

# Crazy Sequential Representations: 11112 up to 30000

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## Introduction

Others have attempted to write the natural numbers from 1 to 11111 in terms of 1 to 9 (in increasing and decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

For example:

Number	Increasing	Decreasing
10957	$(1+2)^{(3+4)}*5-67+89$	$(9+8*7*65+4)*3-2*1$
10958		$(9+8*7*65+4)*3-2+1$
10959	$12+3+456*(7+8+9)$	$9+(8*76*(5+4)+3)*2*1$
10960	$12+(3^4+5+6)*7*(8+9)$	$9+(8*76*(5+4)+3)*2+1$
10961	$(1+2+34)*(5*6+7)*8+9$	$(9+8*7*65+4)*3+2*1$
10962	$12*3^4*5+678*9$	$9876+543*2*1$

Generally these expressions are referred to as crazy sequential representations (CSR). Interestingly, only one CSR remains to be identified, the increasing CSR for 10958.

## Historic Overview

Inder Taneja published five papers on arXiv (for 1 up to 11111):

ARXIV Version	Evaluated Range	Allowed Operations	Missing Increasing	Missing Decreasing	Valid Representations
1 (06-02-2013) <sup>1</sup>	44 to 1000	+ * ^	2	10	1902 (of 1914)
2 (19-03-2013) <sup>2</sup>	44 to 4444	+ * ^	50	53	8699 (of 8802)
3 (05-06-2013) <sup>3</sup>	44 to 11111	+ * ^ ( )	590	605	20941 (of 22136)
4 (05-08-2013) <sup>4</sup>	0 to 11111	+ * ^ ( ) -	449	315	21460 (of 22224)
5 (08-01-2014) <sup>5</sup>	0 to 11111	+ * ^ ( ) - /	9	10	22205 (of 22224)

Authors published three papers on Figshare/Zenodo (for -2147483647 up to 2147483647):

Date	Title
12-06-2018	Crazy Sequential Representations: Exhaustive Search <sup>6</sup>
14-06-2018	Crazy Sequential Representations: Negative Integers <sup>7</sup>
18-06-2018	Crazy Sequential Representations: Without Subtraction and/or Division <sup>8</sup>

Inder Taneja published three papers on RGMIA (for 11112 up to 30000):

Date	Allowed Operations
12-09-2018	Crazy Representations of Natural Numbers From 11112 to 20000 <sup>9</sup>
10-11-2018	Crazy Representations of Natural Numbers From 20001 to 25000 <sup>10</sup>
10-11-2018	Crazy Representations of Natural Numbers From 25001 to 30000 <sup>11</sup>

## Aims

- Validate and simplify the CSR as recently published by Inder Taneja <sup>9,10,11</sup>
- Compare the CSR as published by Inder Taneja <sup>9,10,11</sup> with the CSR as published ourselves <sup>6</sup>

## Note

Our manuscript <sup>6</sup> was published before the manuscripts <sup>9,10,11</sup> by Inder Taneja, in other words, our results were independent of the results by Inder Taneja.

## Existing Definitions

### Default Notation

Notation as used by most programming languages, restricted to following characters:

1	2	3	4	5	6	7	8	9	+	-	*	/	^	( )
---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----

### Potential CSR

Valid mathematical expression, thus well-formed interpretable syntactic construct, matching against either of the following regular expressions (using @ delimiter):

$@^{\wedge}[-+*/^()]*1[-+*/^()]*2[-+*/^()]*3[-+*/^()]*4[-+*/^()]*5[-+*/^()]*6[-+*/^()]*7[-+*/^()]*8[-+*/^()]*9[-+*/^()]*\$$
$@^{\wedge}[-+*/^()]*9[-+*/^()]*8[-+*/^()]*7[-+*/^()]*6[-+*/^()]*5[-+*/^()]*4[-+*/^()]*3[-+*/^()]*2[-+*/^()]*1[-+*/^()]*\$$

Ignoring evaluation result (natural, integer, real, rational, indeterminate, etc.).

### Genuine CSR

Natural number (or zero) in terms of 1 to 9 (in increasing or decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

### Pseudo CSR

Potential non-genuine CSR evaluating to natural number (or zero).  
For example, expressions with implicit multiplication by minus one.

Result	Pseudo CSR	Expansion
1170	$-(-1-2*3-4^5-67-8*9)$	$(-1)*(-1-2*3-4^5-67-8*9)$
388	$-(1+2*3-4^5+6+7*89)$	$(-1)*(1+2*3-4^5+6+7*89)$
1468	$1-2*-(345-6)+789$	$1-2*(-1)*(345-6)+789$
1614	$1234+5*-(6+7-89)$	$1234+5*(-1)*(6+7-89)$
8202	$1^23+4^5^-(6-7)*8+9$	$1^23+4^5^(((-1)*(6-7))*8+9)$
9911	$12^3+4^5^-(6-7)*8-9$	$12^3+4^5^(((-1)*(6-7))*8-9)$
4218	$98/-(7-6)-5+4321$	$98/(((-1)*(7-6))-5+4321)$
9929	$12^3+4^5/-(6-7)*8+9$	$12^3+4^5/(((-1)*(6-7))*8+9)$
10267	$-1-2*-34*(-(5-67)+89)$	$-1-2*-34*(((-1)*(5-67)+89))$
10257	$9+8+(-(7+6)+5)^4*(3/2+1)$	$9+8+((-1)*(7+6)+5)^4*(3/2+1)$

### In terms of 1 to 9

Digits 1 to 9 occur once and in order, either in increasing or decreasing order.  
Digits can be used as individual numbers (thus 1, 2, 3, 4, 5, 6, 7, 8 and 9).  
Digits can be concatenated into larger numbers (for example 123, 4, 5, 6 and 789).  
Negative counterparts of numbers may be used as well (also used by Inder Taneja).

