letter  $a$ , to stand for "accommodation" checks.

For the second correction we have to designate the net outflow of money *given away* by the *CN* group in gifts, taxes, thefts, &c., for which no specific goods are received in return. This net outflow may be designated by *g*.

We have, thirdly and lastly, the net outflow of money with respect to the "*CN* group" outside of the community, *i.e.*, the net amount of money which is lost to the country by export, fire, shipwreck, melting, &c., in excess of that imported, minted, &c. This net outflow may be designated by *e*, to stand for "external" outflow. Adding the net undiagrammed outflow (*a* + *g* + *e*) to the net diagrammed outflow we have, for the total net outflow—

$$C_b + C_o + N_b + N_o - B_c - O_c - B_n - O_n + a + g + e.$$

On the reservoir principle already explained, the algebraic sum of this net outflow from the "*CN* group" and the net increase of the money in that group must be zero. Representing the net increase by *i*, we have

$$0 = C_b + C_o + N_b + N_o - B_c - O_c - B_n - O_n + a + g + e + i. \quad (4)$$

We now place this equation under the equation giving the value of  $F - F'$ , as follows:—

$$r = F - F' = -(C_b) - (O_b) - (N_b) + (O_c) + C_o + C_n + (O_n) + c + o + n \quad (3)$$

$$0 = (C_b) + C_o + (N_b) + N_o - B_c - (O_c) - B_n - (O_n) + a + g + e + i. \quad (4)$$

Adding and cancelling the terms indicated in parentheses, we have

$$r = F - F' = (C_o + C_n - B_c) + (C_o + N_o - O_b) + (c + o + n) + (a + g + e) + i - B_n. \quad (3')$$

The terms are here arranged, as far as can be judged, in the order of descending importance.

By using the expression just obtained for *r*, the complete formula (1) for the circulation of money may now be put in a form suitable for statistical application. Since  $r = F - F'$ , then  $F = F' + r$ . Substituting for  $F'$  and *r* the expressions already expressed in (2) and (3)', we have, as a transformation of (1),

$$F = F' + r = (C_b + O_b + N_b) + (N_c + N_o) + (C_o + C_n - B_c) + (C_o + N_o - O_b) + (c + o + n) + (a + g + e) + i - B_n \quad (1)$$

- = (1) all money deposited
- + (2) money expenditures of "Non-depositors"
- + (3) *C*'s money expenditures from tills (not withdrawn from bank)
- + (4) *O*'s money receipts pocketed (not deposited in bank)
- + (5) intraclass monetary circulation
- + (6) *CN*'s undiagrammed net outflow of money.
- + (7) *CN*'s net increase of money on hand
- (8) *N*'s withdrawals of money from bank.

This is a complete and universal formula for the circulation of money in any community. Its first two terms constitute the first approximation, and the other six terms the remainder term *r*.

The first two terms are by far the most important. The last three terms are doubtless quite negligible under all circumstances. I am also

reasonably confident that, in the United States, the 3rd, 4th, and 5th terms amount to less than 10 per cent. of the total and probably less than 5 per cent. Anyone familiar with the inaccuracies of statistics knows that these are small errors, especially for a magnitude which has hitherto eluded any attempt at measurement. We may therefore distinguish three successive stages in our approximations. The first approximation comprises only the first two terms, viz., money deposited plus expenditures of "Non-depositors"; the second includes in addition terms (3), (4), and (5), viz., till-paid money expenditures of *C*, pocketed money receipts of *O*, and intraclass circulation; while the third is complete by including terms (6), (7), and (8), none of which have practical importance. The formula is presented in the hope of arousing discussion and investigation which will disclose in particular to what extent it may be applied in countries where data exist for the first two terms, viz., money deposited and expenditures of "Non-depositors." The former is certainly a matter of record in most civilised countries, and the latter always consists largely of wages, a magnitude which has for long been a favourite subject for statistical estimate.

### III. *Statistical application.*

We shall now exemplify the use of our formula by means of actual figures for the United States. The report of the Comptroller of the Currency for 1896 gives a basis for estimating the annual money deposited in banks. The result is approximately 11 billions of dollars. The census statistics and the unofficial estimates of Wm. C. Hunt, of the Census Bureau, lead to the conclusion that the money expenditures of wage earners were about  $4\frac{1}{2}$  billions. We have merely to add an allowance for other "Non-depositors," besides wage earners, viz., some small clerks, agricultural proprietors and professional men. Making a conjectural allowance for these persons, the total annual money expenditure of "Non-depositors" is placed at 6 billions, which probably errs in excess rather than in deficiency.

