

Werner's narrative presents a new geographical feature, the warrant for which has probably by this time been confirmed or disallowed. But the chief interest attaching to the book arises from the description of living persons, European or native, who take, or have taken, part in the story of the Congo Free State. The narrative may not always be as gratifying as truthful; nor is the record of that kind of warfare which tells us of the shooting of natives as though they were but large game, quite pleasant reading; but allowance must be made for unrecorded provocations and exigencies, and let us hope that conciliation will have a wider field for exercise when the harder obstacles to peaceful settlement shall have been removed. No civilized being could wish for the renewal of days such as those in which Mr. Werner "saw more than one poor wretch put up his shield, only just in time to receive a ball right through it and himself as well, and come rolling down the clay bank into the river, dead as a door-nail." The author has dwelt, moreover, upon a passage in his own particular career which cannot but leave a painful impression on the mind of his reader. He had been told that Tippoo Tip had threatened mischief to Major Barttelot if certain conditions were unfulfilled, and he had been restrained by circumstances from communicating intelligence of the threat to that gallant and lamented officer. It is easy to understand how this non-revelation of fore-shadowed ill haunted his brain, and how mental distress became aggravated by the sad news of Major Barttelot's death; but he may well derive consolation from the conviction that the reported threat was the outcome of a state of things which must have been fairly appreciated by all Europeans in those days encamped in the neighbourhood of Stanley Falls.

Mr. Werner is no doubt right in assuming that "facility of transport to the coast by means of railways and steamers will do more, by making slave-caravans unprofitable, to put down the curse of Africa," than the extinction of elephants—an hypothesis much favoured by recent writers. Were steam once made available for traffic along the main thoroughfares of the Dark Continent, the necessity for the employment of slaves in the conveyance of ivory would naturally cease, and one source of evil would thus be stopped by the mere force of circumstances—means quite as efficacious as, and more satisfactory than, armed intervention. In the final chapter the author considers with much intelligence and practical sagacity the different lines of communication now being opened out between the coast and the interior of Africa. These are notably, on the east, a land and water route from Uganda to the sea-mouth of the Tana River, passing through or skirting the possessions of the British East African Company: to the westward, the proposed railway to facilitate traffic between Banana and Stanley Falls; and, on the south, communications developing under the far from insignificant agency of the African Lakes Company. Truly, European enterprise is astir in the land; and England, the party most interested in the movement, if she remain content to be the most responsible speculator in its risks, should further seek to become the most important participator in its benefits.

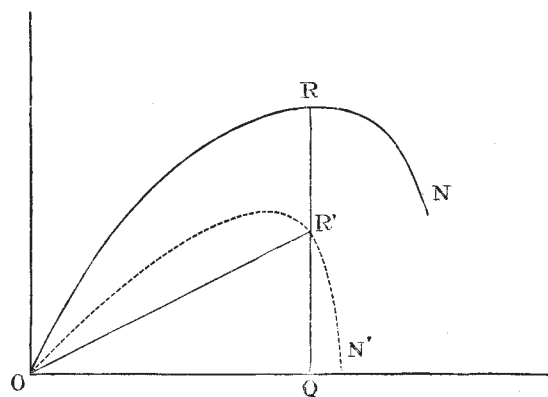
The attraction of this volume is enhanced by photographs and illustrations. Stanley's likeness is excellent, and the sketch of Mata Bwyki highly characteristic.

THE MATHEMATICAL METHOD IN POLITICAL ECONOMY.

Untersuchungen über die Theorie des Preises. Von Rudolf Auspitz und Richard Lieben. (Liepzig: Verlag von Duncker und Humblot, 1889.)

THE usefulness of mathematical reasoning applied to political economy, the value of the methods originated by Cournot and developed by Jevons, may be said to be still *sub judice*. The consideration of Messrs. Auspitz and Lieben's diagrams and symbols tends to confirm the opinion that mathematical analysis is a potent, if not an indispensable, means of obtaining clear general ideas in economics. The metaphysician who twists and turns the terms force and energy without grasping their mathematical signification is not more likely to become entangled in his talk than the practical man who reasons about supply and demand, and cost and value, without having once for all considered the ideas in their clearest and most abstract form. For the purpose of this contemplation Messrs Auspitz and Lieben employ a construction differing from most of their predecessors; namely, a figure in which the abscissa represents the quantity of a certain commodity, the ordinate the amount of some other article—in particular, money—which is exchanged for that which the abscissa represents. We cannot, however, quite admit the statement: "Unsere Kurven unterscheiden sich schon durch die zu Grunde gelegten Koordinaten von Jenen unserer Vorgänger." The same construction is used in the papers of an eminent English Professor, which, though unpublished, have been widely circulated in the learned world. It has also appeared in at least one English publication, Mr. Edgeworth's "Mathematical Psychics," with due acknowledgment to the distinguished originator.