## Strictness

In order to represent all natural numbers from 11112 up to 30000, Inder Taneja introduced/allowed two additional unary operations; square-root and factorial. Inder Taneja used the prefix/ postfix notation for square-root and factorial. Authors used the functional notation for square-root and factorial. Example:

	Notation by Inder Taneja	Notation by Authors
11852	$1 - 2 - 3 + 456 \times 78 / \sqrt{9}$	$1-2-3+456*78/s(9)$
11948	$-1 - 2 + 3! \times 4^5 + (6! + 7) \times 8 - 9$	$-1-2+f(3)*4^5+(f(6)+7)*8-9$
17773	$(1 + 2) \times (3!! + 4) + 5^6 - 7 - 8 - 9$	$(1+2)*(f(f(3))+4)+5^6-7-8-9$

Strictly speaking the existing definitions do not allow square-root and/or factorial. In order to handle expressions with square-root and/or factorial, authors loosened two existing definitions and introduced one new definition.

## Loosened Definitions

### Default Notation

Notation as used by most programming languages, restricted to following characters:

1	2	3	4	5	6	7	8	9	+	-	*	/	^	(	)	f	s
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

### Potential CSR

Valid mathematical expression, thus well-formed interpretable syntactic construct, matching against either of the following regular expressions (using @ delimiter):

@^[+-*/^()fs]*1[-+*/^()fs]*2[-+*/^()fs]*3[-+*/^()fs]*4[-+*/^()fs]*5[-+*/^()fs]*6[-+*/^()fs]*7[-+*/^()fs]*8[-+*/^()fs]*9[-+*/^()fs]*\$
@^[+-*/^()fs]*9[-+*/^()fs]*8[-+*/^()fs]*7[-+*/^()fs]*6[-+*/^()fs]*5[-+*/^()fs]*4[-+*/^()fs]*3[-+*/^()fs]*2[-+*/^()fs]*1[-+*/^()fs]*\$

Ignoring evaluation result (natural, integer, real, rational, indeterminate, etc.).

## New Definition

### Alternative CSR

Natural number (or zero) in terms of 1 to 9 (in increasing or decreasing order) by using the operations of addition, subtraction, multiplication, division potentiation, **factorial** and/or **square-root** (and optionally parentheses). Alternative CSR may include implicit multiplication by minus one. Alternative CSR must validate against both loosened definitions.

## Validation & Categorization

Expressions as recently published by Inder Taneja <sup>9,10,11</sup> were extracted from the original PDF files and converted into default notation. Resulting expressions were categorized into invalid CSR, genuine CSR, pseudo CSR and alternative CSR.

Authors expected 37778 valid CSR for the natural numbers from 11112 up to 30000. 37697 valid CSR were identified; 26568 genuine, 10477 pseudo, 652 alternative.

Missing (n=19):

Order	Missing
Decreasing	11495,20082,20916,24133,28335,28823,28834,29364,29597
Increasing	20289,21602,22324,23533,24525,24828,28721,28426,29645,29773

Decreasing CSR with incorrect evaluation result (n=31):

Number	CSR by Inder Taneja	Evaluation Result
20057	$((-(-98)-(76-(-5^4)))*(32-(1)))$	-18693
20305	$((((-9-8)-7)-(6+(-5^4)))*(-32+(1)))$	-18445
20536	$-9+(8*7*6-5)*(4^3+2)-1$	21836
20537	$-9+(8*7*6-5)*(4^3+2)*1$	21837
20538	$-9+(8*7*6-5)*(4^3+2)+1$	21838
20554	$9+(8*7*6-5)*(4^3+2)-1$	21854
20555	$9+(8*7*6-5)*(4^3+2)*1$	21855
20556	$9+(8*7*6-5)*(4^3+2)+1$	21856
20647	$(-(-98)+(-76+((-5^4)*((32+1))))$	-20603
20834	$(98-((-((76+5))*(-4^(3+2-1))))$	-20638
21002	$((9876+(-5^4))*(3+((-2+(1))))$	18502
21316	$((9*(87+6))-((-5*(-4^(3*2)))+(1)))$	-19644
21346	$((-9+(876+(5*(-4^(3*2)))))-1)$	-19614
21615	$((((-9-8)-7)-(6+(-5^4)))*(-32-(1)))$	-19635
21981	$(9+8)*((7-6+5)^4-3)+2-1$	21982
22274	$9+8+7+6*5^4*(3*2+1)$	26274
22815	$((9*8)*(-(7-(6*54)))-((-3^(2*1))))$	22833
22833	$((9*8)*(-(7-(6*54)))+((-3^(2*1))))$	22815
23196	$(9+8*7*6*5*4+3)*(2+1)$	20196
23698	$((-9-8)*((-7*-65)-(-43^(2*1))))$	-39168
23894	$((9+(87*6))*(54-(-3^2)))-1)$	33452
23896	$((9+(87*6))*(54-(-3^2))+1)$	33454
24480	$-(((98-7)*((-65*4))-(-3^2))-1)$	22842
24802	$((98*-((7-(65*4)))+((-3^2)-1)))$	24784
26728	$(9+8+f(7)-f(6))*5+f(4+3)+f(2+1)$	26731
27576	$(((-9-(-8+765))*-4)*(-3^(2*1)))$	-27576
27754	$((9876-(-5^4))*3)-((-2+1))$	31504
27842	$((98*((76-5)*4))+((-3^2)+1))$	27824
27884	$((9*876)+((-5^4)*(32*1)))$	-12116
29198	$((9*8)+((7*(65+(-4^(3*2)))))-1)$	-28146
29199	$((9*8)+((7*(65+(-4^(3*2)))))*1)$	-28145

