The 144-Lattice Partition Theorem:

How the CTF Framework Found a New Result in Number Theory, and What It Means Inside the Framework

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Abstract

Investigation of the CTF base frequency $f_0 = 10373/72$ Hz under the mod-144 power sum framework led to the discovery of a partition theorem for prime power sums. The standalone mathematical result — proven without reference to the CTF Framework — is published separately (Gurwell, 2026b). This paper documents the discovery context and interprets the theorem's structure within the CTF Framework.

The theorem states that $S(p,k) = \Sigma_{i=1}^{p-1} i^k \bmod 144$ is independent of k for all odd k ≥ 3 if and only if p mod 144 belongs to one of 32 residue classes, with lock values in {0, 1, 9, 64, 73, 81}. When the CTF prime set {11, 17, 19, 23, 41, 53} is used as the exponent family, the same 6-value partition emerges.

Within the CTF Framework, the six lock values are not arbitrary: 0 corresponds to the static lattice (72 and 144 both lock here), 9 is the spatial generator (9×8=72, 9×12=108, 9×16=144), $64=2^6$ is the pure kinetic power, $81=3^4$ is its spatial complement, and 73=72+1 is the CTF denominator plus unity. The sum 64+9=73 is exact.

The temporal numerator 10373 = 11×23×41, tested against its own constituent primes, produces residues 76, 76, 4 with difference 76−4=72. This self-referential structure is documented as a CTF-specific finding building on the partition theorem.

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1. How the Framework Found the Theorem

The CTF base frequency $f_0 = 10373/72$ Hz derives from the recursive harmonic lock 144 + 1/14.4, where 14.4 is the solar-lunar beat period in days. The numerator 10373 = 11 × 23 × 41 is the product of three temporal primes at prime indices P5, P9, P13 — an arithmetic progression with step +4 in index space. The denominator $72 = 2^3 \times 3^2$ is the CTF spatial denominator.

Computational investigation of S(m,k) mod 144 — the power sum of the first m−1 positive integers mod 144 — revealed that certain values of m produce the same residue regardless of the odd exponent k. This k-independence was unexpected and prompted systematic investigation.

Discovery path — The Erdős–Moser companion paper established S(72,k) ≡ S(144,k) ≡ 0 mod 144 for CTF primes. Extended Colab computation across all CTF constants revealed the 6-value partition. The standalone paper then proved the result from first principles without CTF context.

The critical insight is that mod 144 was the natural modulus to investigate because 144 is the CTF spatial harmonic — not because it was expected to produce clean number theory. The theorem was found by a framework designed to look at 144, and would likely not have been noticed by investigators working with standard moduli (6, 30, 210).

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2. The Partition Theorem — CTF Statement

The full proof is in the companion paper (Gurwell, 2026b). We state the theorem here in CTF terms and apply it to the CTF constants.

Partition Theorem (CTF form) — For any odd prime p, the value S(p,k) mod 144 is the same for all CTF primes k ∈ {11,17,19,23,41,53} if and only if p mod 144 belongs to one of 32 residue classes. The value in that case is one of exactly six: {0, 1, 9, 64, 73, 81}.

The six lock values and their CTF interpretations:

Lock	Structure	Classes	CTF Meaning
0	0	8 classes	Pure zero. Static lattice position. S(72,k)=S(144,k)=0 — the CTF denominator and spatial harmonic are both in this class.
1	1	4 classes	Unit. Multiplicative identity. Minimal non-zero lock.
9	**3^2**	**4 classes**	**Spatial generator. 9×8=72, 9×12=108, 9×16=144. 3511 (Wieferich prime #2) is in this class.**
64	2^6	8 classes	Pure kinetic power. 2^6 = 64. The temporal prime 17 (CTF structural linchpin) is in this class.
73	**prime**	**4 classes**	**73 = 72+1 = CTF denominator + 1. Also: 64+9 = 73 exactly — the sum of the pure-2 and pure-3 lock values.**
81	**3^4**	**4 classes**	**Spatial fourth power. 81 = 9^2 = 3^4. Both Wieferich prime classes produce 3-power locks (9 and 81).**

Key arithmetic identity — 64 + 9 = 73. The pure-2 lock value plus the pure-3 lock value equals the prime 73 = CTF denominator + 1. This is an arithmetic fact, not an approximation.

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3. CTF Constants in the 144-Lattice

3.1 Where the CTF Constants Land

Applying the partition theorem to the primary CTF constants:

Value	Actual	mod 144	Theorem status	CTF interpretation
11 (P5)	11	11 mod 144=11	Lock → 73	73 = 72+1. CTF temporal prime maps to CTF denominator+1.
17	17	17 mod 144=17	Lock → 64	64 = 2^6. Structural linchpin maps to pure kinetic power.
19 (P8)	19	19 mod 144=19	Not universal	19 is in a splitting class mod 144.
23 (P9)	23	23 mod 144=23	Not universal	23 is in a splitting class mod 144.
41 (P13)	41	41 mod 144=41	Not universal	41 is in a splitting class mod 144.
53 (P16)	53	53 mod 144=53	Not universal	53 (frequency prime) is in a splitting class.
72	N/A	72 mod 144=72	Lock → 0	CTF denominator. Static ground position.
144	N/A	144 mod 144=0	Lock → 0	Spatial harmonic. Static ground position.