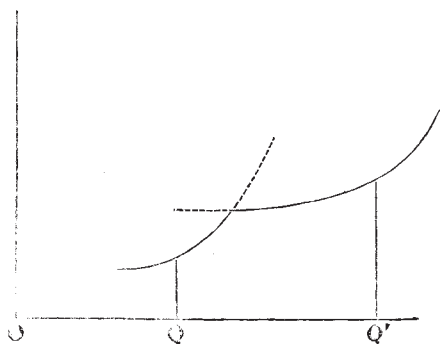
However, our authors have made the construction their own by many important developments. They employ, in addition to demand and supply curves, a less familiar *locus*, which may be thus described. In the accompanying diagram let any abscissa, OQ , represent a quantity of a



certain commodity; and let QR represent the amount of money which a consumer would be just willing to give in exchange for the commodity OQ : in such wise that it would be indifferent to him whether he procured OQ on such terms, or did not consume the article at all. The *locus* of this point ON is called the utility curve. Correlated with this primary curve is the demand curve ON' ,

which indicates the amount which any individual is willing to purchase at the price represented by the tangent of the angle $R'OQ$. The advantage which the individual derives from that purchase is represented by the length of the line RR' —that is, supposing money to be a uniform measure of value, which would cease to be true when the transaction is on a very large scale: for instance, the payment of so much as QR would so cripple a person's resources as to render money a more important object to him, to alter his *Wertschätzung des Geldes*. Still, even in this general case, ON is regarded as a "curve of constant satisfaction," which we may describe as much the same as the "line of indifference" of the English publication to which we have referred. By a parity of construction we have the cost curve to represent the amount of money in return for which one would just be willing to produce a certain quantity of an article; and the offer or supply curve indicating the amount which the producer will offer at a certain price.

The curves which we have described represent primarily the effective dispositions of individuals. By superposition of such individualistic curves, we obtain corresponding *collective* curves. The intersection of the collective demand curve and the collective offer curve gives the price. On this point our authors' analysis throws some new light. They point out that most of the curves with which we have to deal are of the nature of an *envelope*, made up of a number of distinct *loci*. Consider the cost curve of the individual, for instance. His dispositions may be represented by two discrete curves, according as we consider different scales of production; say, hand-work and manufacture by machinery, corresponding to the neighbourhood of Q and Q' respectively, in the accompanying figure. The outer portions of these lines, marked black



in our figure, form the genuine cost curve; from which a similarly composite offer curve may be derived. This sort of discontinuity has not been unnoticed by other writers, in the case of production. But we believe that Messrs. Auspitz and Lieben are the first writers who have maintained that the *locus* on the side of consumption is similarly composite; that the demand curve is made up of several bits, corresponding to different styles of life (*Lebensweise*).

It follows from these conceptions that the demand and supply curve, whose intersection determines price, must be of a simple shape, not re-entrant and crumple-horned, as they have sometimes been represented. Whence we may deduce that—theoretically, and on the supposition of enlightened self-interest—the price which

tends to prevail in an ideal market is not only determinate, but unique. There cannot be, as it were, several solutions of the equations of exchange. The interest of this conclusion will be apparent when it is remembered that the contrary statement is advanced as important by Mill, with respect to international trade, and by Prof. Sidgwick, with respect to trade in general.

The curves employed by Messrs. Auspitz and Lieben assist us in conceiving a subject on which many misapprehensions exist—the gain of foreign trade. It takes Mill and Prof. Sidgwick a good many words to prove that it is possible for a country, by a judicious import or export tax, to benefit itself at the expense of the foreigner. The truth is seen much more easily, and in a higher degree of generality, by a glance at the appropriate mathematical diagrams.

The method also adapts itself to the dealings of a monopolist. The influence of a single large dealer in competition with several small ones is represented by a construction of peculiar beauty and originality. If it is true that we are drifting towards a *régime* of trusts, combinations, and monster establishments, surely any ray of new light on this somewhat unexplored field ought to be welcomed. It may be difficult, perhaps, to estimate the positive practical value of this use of the mathematical method. We might compare, perhaps, the function of the sovereign science in respect to the theory of monopolies, with the duties of government in respect to their management—to exercise a general supervision without attempting to control details.

Messrs. Auspitz and Lieben have also treated the case of monopoly in which an individual or combination deals with another economic unit. They of course see the point, which is often missed by the *littérateur*, that, without perfect competition, the determination of price is, within certain limits, indeterminate. On the question, what basis of arbitration—in the absence of the mechanical principle of competition—should prevail, we venture to regard their answer as much more profound than that which has been given by the most eminent English Professors. An agreement to the terms which afford the greatest sum total of utility will tend to come to pass. The utilitarian position thus indicated would coincide with the settlement towards which perfect competition tends, upon a certain condition which our authors have introduced. The condition may be described as proper to perfect competition; namely, that every portion of an article should be exchanged at the same rate. We are not satisfied that our authors are justified in predicating this condition of a bargain, such as that between an employer and a combination of workmen. Nor do we accept the implied optimistic conclusion that, in the abstract at least, the play of competition in the labour market tends to the arrangement which is the best possible for all concerned.