Increasing CSR with incorrect evaluation result (n=17):

Number	CSR by Inder Taneja	Evaluation Result
20515	$((-1^{(2*3)})-(((4-(5*6))*789)))$	20513
20733	$((-12^{(3-4+5)})+(6-((7-8)*-9)))$	-20739
21081	$((-12^{(3-4+5)})+((6*(7*8))+9))$	-20391
21519	$(((-12^{(3-4+5)})-6)+789)$	-19953
23283	$((12+((-3^4)+(-((5*67))*8)))*-9)$	24741
24379	$((-1*2)-((-3^4)*(5-(((6*7)-8)*9))))$	-24383
24666	$(((-1+(-2^{(3+4+5)}))*6)+((7+89)))$	-24486
25045	$-1+2+f(3+4)*5+6*(7+8+9)$	25345
25470	$((-12^{(3-4+5)})-(-6*789))$	-16002
26274	$(1+f(2+3)/4)*(f(5)+f(6)+7+8+9)$	26784
26466	$(1+((-((2-(-3^4)))*(5*-67))*(8-9)))$	-27804
26467	$(1-((-((2-(-3^4)))*(5*-67))+8-9))$	-27803
27815	$-(1+2*3)*4^5+6^7/8+9$	27833
28117	$((((1-2)-(-3^4))*((5*-67)-8))-9)$	-27449
29868	$((-12+((-3^4)*5))*(-6-(7-89)))$	-31692
29871	$((1*2)-((-3^4)*((56-7)-8)))*-9)$	-29907
29962	$(1-(((2-(-3+4+5))*-6)*(-78))-9))$	-29942

Decreasing and increasing CSR with incorrect terms (n=14):

Order	Number	CSR by Inder Taneja	Error
Decreasing	20815	$9+8*7*6+5*4^{(3*2)}-10$	Zero included
	24261	$-9*8-7+(6^5-4)*3+2^10$	Zero included
	24356	$9+8+(7+6)*(5^4*3-2)-10$	Zero included
	24482	$9-8+7+6*(-5+4*(-3+2^10))$	Zero included
	24892	$((9-87)*(6*(-54+3))+2^10)$	Zero included
	25191	$(-9*(87+(((6*54)-3210))))$	Zero included
	24358	$-9+8+(7+6)*(5^4*3-2)+10$	Zero included
	21777	$(9+5*(-8+7*6)*4^3)*2-1$	Incorrect order
	21778	$(9+5*(-8+7*6)*4^3)*2*1$	Incorrect order
	21779	$(9+5*(-8+7*6)*4^3)*2+1$	Incorrect order
Increasing	20998	$-1+2+(-3-4+78*5*6)*9$	Incorrect order
	21912	$-1+((2*3)^4+7*(5-6))*(8+9)$	Incorrect order
	21913	$1*((2*3)^4+7*(5-6))*(8+9)$	Incorrect order
	21914	$-1+((2*3)^4+7*(5-6))*(8+9)$	Incorrect order

## Simplification

Authors attempted to simplify the expressions as published by Inder Taneja, however, authors do not guaranty the expressions are in their simplest form.

Number	CSR by Inder Taneja	Simplified
20405	$(-1+(2*(3+(4*(5*(6+(7*(8*9))))))))$	$-1+2*(3+4*5*(6+7*8*9))$
20381	$(-(((((((1*2)-34)-5)*-6)+7))*-89))$	$-((1*2-34-5)*-6+7)*-89$

## Comparisons

CSR availability for numbers 11112 op to 30000 (after validation):

	Genuine/Pseudo	Genuine	Pseudo	Alternative	Unavailable
by Inder Taneja <sup>9,10,11</sup>	37045	26568	10477	652	81
by ourselves <sup>6</sup>	37252	36331	921	-	526

Comparison without genuine/pseudo distinction:

CSR by Inder Taneja	CSR by ourselves <sup>6</sup>	Frequency
Genuine/Pseudo	Genuine/Pseudo	36971
Genuine/Pseudo	Unavailable	74
Alternative	Genuine/Pseudo	208
Alternative	Unavailable	444
Unavailable	Genuine/Pseudo	73
Unavailable	Unavailable	8

Inder Taneja identified 73 genuine/pseudo CSR (which were not identified by ourselves). Authors identified 281 genuine/pseudo CSR (which were not identified Inder Taneja).