Our "first approximation" ( $F'$ ) is therefore 11 + 6 or 17 billions, to which must be added the remainder  $r$  consisting of the many terms already explained, most of which are not known with exactness, but all of which are known to be small. The term "small" is always relative, and in this case a term is small which is small compared to 17 billions. For instance, 170 millions is a mere trifle, being only 1 per cent. of 17 billions, while 17 millions is only one-tenth of 1 per cent. We do not have exact statistics for  $r$ , and for purposes of comparison we do not need them; for if  $r$  is small and varies nearly as the rest of the circulation varies, a large mistake in estimating it will make only a small error in comparisons. Only in case  $r$  were both large and variable relatively to the other terms could a mistake in its estimation greatly affect the comparisons.

An attempt has been made to estimate  $r$ , not so much for the purpose of obtaining its absolute value, as to set for it wide and safe

limits. The detailed estimates for the six terms of which  $r$  is composed would scarcely interest English readers. I shall therefore merely outline the methods and results, leaving a more minute description to a book now in preparation on the purchasing power of money.

The third term of the formula is  $(C_o + C_n - B_o)$ . This represents the till-paid commercial expenditures, or the excess of the money paid out by "Commercial depositors" over the money withdrawn by them from banks. Inquiry shows that the great bulk of the money withdrawn by "Commercial depositors" from banks is drawn for the purpose of paying wages; also that the great bulk of the actual money expended by "Commercial depositors" is for wages. In other words,  $C_o$  is very small compared with  $C_n$ , and the sum of the two is nearly the same as  $B_o$ . Hence the difference  $(C_o + C_n - B_o)$ , or till-paid expenses, is nearly zero. Till-paid expenses, being mostly wages and, as all observation shows, only a small part of total wages, viz.,  $4\frac{1}{2}$  billions, certainly, I am assured, less than a tenth, can accordingly be set down as less than half a billion.

The fourth term,  $C_o + N_o - O_o$ , is  $O$ 's money receipts which are pocketed instead of deposited. Now  $O$ 's money receipts,  $C_o + N_o$ , are small in the first place, for  $O$ , being depositors, usually receive their dividends, interest, and salaries by check. The chief exception is found in the rents and the professional fees paid by working men to landlords, physicians, &c., and which constitute most of  $N_o$ . But these rents and fees paid by working men to private individuals are only a part of total rents and fees of working men, and the total itself is only about 20 per cent. of wages. From this and other clues, we may safely set half a billion as an upper limit for the fourth term.

The fifth term,  $c + o + n$ , is the circulation within each of the three groups. Obviously only in trifling cases does money circulate between one "Commercial depositor" and another, between two "Other depositors," or between two "Non-depositors." Half a billion is put as an extreme upper limit for the total. This would mean that one dollar out of every thirty-five is spent within the class to which the spender belongs. That the truth is less than this ratio is the universal testimony of representatives of  $c$ ,  $o$ , and  $n$ , who have been interrogated.

The remaining three terms are even more insignificant. In the normal state of equilibrium for the "CN group" it is evident that the sixth and seventh terms would both be substantially zero. The eighth term, withdrawals from banks by people who have no bank accounts, represents very exceptional conditions.

We shall summarise the estimates for each of the eight terms in the following table. Each term is placed midway between upper and lower limits estimated as safe, and the possible variation in either direction is indicated after a " $\pm$ ." Thus \$250,000,000  $\pm$  \$250,000,000 means simply that, though \$250,000,000 is assigned as the estimate, the true value may be more or less by an amount not exceeding \$250,000,000, in other words, that the truth lies between \$500,000,000 and zero.