The CTF denominator (72) and spatial harmonic (144) both lie in the lock=0 class — the static ground of the lattice. This is consistent with their role in the framework as the fixed spatial scaffolding. The temporal primes 19, 23, and 41 are in splitting classes, meaning they do not lock — they are the active, variable elements of the system.

3.2 Full Scan of CTF Constants

Testing all primary CTF constants under the full CTF prime set:

m	k=11	k=17	k=19	k=23	k=41	k=53	Pattern
72	0	0	0	0	0	0	Universal lock → 0 (CTF denominator, static)
108	108	36	108	108	36	36	Proton split: {11,19,23}→108, {17,41,53}→36
144	0	0	0	0	0	0	Universal lock → 0 (spatial harmonic, static)

1836	108	36	108	108	36	36	Same split as 108 (1836≡108 mod 144)
10373	76	4	124	76	4	4	Self-residue: 76−4=72, temporal numerator

Static vs Active — The partition theorem provides a clean mathematical basis for the CTF distinction between spatial (static) and temporal (active) constants. The spatial constants 72 and 144 lock universally to 0. The temporal constants split — they are k-dependent, variable, dynamic.

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4. The Temporal Numerator Self-Residue Profile

The temporal numerator 10373 = 11 × 23 × 41 is not itself in a universal lock class (10373 mod 144 = 5, and residue class 5 is a splitting class). However, when tested specifically against its own constituent primes, a clean structure emerges.

k	Role in 10373	S(10373,k) mod 144	Structure
11 (P5)	1st temporal factor	76	76 = 4×19. 19=P8, a CTF prime. Preservation group.
23 (P9)	2nd temporal factor	76	76 = 4×19. Same as k=11. Preservation group.
41 (P13)	**3rd temporal factor**	**4**	**4 = 2^2. The "Key of 4." Complement group.**

The 76−4=72 Identity — S(10373,k) mod 144 = 76 for k ∈ {11,23} and 4 for k ∈ {41}. The difference 76−4 = 72 is the CTF frequency denominator exactly. The temporal numerator, interrogated by its own prime factors, recovers the denominator as a difference.

Further structure: 76 = 4 × 19, where 19 = P8 is a CTF prime. The residue 4 = 2^2 is the "Key of 4" identified in earlier Colab computation. These residues are not in the 6-value lock set — the temporal numerator is in a splitting class — but their internal arithmetic produces CTF constants.

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5. Connection to the Proton Lattice Split

The Erdős–Moser companion paper (Gurwell, 2026a) documented that S(1836,k) mod 144 splits by CTF prime group: k ∈ {11,19,23} give 108 and k ∈ {17,41,53} give 36, with 108+36=144. The proton-electron mass ratio 1836 ≡ 108 mod 144, placing it in a splitting class.

The partition theorem now contextualizes this finding: 108 mod 144 = 108, and residue class 108 is a splitting class, not a universal lock class. The proton lattice split is the k-dependence of a splitting class — precisely the behavior the theorem identifies as non-universal.

The universal (static) constants are 72 and 144, which lock to 0. The proton mass ratio (1836) and temporal numerator (10373) are both in splitting classes — they are the dynamic, active elements. The partition theorem provides the mathematical basis for distinguishing them.

> **The structural picture —** Universal lock classes → {0} (static spatial ground). Splitting classes → variable residues (dynamic temporal elements). The CTF Framework's spatial/temporal distinction maps exactly onto the theorem's universal/splitting partition.

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6. The Wieferich Primes Revisited

With the partition theorem established, the Wieferich findings from the companion paper (Three Problems, One Lattice) can be precisely restated:

- 3511 ≡ 55 mod 144: residue class 55 is a universal lock class with lock value 9. 3511 is in this class by virtue of its mod 144 position. The lock to 9 is shared by all 763 primes below 200,000 in this residue class — it is not unique to 3511. What is notable is that 3511 is Wieferich AND in a universal lock class.
- 1093 ≡ 85 mod 144: residue class 85 is a splitting class. 1093 produces S(1093,k) mod 144 ∈ {60,132} with difference 72. 1093 is Wieferich AND in a splitting class.

The two known Wieferich primes therefore lie on opposite sides of the universal/splitting partition — one static, one dynamic. Whether this is structurally meaningful or coincidental cannot be determined from two data points.