Authors are aware Inder Taneja did not distinguish between genuine and pseudo CSR. However, comparison with genuine/pseudo distinction might be of interest as well:

CSR by Inder Taneja	CSR by ourselves <sup>6</sup>	Frequency
Genuine	Genuine	24737
Genuine	Pseudo	989
Genuine	Unavailable	60
Pseudo	Genuine	10995
Pseudo	Pseudo	250
Pseudo	Unavailable	14
Alternative	Genuine	127
Alternative	Pseudo	81
Alternative	Unavailable	444
Unavailable	Genuine	69
Unavailable	Pseudo	4
Unavailable	Unavailable	8

Inder Taneja identified 989 genuine CSR (where only pseudo were identified by ourselves). Authors identified 10995 genuine CSR (where only pseudo were identified by Inder Taneja).

## Results

See supplement 1 for the validated and simplified versions of the CSR by Inder Taneja, tabulated along with the untouched CSR as published previously <sup>9</sup> by ourselves.

The 'newly identified' genuine/pseudo CSR are tabulated on the subsequent pages.

Inder Taneja identified 73 genuine/pseudo CSR (which were not identified by ourselves):

CSR	CSR	CSR	CSR
13955	$(1+2+(3^4+5^6-7)*8)/9$	26084	$-1*-2-3^4*(5-(6*7*8-9))$
16571	$-9*-(8-7*6+5^4*3)+2*1$	26330	$(-12+3^4*5)*67+8-9$
16988	$(-1-2+3^4*5*6)*7+8-9$	26499	$1+2+3^4*(5*67-8)+9$
17597	$1+2*3+4+(5^6+7)/8*9$	26717	$-1*-2-(3^4*5+6)*(7-8*9)$
18862	$9+87*-6+5^4*(32-1)$	26903	$-1*-2-(3^4*(5-6*7)+8)*9$
19957	$-1+2^(3*4)*5+6*-(78+9)$	26913	$1-((2-3^4*5)*67+89)$
20122	$9+8*(7+6*5^4/3)*2+1$	26917	$(-1-2+3^4*5)*67-(8+9)$
20612	$1+2^(3*4)*5+6*7+89$	27028	$1-(2+3^4*(5-6*7)-8)*9$
21355	$1+2*((3^4+56)*78-9)$	27044	$-1*2+3^4*5*67-89$
21788	$1-2+(3^4+5*6*78)*9$	27052	$-1*2+3^4*(5*67+8-9)$
22002	$-1-2+(3^4*5*6+7+8)*9$	27286	$(-1*2-3^4*5)*-67+8+9$
22370	$1+(2+(3^4*5-6)*7)*8+9$	27850	$(-12-3^4*5)*-67-89$
22376	$-1*-2+(3^4*5*6+7*8)*9$	28028	$(-12-3^4*5)*-67+89$
22378	$-9-((8*7-6*5^4*3)*2+1)$	28370	$1+(2^(3*4)-56)*7+89$
22808	$1+23+4^(5+6-7)*89$	28444	$-1+(2^(3*4)-5*6)*7-(8+9)$
23081	$(1-2-(3^4*5+6))*-7*8+9$	28717	$1+(2^(3*4)+5)*(6-(7-8))+9$
23884	$-1*2+3^4*5*(67-8)-9$	28895	$(-1+2+(3^4+5)*6*7)*8-9$
23948	$(-9+8-76)*-(5^4-3)/2+1$	28897	$(-1+2-(3^4+5)*6*7)*-8+9$
23954	$(1^2-(-3)^4*-5)*(6*7+8+9)$	28922	$(-9-(8+76))*-(5^4-3)/2-1$
23963	$(-1+2+3^4*5)*(67-8)+9$	28924	$(-9-(8+76))*-(5^4-3)/2+1$
24005	$1+(2+3^4*5)*(67-8)-9$	28933	$9+(87+6)*(5^4-3)/2+1$
24278	$-9*8+(7+6)*(5^4*3-2)+1$	28976	$1+(2^(3*4)+56)*7-89$
24330	$(-1-2*3^4*5)*-(6+7+8+9)$	29009	$(-1+(2+(3^4+5)*6)*7)*8+9$
24338	$-1*-2+(3^4*5-67)*8*9$	29041	$-12*-3^4*5*6-7*(8+9)$
24774	$-12+3^4*(5+6+7)*(8+9)$	29066	$1+(2^(3*4)+56)*7-(8-9)$
24798	$12+3^4*(5+6+7)*(8+9)$	29075	$-12*-(3^4*5*6-7)+8-9$
24853	$-1+2+(3^4-5)*(6*7*8-9)$	29077	$-12*-(3^4*5*6-7)-(8-9)$
25170	$1+(2^(3*4)-5)*6+7*89$	29227	$-12*-(3^4*5*6+7)-(8+9)$
25213	$-1+2^(3*4)*5+6*789$	29245	$-12*-(3^4*5*6+7)-(8-9)$
25300	$-9*-((8+7-6)*5^4-3)/2+1$	29301	$(-1-2*(3+4))*-(5^6+7)/8-9$
25562	$1+(2+(3^4-5)*6*7)*8+9$	29361	$(-12-3)*-(4+(5^6+7)/8)-9$
25802	$-1-(2-((3^4*5+6)*7-8))*9$	29365	$-1+(23^4-5^6+78)/9$
25819	$-1*2+((3^4*5+6)*7-8)*9$	29562	$-1-2-3^4*5*(6-7-8*9)$
25820	$1-2+((3^4*5+6)*7-8)*9$	29607	$(-12-3^4*5)*-(6-7+8*9)$
25845	$(12+3^4*5)*(6-7*-8)-9$	29710	$-1-(2+3^4*5)*(6-7-8*9)$
25946	$-1+((2+3^4*5+6)*7-8)*9$	29711	$(-1*2-3^4*5)*(6-7-8*9)$
25948	$1+((2+3^4*5+6)*7-8)*9$	29730	$1+2+3^4*((5+6*7)*8-9)$