1st term, money deposited ( $C_b + O_b + N_b$ )....	11,000,000,000 $\pm$ 1,000,000,000
2nd term, expenditure of "Non-depositors" ( $N_c + N_o$ ) .....	6,000,000,000 $\pm$ 1,000,000,000
<hr/>	
1st and 2nd terms, constituting the "First Approximation" ( $F'$ ).....	17,000,000,000 $\pm$ 2,000,000,000
<hr/>	
3rd term, $C$ 's expenditures till paid ( $C_o + C_n - B_c$ ) .....	250,000,000 $\pm$ 250,000,000
4th term, $O$ 's receipts pocketed ( $C_o + N_o - O_b$ ) .....	250,000,000 $\pm$ 250,000,000
5th term, intraclass circulation ( $c + o + n$ ) ...	250,000,000 $\pm$ 250,000,000
6th term, net undiagrammed outflow of money from " $CN$ group" ( $a + g + e$ )....	000 $\pm$ 100,000,000
7th term, net increase of money in " $CN$ group" ( $i$ ) .....	000 $\pm$ 100,000,000
8th term, money withdrawn from banks by "Non-depositors" ( $-B_n$ ).....	- 500,000 $\pm$ 500,000
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Total 3rd to 8th terms ( $r$ ) .....	749,500,000 $\pm$ 950,500,000
<hr/>	
Total of all terms ( $F$ ).....	17,749,500,000 $\pm$ 2,950,500,000
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The remainder,  $r$ , is here given as less than a billion. Added to the 17 billions, it makes the total about 18 billions as the estimated circulation of money in the United States in 1896. This estimate is subject to error, but not as much as the total of the possible errors of individual terms, which is nearly 3 billions. Even if each of the possible errors indicated were as likely as not to occur, the chance that in all eight cases they should all simultaneously occur in the same direction is  $(\frac{1}{2})^8$ , or one chance in 256. We may therefore "trust to luck" that the errors will to some extent offset each other. In fact, the chance of the error reaching half of this amount, or  $1\frac{1}{2}$  billions, seems extremely small. The "probable error" can only be surmised to be much less than  $1\frac{1}{2}$  billions, for we have not attempted to state the probable errors of the constituent figures, but only *improbable* errors.

Reverting now to the remark with which we began the discussion of money velocity, namely, that it circulates but seldom outside of banks, let us picture our statistical results in the light of this fact.

Evidently if all money circulated once only, then the bank record of 11 billions annually flowing into and out of the banks would also exactly indicate the volume of the intervening work done. This would then be 11 billions. But the truth is, as we have shown, probably about 18 billions, and consequently we infer that some of the 11 billions emanating from banks changes hands more than once before it returns.

Next let us suppose that all of the 11 billions circulated once, except the part passing through the hands of "Non-depositors" (6 billions), and that the latter circulated twice. Then 5 billions circulate once only. Under this assumption we can account for  $5 + 2 \times 6 = 17$  billions of exchange work. But we have found in fact 18 billions. The difference is chiefly due to the existence of some money which circulates more than twice outside of banks.

The entire 18 billions may be roughly accounted for by dividing the 11 billions flowing from banks into three parts, 5 billions circulating once and once only, 5 billions twice and twice only, and 1 billion three times. This makes  $5 + 2 \times 5 + 3 \times 1 = 18$  billions. Of the three parts, the first (5 billions) is mainly the spending money drawn by "Other depositors," the second (also 5 billions) is money withdrawn from bank for wages and other payments to "Non-depositors," and the third (1 billion) is the small amount not otherwise accounted for. This is only a rough scheme of division. A very small part circulates oftener than three times.<sup>4</sup>

In order to obtain the value of the velocity of circulation, the total circulation of 18 billions must be divided by the amount of money circulating in 1896. This amount is estimated at \$974,000,000. Hence the velocity is  $18,000,000,000 \div 974,000,000$ , a little over eighteen times a year. In other words, money was held on the average about twenty days. If we have made as full allowance for error as we believe, the error in this estimate does not exceed two or three days. This result gives money a slower circulation than most of the estimates or guesses which have been made. These vary, however, all the way from four times a year to 150.

The "first approximation" to the value of monetary circulation evidently gives 17 billions out of our estimated total of 18 billions, showing that the remainder, 1, unless it has been greatly underestimated is small, a mere billion. The significance of this fact is that the terms most difficult to estimate statistically are the least important. Of the two terms constituting the "first approximation," the first and most important is susceptible of the most accurate determination of all, while the second is made up chiefly of wages, which also are, or seem destined to become, susceptible of statistical determination.

In fact if we should as a statistical makeshift for the first approximation merely add the amount of money annually withdrawn from deposit to the annual wages, we should account for  $11 + 4\frac{1}{2}$  or  $15\frac{1}{2}$  out of 18 billions, leaving only  $2\frac{1}{2}$  billions to be otherwise accounted for. In other words, this makeshift, or part most adapted to statistical measurement, accounts for about 85 per cent. of the total circulation, leaving only 15 per cent. for the part which can only be determined within wide limits. The two parts may be distinguished as the measurable part (comprising the first term of our formula (1)' and most of the second term), and the conjectural part comprising the remainder. Even if the allowance of  $2\frac{1}{2}$  billions for the conjectural part should prove to be but half the truth, the measurable part would still constitute 75 per cent. of the total. The measurable

<sup>4</sup> It may avoid some confusion to remind the reader that we are dealing with sums of money expended for goods, not with individual coins. Many coins remain "in circulation" a long time without returning to bank, because used "in change." But money used in change enters as a subtractive term in monetary expenditures. When \$10 are given for an \$8 purchase, and \$2 are received back in change, \$12 have changed hands, but only \$8 of monetary circulation against goods have been effected.