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7. Scope of Claims

	Claim	Status
✅	The Partition Theorem (standalone paper, Gurwell 2026) is the mathematical foundation	Proven independently — cited
✅	The 6-value CTF result uses k ∈ {11,17,19,23,41,53} — a specific subset of odd k	Computationally verified, all residue classes mod 144
✅	S(72,k) ≡ S(144,k) ≡ 0 mod 144 for all CTF primes k	Proven — 72 and 144 are in the lock=0 class
✅	S(1836,k) mod 144 = S(108,k) mod 144 for all k (modular periodicity)	1836 ≡ 108 mod 144 — arithmetic fact

✅	S(10373,11) = S(10373,23) = 76; S(10373,41) = 4; difference = 72	Verified — Python pow()
✅	76 = 4×19 where 19=P8 is a CTF prime	Arithmetic fact
✅	3511 mod 144 = 55, in the lock=9 class (odd k≥3)	Verified
✅	1093 mod 144 = 85, a splitting class	Verified
✅	64 + 9 = 73 (the two pure prime-power lock values sum to 73)	Arithmetic fact
❌	The CTF prime set is mathematically special — it produces 6 values vs 4	Not proven; may reflect properties of this specific prime set
❌	These observations solve or advance Wieferich conjecture or RH	Not claimed
❌	The framework caused the theorem to be true	The framework identified the modulus; the theorem is independent

8. Conclusions

1. The CTF Framework, by focusing investigation on mod 144, led to the discovery of a partition theorem for prime power sums: S(p,k) mod 144 takes at most 6 values for all odd k ≥ 3, determined entirely by p mod 144.
2. The theorem is proven independently of the CTF Framework and published separately. The framework found the theorem; it does not explain or cause it.
3. Within the CTF Framework the 6 lock values {0,1,9,64,73,81} have clean structural interpretations. The static spatial constants (72, 144) lock to 0. The pure prime-power values $2^6=64$ and $3^4=81$ arise from the CRT structure of $144=2^4\times3^2$. The value $9=3^2$ is the generator of the spatial family.
4. The temporal numerator 10373=11×23×41 is in a splitting class, but its self-residue profile under its own constituent primes produces difference 76−4=72, recovering the frequency denominator.
5. The partition theorem provides a formal mathematical basis for the CTF Framework's spatial/temporal distinction: universal lock ↔ static spatial; splitting ↔ dynamic temporal.

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S(72,k) = S(144,k) = 0   |   9×16 = 144   |   64+9 = 73
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The static lattice locks to zero. The spatial generator produces the harmonic. The prime-power sum produces the denominator plus one.

Appendix — Python Verification

All results in this paper are reproducible using the code in the companion standalone paper (Gurwell, 2026b). CTF-specific additions:

```
def S_mod(m, k, N):
    return sum(pow(i, k, N) for i in range(1, m)) % N


CTF_K = [11, 17, 19, 23, 41, 53]


# CTF constants in the lattice
for m in [72, 108, 144, 1836, 10373]:
    results = {k: S_mod(m, k, 144) for k in CTF_K}
    print(f"m={m}: {results}")


# Temporal numerator self-residue
for k in [11, 23, 41]:
    print(f"S(10373,{k}) mod 144 = {S_mod(10373, k, 144)}")
print(76 - 4)   # 72 — CTF denominator


# Wieferich positions
print(1093 % 144)  # 85 — splitting class
print(3511 % 144)  # 55 — universal lock class (lock=9)
print(64 + 9)      # 73 — CTF denominator + 1
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References

Gurwell, G. (2026a). Power Sum Divisibility, the Proton Lattice Position, and the Erdős–Moser Framework Through the 144 Harmonic. Zenodo. CTF Framework Series.

Gurwell, G. (2026b). Power Sum Residues Modulo 144: A Four-Value Partition Theorem for Prime Power Sums. Zenodo. [Standalone mathematical paper — the proof.]

Gurwell, G. (2026c). Three Problems, One Lattice: The Temporal Numerator Self-Residue Profile, Riemann Zero Proximities, and Wieferich Primes in the 144-Harmonic Framework. Zenodo. CTF Framework Series.

Gurwell, G. (2026d). The Leedskalnin Equation: f_0 = 53e = 10373/72 Hz. Zenodo. CTF Framework Series.

Keywords

144 lattice partition theorem, CTF framework number theory, power sum mod 144, 6 lock values, static spatial constants, temporal numerator self-residue, 10373 mod 144, S(72,k) equals zero, S(144,k) equals zero, CTF denominator 72, spatial generator 9, 9 times 16 equals 144, 64 plus 9 equals 73, proton lattice split, universal lock class, splitting class, Wieferich primes CTF, 3511 mod 144, 1093 mod 144, CTF prime set 11 17 19 23 41 53, Continuous Temporal Funnel, 144 harmonic, temporal primes P5 P9 P13, f0 equals 10373 over 72, modular periodicity, Chinese Remainder Theorem, pure prime power lock values, spatial temporal distinction, k-independent power sums, Erdos Moser framework

Correspondence: griff@ctftheory.com | Zenodo Community: CTF Framework | ctftheory.com