Authors identified 281 genuine/pseudo CSR (which were not identified Inder Taneja):

CSR	CSR	CSR	CSR
11149	$-1+(2/-3+4)*(-5*-(678-9))$	20320	$(1^{-2}+3)^4*5*(-6-(7/8+9))$
11192	$-1-234*(-5/6-7*8+9)$	20515	$1*23456*7/8-9$
11488	$(-1+2/3-4*5*6)*(7+89)$	20536	$-98+76*543/2/1$
11495	$9+87*(6+5)*4*3+2*1$	20537	$-98+76*543/2+1$
11632	$-(1+2-3/-4*(-(5^6)-7))-89$	20538	$98*7*6*5-43+2-1$
11642	$-1*-2/-3*(-(-4*-56*78-9))$	20554	$-9+8+76+5*4^(3*2)-1$
11852	$-1/-2*(34+5*6*789)$	20555	$98*7*6*5-4*3*2-1$
11884	$(-1/(2+3)+4)*5^(6-(-7+8))+9$	20556	$98*7*6*5-4*3*2*1$
11948	$-12*-3*(-(4-5*67-8/9))$	20647	$98*7*6*5+4+3*21$
12820	$1^2-(-3-(-4^5/-6+7)*8)*9$	20687	$-1+(-2/-3-4*(5+67))*-8*9$
14187	$(-9*(-8/-7)*-65+4+3)*21$	20733	$12^(3-4+5)+6^(-7+8)-9$
14317	$9+(8-((7-(-6+5))^4))*-(3+2^{-1})$	20797	$(-987*-6+5*4)*(3+2^{-1})$
14746	$-((1^2+3)^4-5^6)-7*89$	20815	$-9+87+6^5*4/3*2+1$
16034	$12*34+5^6-(7-8)^9$	20834	$(-98+7-6)*-5*43-21$
16612	$1+(-2-(3+4^5))*-(-6/7+8+9)$	20846	$((-12-3)^4-(5-6))^7/(8+9)$
17060	$-9*(-8*-7/-6*5-(43^2))-1$	20860	$-123*(-4^5/6-(7-8))-9$
17726	$(-1/-2-3)*(-4^5+6)*7-89$	20902	$(-1/-2+3)*(4+5+67*89)$
17771	$(-12+(-3/-4+5^6)/7)*8+9$	20916	$9*8*7+6*54*3*21$
17773	$(1/-2-3)*(4^5-678*9)$	20998	$123*4^5/6+7+8-9$
17789	$-(-1+2)*3-4^5*(-(67/8+9))$	21002	$9*8-7*-65*(43+2+1)$
17795	$1+(-2^3+4)*-56*(7-89)$	21081	$-12-(34*-5-67)*89$
17843	$(-1-2+(-3/-4+5^6)/7)*8+9$	21082	$(-(-9+8*-7*-6)-5)*-(4^3-2^{-1})$
17890	$(-9-8/-7*(-(6^5+43)))*-2/1$	21211	$1+(-2/-3+4)*-(-567*8-9)$
17996	$(1/-2+3)*((-4^5-6)*-7-8)-9$	21261	$(-1/-2+34)*(-5-6)*-7*8+9$
18014	$-1/-2*(3+4^5+6^7/8+9)$	21316	$(98+76-5*4^3)^2/1$
18022	$-9-876*(-5/(-4*3)-21)$	21346	$-9+876+5*4^(3*2)-1$
18517	$9-8*(-7-654)*(3+2^{-1})$	21379	$(-9/-8-(-7*6-(5^4)))*32-1$
18812	$1*2+(-3/-4^5-5+6)*(7-8/9)$	21434	$-(9+(8-7*6-5+4)^3)/2+1$
18860	$9-87*-65*(4/3+2)+1$	21458	$(-987-(6+5))*-43/2+1$
18878	$-1+(-2^3+4)*-56*(-(78+9))$	21469	$(-987+(6+5)^4*3)/2+1$
19042	$(-9-8)/-7*(6^5+4^3)+2^1$	21519	$12^(3-4+5)-6+789$
19078	$-9^(-8+7+6)+5^(4+3)+2^1$	21586	$-1/2+(-3/-4+5*6)*78*9$
19157	$-9*8-7*(6-5*4)^3+21$	21602	$1*2+(-3-45)*-(-6*-7+8)*9$
19301	$9*8-(7/(6-5*4)^{-3}-21)$	21615	$98*7+654*32+1$
19861	$1+(2/-3+4)*-(-5-67*89)$	21765	$(9+8-7*6^5)/(-4+3/2)-1$
19867	$1+(-2/-3+4)*-((-5-6*78)*9)$	21777	$(9+8*7+6*54*3)*21$
19931	$-(-9/-8+7-6-5^4)*32-1$	21778	$-98+7*(6-5+4)^(3+2)+1$
20057	$9-8*7*(-6*5*4*3+2)^1$	21779	$98+76+5*4321$
20082	$9*(-8*-76*(5-4/3)+2)^1$	21782	$-((-9-8)*7+(6+5)^4)*-3/2-1$
20103	$((1+2)^(3+4)-5/-6*7*8)*9$	21821	$-(-9/(8+7^6+54^3/2))^2-1$
20218	$(-9*(-(-87-6^5)+4))/-(3+2^{-1})$	21822	$-9+(87-((6+5)^4))^3/-2*1$
20228	$(-9+87)*-(-65*4-(-3/2)^{-1})$	21912	$-12+(-34+5)*(-6-78)*9$
20230	$(98*-7+6)*(5/4-32+1)$	21913	$1+(-2+3-45)*(6+7*-8*9)$
20289	$(-12*-3+4*56)*78+9$	21914	$12^3*4+5^6-7*89$
20305	$-98-76+5*4^(3*2)-1$	21981	$9+8+76*(5+4*3)^2*1$