But we are sensible that on points so abstruse it is hardly possible to make our own meaning, or that of our authors, clear, without a more copious use of symbols and verbal explanation than would be here admissible. We regret, also, that, while indicating some salient features of the work before us, we have not been able to bring out the beauty and completeness of the whole. Perhaps no other piece of reasoning which has issued from the mathematical school of economics is so perfectly fitted

together. No other of equal originality is equally easy to understand. The intellectual pleasure which is compounded of mathematical exertion and the interest in human affairs is here enjoyed must purely. F. Y. E.

PROFESSOR VON "CRANK."

Richtigstellung der in bisheriger Fassung unrichtigen Mechanischen Wärmetheorie und Grundzüge einer allgemeinen Theorie der Aetherbewegungen. Von Albert R. von Miller-Hauenfels, Professor a. D. in Graz. Pp. 256. (Wien: Manz'sche k.k. Hof-Verlags- und Universitäts-Buchhandlung, 1889.)

IT is quite refreshing to come across a real "crank" among the sober Germans. As might be expected, there is a good deal of irregular metaphysics involved in the lucubrations of a German "crank." One would not, however, expect an entire ignorance of the first principles of the mathematics involved. The author of this hardly sufficiently ingenious to be even curious work begins by objecting to the well-known thermodynamic equation for perfect gases—

$$J(C - c) = p \frac{dv}{dt} = R = \frac{pv}{T},$$

because, forsooth, it is not identical with the general differential equation—

$$R = p \frac{dv}{dt} + v \frac{dp}{dt};$$

forgetting that the definition of C , as he himself gives it, assumes that, in the first equation, p is constant. In order to escape this invented difficulty, he loads himself with an equation—

$$JQ = Jcdt + vdp + pdv,$$

which involves the remarkable result that the heat required to warm a gas at constant volume is $JQ = Jcdt + vdp$, while by definition it is $= Jcdt$. It is not necessary to remark that the author carefully neglects to draw this conclusion. His equation is founded on the interesting principle that, when any event produces two different effects on the same organ of sense, each effect must be due to a separate flow of energy. When a mass of gas is warmed at constant volume, and one resists its expansion, one feels two effects with the same organ of sense: (1) the increase of pressure, and (2) the increase of temperature; and it is argued that each must be due to a separate flow of energy. This interesting principle leads to the startling conclusion that the heat required to change a body from one state to another, is independent of the states through which the body passes, and depends only on the initial and final states; and this startling conclusion involves the equally startling inference that the internal energy of a body is a function of the states it has passed through. It would be very interesting to study the difference between water that had frequently passed through some cyclic process, and water which had not: no chemist has yet detected the difference.

It would be multiplying words without wisdom to go through the elaborate bolstering of hypothesis by assumption and unreason required to deduce any semblance to fact from these beginnings. It may however, be

worth while to notice something in the second part of this work on the nature of the ether. It is assumed that Fresnel has conclusively proved that an ether consisting of molecules which repel one another would transmit transverse vibrations like light; and, in order to turn the difficulty of the existence of longitudinal vibrations to useful account, it is assumed that these latter exist and are heat. It is hardly necessary to investigate a theory of electric and magnetic forces founded upon such an ether, and upon some curious ideas as to forces flowing from place to place.

And what is the use of spending time looking into such a work as this? It is by studying extraordinary and startling departures from reason, and not the ordinary and familiar ones, that we learn the causes of our aberrations and how to avoid them. It is the same unreasoning prejudice for "I can hardly believe it otherwise," the same neglect to study the meanings of symbols, whether words or letters, the same satisfaction with a theory that leads to some true conclusions, which bristle upon every page of this book, and which are some of the most important factors in the prejudice that ignores the necessity for verification, the muddle-headedness that is content with vague notions, the clinging to an incomplete hypothesis that stands in the way of a true theory, all and each of which are in all and each of us such bars to progress. If the study of Prof. Miller-Hauenfels' errors leads to even a state of preparedness to look out for similar errors in our own work, the study will have been fruitful. G. F. F. G.

OUR BOOK SHELF.

Traité Encyclopédique de Photographie. By Charles Fabre. Vol. I., Part I. (Paris: Gauthier-Villars, 1889.)

THIS is the first part of an encyclopædic work on the history and development of photographic methods. Its general object is to set forth, not only full particulars of the methods now in actual use, but also a complete story of the gradual improvements which have led up to them. There is little doubt that the rapid progress which has been made in photography has been largely due to the fact that the whole subject is so new, that every investigator who sets himself to work soon becomes familiar with what has gone before, and is thus in a position to consider what further advances are possible. It is certainly not too much to expect that such a work as is contemplated by M. Fabre will do a great deal towards simplifying the acquirement of this knowledge.

It is proposed to issue the work in twenty monthly parts, of which five parts will constitute a volume. The first volume will treat more particularly of the general history of photography and photographic apparatus, special attention being given to the subject of lenses. The second volume will deal with the production of negatives, and the third with positives of every description. The fourth volume will first treat of the methods of enlargement, and then of photographic chemistry and theories of the formation of photographic images. This comprehensive scheme, if well followed out, as no doubt it will be, if we may judge by the excellence of the first part, will obviously constitute a valuable addition to photographic literature.

In the first part the whole subject of lenses is considered, from the chemical composition of the various kinds of glass employed in their construction to the various combinations now used. Spherical aberration, distortion,