

A DIAGRAM ILLUSTRATING UNIFORMLY ACCELERATED MOTION.

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Fig. 1 contains three right triangles. The sides AB , BC and BD are represented arbitrarily by the symbols $\frac{2}{a}$, S and T . The geometry of the figure shows that

$$S = \frac{1}{2} a T^2 \quad (1)$$

Let the points A and B be fixed; $\frac{2}{a}$ is then fixed. Let the point D start at B and move uniformly in the direction BD with

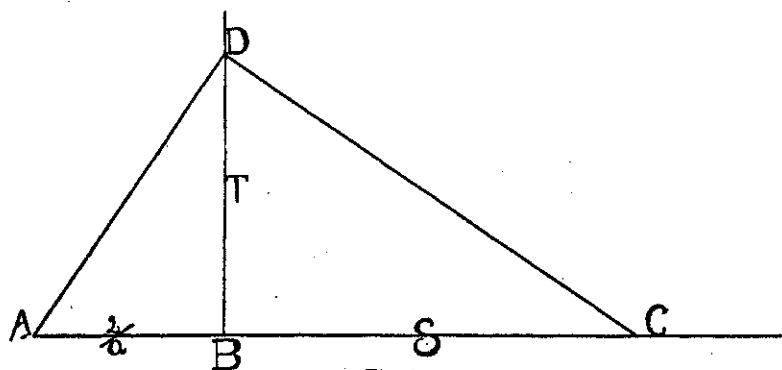


Fig. 1.

unit velocity. BD , measured in units of length, is then numerically equal to the time T measured in seconds and the purpose of the symbol T is plain.

During the motion of D let the point C move in the direction ABC so that the angle ADC shall always be a right angle. To fulfill this condition C and D must leave B at the same instant and T represents the time required by the point C in moving the distance BC or S .

Equation (1) now expresses the space passed over by the point C in terms of the time T and a fixed quantity a and the equation is seen to be that of uniformly accelerated motion, the rate of acceleration being a .

Two simple cases will serve for illustration:

Suppose that A is a powerful source of light and that a uni-

formly moving car D carries a mirror that rotates in such a way as to reflect a beam of light from A at right angles to AB and against a wall BC . The spot of light will move along the wall with uniform acceleration.

Suppose, again, that A in Fig. 2 is a knife edge and that a carpenter's square moves so that one arm slides against A while the vertex of the right angle moves uniformly along BD . If the other arm drives a particle C along a grooved track BC , C will move with uniform acceleration.

In both these cases the rate of acceleration will be a if D move

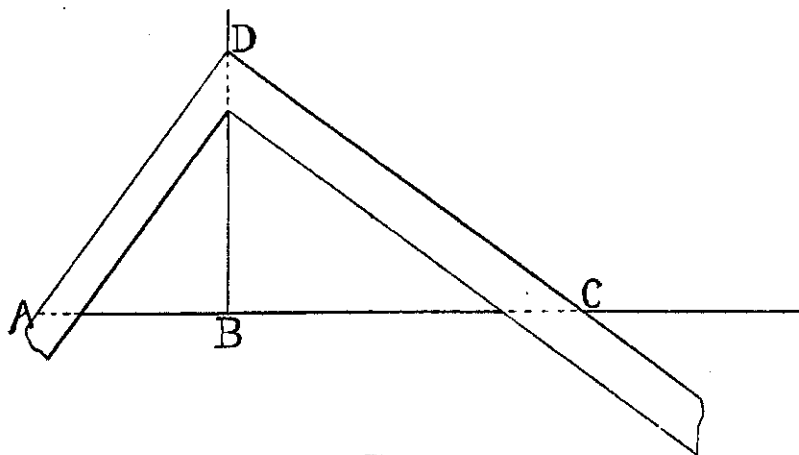


Fig. 2.

with unit velocity. If D move with any other uniform velocity the rate of acceleration will be proportional to a .

If this geometrical method were used in presenting the subject of uniformly accelerated motion to a student for the first time, it would be necessary to show that equation (1) represents such motion. The usual algebraic method is to express the mean velocity \bar{v} during the time $T_2 - T_1$ by the equation

$$\bar{v} = \frac{S_2 - S_1}{T_2 - T_1} = \frac{1}{2} a \frac{T_2^2 - T_1^2}{T_2 - T_1} = a \frac{T_2 + T_1}{2}$$

Hence,

$$\bar{v} = a T, \quad (2)$$

where T is midway between T_1 and T_2 .

Since the same value is obtained for \bar{v} for all values of

the interval $T_2 - T_1$ if the mid-point T remain unchanged, it follows that the mean velocity \bar{v} is the actual velocity v at the instant T .

Hence,

$$v = aT. \quad (3)$$

Equation (3) contains the definition of uniform acceleration.

However, equations (2) and (3) may be approximately obtained from the diagram without the use of algebraic methods.

Draw Fig. 3 as accurately as possible and locate a point T midway between two points T_1 and T_2 . Locate the corresponding points S_1 and S_2 . Measure the lines $S_2 - S_1$ and $T_2 - T_1$ and find the value of $\bar{v} = \frac{S_2 - S_1}{T_2 - T_1}$.

Proceed in the same way with other values of T_1 and T_2 , keeping the mid-point T unchanged. This will lead to the determina-

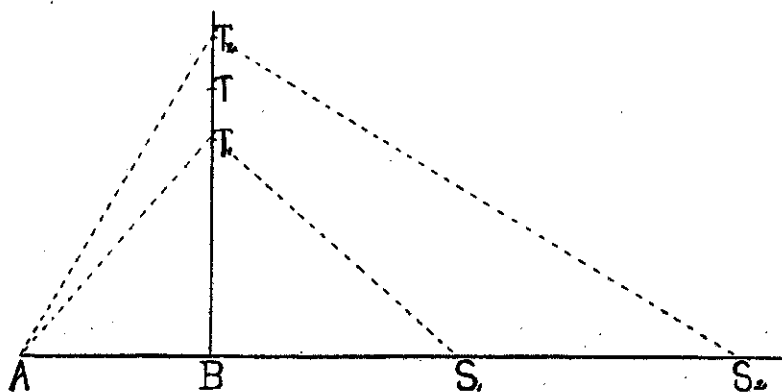


Fig. 3.

tion of v . Now take the other positions for T and proceed as before. Equations (2) and (3) will soon appear if the drawing and measuring are at all accurate and this work will make clearer the algebraic solution.

The diagram is applicable, of course, to any units of length and of time, and any rate of acceleration can be represented by making the line AB the proper length.

For the case of a freely falling body the decameter (ten meters) and the second are convenient units.