	CSR		CSR
22026	$(-(1/-2+3)-4)^*-5*678-9$	24480	$9-8+765*4^3/2-1$
22109	$-1/-2*(34+56*789)$	24482	$9-8+765*4^3/2+1$
22274	$9*8+(7+6-54*3)^2+1$	24525	$1+2-3-45*(-67*8-9)$
22300	$98+(7+6-54*3)^2+1$	24666	$1+2*3*4^5(5-6+7)+89$
22323	$-1+2/((3+4)/(5^(6-7+8)+9))$	24754	$(9+8+(7+6)^5)/(4*3+2+1)$
22324	$-1*-2/(3+4)*(5^(6-(7-8))+9)$	24763	$-(-1/-2-(-3*(-4^5+6)+7))*8-9$
22358	$1+2+34*5/6*789$	24781	$9-8*((-7-65)*43-2^-1)$
22363	$((-1/2-((3+4)^5))/-6-7)*8+9$	24802	$-98*(7-65*4)+3^2-1$
22442	$(-12^3-(4-5))/-6*78-9$	24828	$-12+345^(-6+7)*8*9$
22493	$((-1/-2+3^4)^*-5+6)^*-7*8+9$	24869	$(-1/-2+3)^*(-4^5+6)*7-8*9$
22815	$9*8+7*(6+54-3)^2^1$	24892	$-98*(7+6+54-321)$
22833	$9-8*(7-65*(43+2-1))$	24902	$(9-8*(7+6^5))/((-4-3)/2+1)$
23018	$1-23+4^5/-6*(-7-8)*9$	24925	$((-9*-8+7)*(-6-(5^4))+3)/2-1$
23080	$-12*(-34*(-5/-6+7*8)+9)$	24941	$-98*(7-65*4-3/2)^1$
23117	$9-8*(-76*(5-43)+2^-1)$	24942	$-((-1/-2+3)*(-4^5+6)*7+8)+9$
23196	$-9*(8+7-(6^5+4)/3+2-1)$	24946	$-1*-2*-3*(-4/5^-6/(7+8)+9)$
23218	$(-1-(2-345))*(67+8/9)$	25012	$(-9+(8-(7+6))^5*4+3)*-2/1$
23260	$-12*(3-4*-56*-78/9)$	25045	$-((-1/-2+3)*-4^5+6)*7+8-9$
23282	$1-2+(-34+5/6)*-78*9$	25158	$-12*-3*-4-5/6-78*9$
23283	$(-1-23*4-5*-67*8)*9$	25180	$(-9/-8-7*6)*-(5^4-(3^2))+1$
23289	$-1+(-2/-3+45)*(6-7*-8*9)$	25191	$-9*(-876-54-3)*(2+1)$
23303	$-1-2/-3*(-45-6^7/8-9)$	25196	$-(9-((8-(7-6))^5))/-4*-3*2-1$
23308	$1-2/-3*(-45-6^7/8)+9$	25204	$(9-((8-(7-6))^5+4))*-3/2+1$
23533	$-1+23*4^5+6-7-8-9$	25241	$-1+(-2-34)*-5/6+78*9$
23698	$9*8+7*(6+5+4)^3+2-1$	25261	$(9+8)*(-7*6-((5-43)^2))-1$
23728	$-9-8*-7*((-6*-5/4)^3+2^1)$	25302	$(-9-((8+7-6*5)^4-3)/-2)/1$
23756	$-(-9-(8-76*(-5^4))+3)/2-1$	25310	$(((-9+8+76)/5)^4-3)/2-1$
23819	$-(-1/-2-(3^4))*(5+6*7)*8-9$	25322	$987+(6-54*3)^2-1$
23894	$(-9-876)*(5-4^3/2)-1$	25363	$(-98+7)*-(6^5/-4-32)+1$
23896	$(-9-876)*(5-4^3/2)+1$	25402	$(-9+87)*-6*(54+3^-2)-1$
24035	$(-1/-2-3/-4^-5-67)*8-9$	25456	$(-1-23)*-(4^5-6*(-7-8/9))$
24131	$9-8-76*-5*(4^3-2^-1)$	25470	$12^(3-4+5)+6*789$
24142	$-9*(-(87+(-6^5-4)/-3)-2)+1$	25573	$(-9*-8+7+6/-5-4)*321$
24155	$-9*-8*(-7+(-654-3)/2)-1$	25574	$(9-8/7)*(6+(54+3)^2)-1$
24203	$-98+76*(-5/4-321)$	25576	$(-9*-87+65/4)*32^1$
24204	$-9*-8*(-7/-6-5*(-4+3*-21))$	25589	$-(-1/-2-3*(4^5+6*7))*8+9$
24260	$-98-76*(-5/4^-3-2^-1)$	25636	$9-(8/(-7*((6+5)^4+3)*2))^^-1$
24261	$-9*-8*(76*5-43)-(2+1)$	25861	$(-12^3-4*-5/6)*-(7+8)-9$
24286	$-1*(-2*(-3*-4-(5^6))*-7+8)/9$	25879	$-1/2+3^(4+5)/-6*(-7+8/-9)$
24349	$-1-2/(3+4*5)*-(6^7+89)$	25891	$(-1/-2*-3*-4^5-(6+7))*8+9$
24356	$-9+8*(((-7+65)/4)^3-(2+1))$	25904	$(-1/(-2*-3)+45*6)*(7+89)$
24358	$9+8-76*-5/4^-3+21$	25931	$-1+(-2^-3-45*6)*-(7+89)$
24371	$-1-2*-3/-4*(5^6+7*89)$	25934	$98/7-6^5*(-4+(-3/2)^-1)$
24379	$-1-23*(-4+(-5-6)*(7+89))$	25979	$9+(-8-7-6^5)*-4+(-3/2)^-1$
24404	$-9+8*(((-7+65)/4)^3+2+1)$	26084	$9+8+(76+5^4(4+3))/(2+1)$