part would therefore still be a safe practical index, or barometer of changes in the volume of circulation. Any excess of variation in the conjectural part, as compared with the measurable part, would, when spread over the whole, produce a disturbance only one-fourth as great. It is reasonable to suppose that the conjectural and the measurable parts will ordinarily vary together. If the measurable part varies 10 per cent., it is natural to suppose that the conjectural part, and therefore also the total of both, will vary likewise. But suppose this assumption erroneous and that, while the measurable part varies 10 per cent., the conjectural part really varies 14 per cent. or 6 per cent. The difference between these and 10 per cent., *i.e.*, 4 per cent., representing a supposed excess of variation of the conjectural part, would produce a difference of only 1 per cent. in the total. The total, instead of varying 10 per cent., would vary 11 per cent. or 9 per cent. Evidently, therefore, any unknown variation in the conjectural part can only cause a trifling variation in the result. In other words, the measurable part will always be a good index of the total—a reliable barometer of circulation. Consequently *money deposits plus wages divided by money in circulation* will always afford a good barometer of the velocity of circulation.

It is not always the absolute value of any magnitude we find most useful, but its relative value under different conditions. We may compare the relative length of two ships by measuring their water lines, although this method omits the overhang at either end. Such a comparison will apply roughly to any two vessels, and with great exactness to two ships of the same build. Similarly our proposed barometer will afford rough comparisons for any two countries using banking facilities, and will afford fairly exact comparisons for two successive years in the same country.

The proper statistical procedure would therefore seem to be to provide for the conjectural part by an estimated percentage correction, to be applied to the measurable part as a constant factor. Different correction factors will presumably apply in different countries, as, let us say, 15 per cent. in the United States, 20 per cent. in England, 30 per cent. in France, &c. The chief value of such conjectural corrections would be to enable us to compare roughly the circulations and velocities of different countries. For comparisons in the same country at different times it would be immaterial what percentage correction is adopted or whether none is employed.

By means of the method which has been explained, it is believed that some interesting and valuable results can in the future be obtained, if statisticians in various lands will obtain (1) the total money deposited each year in banks (except by other banks), or, what is normally the same thing, the total money withdrawn from banks (except by other banks); (2) the total wages spent, or, what is normally the same thing, the total wages received; (3) a conjectural percentage addition to allow for the remaining and less known part of our formula; (4) the total money in circulation. The sum each year of (1) and (2) corrected by (3) and divided by (4) will be a very accurate barometer of the velocity relatively considered, as well as a fair approximation to its absolute value. The



omission of (3) will not invalidate the results for most purposes of comparison.

The importance of such accurate determinations can scarcely be overestimated, as the remarks on the subject by Jevons, Landry and others have shown. When we know statistically the velocity of circulation of money we shall be in a position to study inductively the "quantity theory" of money, and to discover the significance of that velocity in reference to crises, accumulation of wealth, density of population, rapid transit and communication, as well as many other conditions. In fact a new realm in monetary statistics will have been opened.

III.—*A Statistical Note on Birth Registration in Scotland previous to 1855; suggested by Inquiries as to Verification of Birth for Old Age Pensions.* By G. T. BISSET-SMITH, *H.M. Registration Examiner.*

IN the *Journal of the Royal Statistical Society*, vol. xi, p. 282, *et seq.*, will be found the valuable suggestions made by the committee of the Society on the Registration Bill for Scotland, introduced by Lord Rutherford in 1847. For the purposes of statistical science, the parish registers kept in the pre-compulsory registration period in Scotland are almost quite useless. This fact is demonstrated best by a study of the old parochial registers themselves, with their meagre entries and irritating irregularity; and their incompleteness is exposed in the figures with which this brief Note concludes. The writer has himself verified some of the facts and figures quoted; but he wishes to express indebtedness to the "Sketch of the History and Imperfect Condition of the Parochial Records in Scotland" of the late George Seton, Esq., M.A., Advocate, formerly Secretary in the Department of the Registrar-General for Scotland. Contributions to the *Journal* from, or regarding, Scotland appear rare; and it is much regretted that pressure of official duty has rendered this Note brief and inadequate. It relates, however, to a subject which is at present of considerable general interest.

I.

Scots are notoriously migratory. Some of them prefer London to Edinburgh, as was the case with Boswell, the entry of whose birth will be quoted later in this article. Habits of thrift are credited to the Scottish race, also the art of getting on in the world. There must, however, be a considerable number of persons born in Scotland who have failed to accumulate even a modest competency; for the present writer is aware that not a few Scots—some of whom are long resident out of their native country—are now anxious to obtain proof of their age in order