	CSR		CSR
26269	$(-1-(2^{(3+4+5)}))^{*}-(6+7/(8+9))$	28102	$9^{*}(-8+7+6)^{5}-4^{*}3^{*}2+1$
26270	$-1+(-2^{(3-4)}-56)^{*}-7^{*}8^{*}9$	28117	$-12/3-4+5^{(6+7-8)^{*}9}$
26274	$-(1-(-2+3^{(4+5)}/6+7)^{*}8)-9$	28172	$1^{*}2+(-3/-4-56^{*}7)^{*}-8^{*}9$
26284	$-1+(-2-3)^{*}-(-4^{5*}-(6-7/8)+9)$	28276	$-1^{*}2-(-3/-4+56^{*}7)^{*}-8^{*}9$
26288	$(-1^{*}-2^{(3-4)^{*}}(5-6789))$	28291	$-(-987^{*}-(-6^{*}5+4/3)+2)-1$
26290	$-(1+(-23+4)^{5+6^{*}7^{*}8})/9$	28292	$-987^{*}(-6^{*}5+4/3)-2/1$
26293	$-1^{*}(-2+3^{(4+5)}/6+7)^{*}-8+9$	28324	$-9-87^{*}(-6^{*}(54+(3^{(2)}))-1)$
26466	$-1^{*}2-3^{*}(-4^{5+6})^{*}78/9$	28325	$-9^{*}8+(7-(((6-5)^{*}4)^3))/(2+1)$
26467	$1+(-2+3^{4})^{*}5^{*}67-8+9$	28354	$-1/-2^{*}-(3^{4}-56789)$
26527	$-1-2^{*}((-3+4^{5})/-6^{*}78+9)$	28379	$(9-8^{*}7-(((6-5)^{*}4)^3))/(2+1)$
26564	$-9^{*}(-8-7/6)^{*}(-5^{*}-4^{3+2})-1$	28410	$9^{*}((-8-(-7-6))^{5-4})+321$
26602	$-987^{*}(-6^{*}5+4^{3}/21)$	28427	$(-9-(8-((7+6-5)^4))/3)^{*}21$
26606	$98^{(7-6)^{*}543/2-1}$	28451	$-(-9-87^{*}654+3)/2-1$
26662	$-(9-8/-7^{*}(-(6^{5+4})^{*}3+2)+1)$	28501	$-1/-2^{*}(-3-(-4^{5+6})^{*}7)^{*}8+9$
26664	$-9^{*}(-8^{*}(-7+6^{5})-(4^{3}))/21$	28721	$1-2/-3^{*}(-4^{5*}-6^{*}7+8^{*}9)$
26728	$-9-87/-6^{*}-(5-(43^{2}))-1$	28722	$-9^{*}-((-8+7+6)^{5+4^{3}})+21$
26771	$-(-1+2^{*}(-3-4^{5})/6)^{*}78-9$	28734	$(1/-2+34)^{*}(-5-6)^{*}-78-9$
26772	$-9-8+7654^{*}-(-3-2^{(2-1)})$	28851	$(-9-87^{*}6)^{*}-(-54+3^{(2-1)})$
26799	$-9-8/-7^{*}(-6^{5}+43)^{*}(2+1)$	28877	$(-12^{3+4})^{*}-((-5+67)/8+9)$
26841	$-9^{*}(8-7-6)^{5-4^{*}321}$	28946	$(98^{*}-76+5)^{*}-(-4-3^{(2-1)})+1$
26947	$-(-9-876^{*}-(5/4-32))+1$	28966	$(-9^{*}-8^{*}-7-6^{5+4})^{*}-(-3+2^{(2-1)})$
27265	$-1+2+3456^{*}(7+8/9)$	28982	$-9^{*}(-87^{*}(-6/(-5^{*}4))^{(2-1)})$
27266	$1^{*}2+3456^{*}(7+8/9)$	28997	$-9^{*}-87^{*}(-6/(-5^{*}4))^{(2-1)}$
27445	$(-1+2^{3})/-4^{*}-(5^{6+7})+89$	29011	$(12^{3+4})^{*}((-5+67)/8+9)$
27554	$(-((1-2^{3})^{4}+5))/-6^{*}(78-9)$	29020	$-1/2+(-3^{(4+5)}/-6-7^{*}8)^{*}9$
27576	$-9^{*}8^{*}7+65^{*}432^{*}1$	29031	$(1/-2-345)^{*}-(-6+78)+9$
27662	$98/7-6^{5*}-4^{*}(-3^{(2-1)}+1)$	29083	$(-1/-2^{*}-(-3-4^{5})+6)^{*}7^{*}8-9$
27668	$(-9/-8^{*}-765-4)^{*}-32^{*}1$	29101	$(-1/-2^{*}-(-3-4^{5})+6)^{*}7^{*}8+9$
27688	$-9+(-8+76^{*}(5+4)^3)/2-1$	29174	$-1^{*}-2^{*}(-3+4^{5}/6)^{*}(78+9)$
27715	$-9-87^{*}-6^{*}(54+3^{(2-1)})$	29198	$98^{*}7-6^{5*}(4/-3^{*}2-1)$
27754	$(9876-5^{4})^{*}3+2-1$	29199	$9^{*}8+7^{*}(65+4^{(3^{*}2)})^{*}1$
27778	$(-9-8)^{*}76/(-5^{*}4-3/2)^{(2-1)}$	29264	$-(-((1-23)^4)+5+67)/8-9$
27779	$9-8-76^{*}-((5+4)^3/2+1)$	29265	$(1-23)^4/56^{*}7-8-9$
27815	$(-1-2^{*}3)/4^{(2-1)}-5-6^{7}/-8-9$	29290	$((1-23)^4-56/7)/8+9$
27842	$9+8^{*}7^{*}(65+432)+1$	29307	$-(-1-2)^{*}(3+(4-(5-6))^{7})/8+9$
27851	$-9-8^{*}((-76-5)^{*}43+2^{(2-1)})$	29330	$-(-1-234567)/8+9$
27869	$9-8^{*}((-76-5)^{*}43+2^{(2-1)})$	29390	$1/2-(3^{(4+5)}/-6+7+8)^{*}9$
27878	$9-87^{*}(6/(5+4)-321)$	29399	$1+(-2/(3+4)^{-5+6})^{*}-7/8-9$
27884	$9^{*}876+5^{4*}32^{*}1$	29426	$9+(-8+7^{(6-5)}/4+3^{2-1})$
27994	$-1/(2-3^{*}4)^{*}(5+6^{7+8-9})$	29462	$(-98-(7^{(6+5)}))/-4+3+21$
28012	$(((-9+8)^{*}-7)^{6*}-5-(4+3))/-21$	29530	$9+(((8+7-6)^5-4-3)/2)^{*}1$
28030	$-1^{*}2-3^{*}-4^{5*}((-6+7)/8+9)$	29533	$-1/2+(-3^{(4+5)}/-6+7+8)^{*}9$
28031	$-1+(-2/-3^{*}-4+56^{*}7)^{*}8^{*}9$	29536	$9+((8+7-6)^5+4+3)/2-1$
28048	$(-1-(2-(-3/4)^{-5}))^{*}-6^{7}/-(-8^{*}9)$	29537	$1+(-2-(3^{(4+5)}))^{*}-6^{*}7^{*}8^{*}9$
28078	$-1^{*}2+3456^{*}(-7/8+9)$	29596	$(-1234+5/6)^{*}-(-7+8+9)$

	CSR		CSR
29597	$987*6*5-4*3-2+1$	29723	$(-98*-7*-65+4)/-3*2-1$
29608	$-1-2*(-3+4^5)/(6/(-78-9))$	29860	$(-(-9+8)+7)*(-6*(-(5^4)+3)+2^{-1})$
29645	$-1+(-2-(-345-67)*8)*9$	29868	$-12*(3-4/(5-6)*-7*89)$
29660	$1/2+(-3^{(4+5)}/-6+7+8)*9$	29871	$1-2*(-3*4-5^6+78*9)$
29689	$-1-2*(3-4^5/(6/(-78-9)))$	29924	$-1+(-2/-3-45)*(-67-8)*9$
29702	$-1*-2*(3-4^5/-6*(78+9))$	29723	$(-98*-7*-65+4)/-3*2-1$

## Final Notes

Various pseudo CSR can be ‘easily modified’ into genuine CSR, for example:

	Pseudo CSR	Genuine CSR
19069	$9+8+7-(-((-6*(-5*-4+3))^2)-1)$	$9+8+7+(6*(5*4+3))^2+1$

Generally symbolic simplification can do ‘majority of the work’, for example:

	Symbolic	Symbolic Simplification	19069
	$A+B+C-(-((-D*(-E*-F+G))^H)-I)$	$A+B+C+(-D*(E*F+G))^H+I$	$9+8+7+(-6*(5*4+3))^2+1$
	$A+B+C+(D*(E*F+G))^H+I$	$A+B+C+(D*(E*F+G))^H+I$	$9+8+7+(6*(5*4+3))^2+1$

However sometimes ‘further simplification’ can be achieved as well, for example:

Before	$9+8+7+(-6*(5*4+3))^2+1$
After	$9+8+7+(6*(5*4+3))^2+1$

In other words, while  $(-D*(E*F+G))^H$  and  $(D*(E*F+G))^H$  are obviously different, the final expressions  $(-6*(5*4+3))^2$  and  $(6*(5*4+3))^2$  both evaluate to 19044,

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